



## Score lists in multipartite hypertournaments

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**Abstract.** Given non-negative integers  $n_i$  and  $\alpha_i$  with  $0 \leq \alpha_i \leq n_i$  ( $i = 1, 2, \dots, k$ ), an  $[\alpha_1, \alpha_2, \dots, \alpha_k]$ - $k$ -partite hypertournament on  $\sum_1^k n_i$  vertices is a  $(k+1)$ -tuple  $(U_1, U_2, \dots, U_k, E)$ , where  $U_i$  are  $k$  vertex sets with  $|U_i| = n_i$ , and  $E$  is a set of  $\sum_1^k \alpha_i$ -tuples of vertices, called arcs, with exactly  $\alpha_i$  vertices from  $U_i$ , such that any  $\sum_1^k \alpha_i$  subset  $\cup_1^k U'_i$  of  $\cup_1^k U_i$ ,  $E$  contains exactly one of the  $(\sum_1^k \alpha_i)! \sum_1^k \alpha_i$ -tuples whose entries belong to  $\cup_1^k U'_i$ . We obtain necessary and sufficient conditions for  $k$  lists of non-negative integers in non-decreasing order to be the losing score lists and to be the score lists of some  $k$ -partite hypertournament.

### 1 Introduction

Hypergraphs are generalizations of graphs [1]. While edges of a graph are pairs of vertices of the graph, edges of a hypergraph are subsets of the vertex set,

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consisting of at least two vertices. An edge consisting of  $k$  vertices is called a  $k$ -edge. A  $k$ -hypergraph is a hypergraph all of whose edges are  $k$ -edges. A  $k$ -hypertournament is a complete  $k$ -hypergraph with each  $k$ -edge endowed with an orientation, that is, a linear arrangement of the vertices contained in the hyperedge. Instead of scores of vertices in a tournament, Zhou et al. [13] considered scores and losing scores of vertices in a  $k$ -hypertournament, and derived a result analogous to Landau's theorem [6]. The score  $s(v_i)$  or  $s_i$  of a vertex  $v_i$  is the number of arcs containing  $v_i$  and in which  $v_i$  is not the last element, and the losing score  $r(v_i)$  or  $r_i$  of a vertex  $v_i$  is the number of arcs containing  $v_i$  and in which  $v_i$  is the last element. The score sequence (losing score sequence) is formed by listing the scores (losing scores) in non-decreasing order.

The following characterizations of score sequences and losing score sequences in  $k$ -hypertournaments can be found in G. Zhou et al. [12].

**Theorem 1** *Given two positive integers  $n$  and  $k$  with  $n \geq k > 1$ , a non-decreasing sequence  $R = [r_1, r_2, \dots, r_n]$  of non-negative integers is a losing score sequence of some  $k$ -hypertournament if and only if for each  $j$ ,*

$$\sum_{i=1}^j r_i \geq \binom{j}{k},$$

*with equality when  $j = n$ .*

**Theorem 2** *Given positive integers  $n$  and  $k$  with  $n \geq k > 1$ , a non-decreasing sequence  $S = [s_1, s_2, \dots, s_n]$  of non-negative integers is a score sequence of some  $k$ -hypertournament if and only if for each  $j$ ,*

$$\sum_{i=1}^j s_i \geq j \binom{n-1}{k-1} + \binom{n-j}{k} - \binom{n}{k},$$

*with equality when  $j = n$ .*

Some recent work on the reconstruction of tournaments can be found in the papers due to A. Iványi [3, 4]. Some more results on  $k$ -hypertournaments can be found in [2, 5, 9, 10, 11, 13]. The analogous results of Theorem 1 and Theorem 2 for  $[h, k]$ -bipartite hypertournaments can be found in [7] and for  $[\alpha, \beta, \gamma]$ -tripartite hypertournaments in [8].

Throughout this paper  $i$  takes values from 1 to  $k$  and  $j_i$  takes values from 1 to  $n_i$ , unless otherwise stated.

A  $k$ -partite hypergraph is a generalization of  $k$ -partite graph. Given non-negative integers  $n_i$  and  $\alpha_i$ , ( $i = 1, 2, \dots, k$ ) with  $n_i \geq \alpha_i \geq 0$  for each  $i$ , an  $[\alpha_1, \alpha_2, \dots, \alpha_k]$ - $k$ -partite hypertournament (or briefly  $k$ -partite hypertournament)  $M$  of order  $\sum_1^k n_i$  consists of  $k$  vertex sets  $U_i$  with  $|U_i| = n_i$  for each  $i$ , ( $1 \leq i \leq k$ ) together with an arc set  $E$ , a set of  $\sum_1^k \alpha_i$ -tuples of vertices, with exactly  $\alpha_i$  vertices from  $U_i$ , called arcs such that any  $\sum_1^k \alpha_i$  subset  $\cup_1^k U'_i$  of  $\cup_1^k U_i$ ,  $E$  contains exactly one of the  $\binom{\sum_1^k \alpha_i}{\sum_1^k \alpha_i}$   $\sum_1^k \alpha_i$ -tuples whose  $\alpha_i$  entries belong to  $U'_i$ .

