Density of safe matrices (Winkler modell of percolation): FSU Jena, November 7, 2005

(downloadable from: http://compalg.inf.elte.hu/∼tony/Percolation)

A binary matrix $B = [B_{ij}]_{m \times n}$is called ***good***, if it contains at most one 1's in each column; the matrix is called ***schedulable,*** if it can be transformed into a good matrix repeating the following operation: we remove any zero element $m_{i,j} = 0$, shift the elements $m_{i,j+1}, \ldots m_{i,n}$ to left and add a new element $m_{i,n} = 0$ to the i-th row of the matrix; the matrix is called *safe,* if for any k ($k = 1, \ldots, n$) holds that the first k columns of the matrix contain at most k 1's.

Let $\mathbf{Z} = [z_{ij}]_{m \times n}$ be a matrix containing independent random variables having the join distribution $P(z_{i,j} = 1) = p$ and $P(z_{i,j} = 0) = 1 - p$. In the case $m \geq 1$ we give lower and upper bound of the asymptotic probability of the event that a concrete realisation of the matrix Z is schedulable.

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