Let  $e = (u_{11}, u_{12}, \dots, u_{1\alpha_1}, u_{21}, u_{22}, \dots, u_{2\alpha_2}, \dots, u_{k1}, u_{k2}, \dots, u_{k\alpha_k})$ , with  $u_{ij_i} \in U_i$  for each  $i$ , ( $1 \leq i \leq k, 1 \leq j_i \leq \alpha_i$ ), be an arc in  $M$  and let  $h < t$ , we let  $e(u_{1h}, u_{1t})$  denote to be the new arc obtained from  $e$  by interchanging  $u_{1h}$  and  $u_{1t}$  in  $e$ . An arc containing  $\alpha_i$  vertices from  $U_i$  for each  $i$ , ( $1 \leq i \leq k$ ) is called an  $(\alpha_1, \alpha_2, \dots, \alpha_k)$ -arc.

For a given vertex  $u_{ij_i} \in U_i$  for each  $i$ ,  $1 \leq i \leq k$  and  $1 \leq j_i \leq \alpha_i$ , the score  $d_M^+(u_{ij_i})$  (or simply  $d^+(u_{ij_i})$ ) is the number of  $\sum_1^k \alpha_i$ -arcs containing  $u_{ij_i}$  and in which  $u_{ij_i}$  is not the last element. The losing score  $d_M^-(u_{ij_i})$  (or simply  $d^-(u_{ij_i})$ ) is the number of  $\sum_1^k \alpha_i$ -arcs containing  $u_{ij_i}$  and in which  $u_{ij_i}$  is the last element. By arranging the losing scores of each vertex set  $U_i$  separately in non-decreasing order, we get  $k$  lists called losing score lists of  $M$  and these are denoted by  $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$  for each  $i$ , ( $1 \leq i \leq k$ ). Similarly, by arranging the score lists of each vertex set  $U_i$  separately in non-decreasing order, we get  $k$  lists called score lists of  $M$  which are denoted as  $S_i = [s_{ij_i}]_{j_i=1}^{n_i}$  for each  $i$  ( $1 \leq i \leq k$ ).

## 2 Main results

The following two theorems are the main results.

**Theorem 3** *Given  $k$  non-negative integers  $n_i$  and  $k$  non-negative integers  $\alpha_i$  with  $1 \leq \alpha_i \leq n_i$  for each  $i$  ( $1 \leq i \leq k$ ), the  $k$  non-decreasing lists  $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$  of non-negative integers are the losing score lists of a  $k$ -partite hypertournament if and only if for each  $p_i$  ( $1 \leq i \leq k$ ) with  $p_i \leq n_i$ ,*

$$\sum_{i=1}^k \sum_{j_i=1}^{p_i} r_{ij_i} \geq \prod_{i=1}^k \binom{p_i}{\alpha_i}, \quad (1)$$

*with equality when  $p_i = n_i$  for each  $i$  ( $1 \leq i \leq k$ ).*

**Theorem 4** Given  $k$  non-negative integers  $n_i$  and  $k$  non-negative integers  $\alpha_i$  with  $0 \leq \alpha_i \leq n_i$  for each  $i$  ( $1 \leq i \leq k$ ), the  $k$  non-decreasing lists  $S_i = [s_{ij_i}]_{j_i=1}^{n_i}$  of non-negative integers are the score lists of a  $k$ -partite hypertournament if and only if for each  $p_i$ , ( $1 \leq i \leq k$ ) with  $p_i \leq n_i$

$$\sum_{i=1}^k \sum_{j_i=1}^{p_i} s_{ij_i} \geq \left( \sum_{i=1}^k \frac{\alpha_i p_i}{n_i} \right) \left( \prod_{i=1}^k \binom{n_i}{\alpha_i} \right) + \prod_{i=1}^k \binom{n_i - p_i}{\alpha_i} - \prod_{i=1}^k \binom{n_i}{\alpha_i}, \quad (2)$$

with equality when  $p_i = n_i$  for each  $i$  ( $1 \leq i \leq k$ ).

We note that in a  $k$ -partite hypertournament  $M$ , there are exactly  $\prod_{i=1}^k \binom{n_i}{\alpha_i}$  arcs and in each arc only one vertex is at the last entry. Therefore,

$$\sum_{i=1}^k \sum_{j_i=1}^{n_i} d_M^-(u_{ij_i}) = \prod_{i=1}^k \binom{n_i}{\alpha_i}.$$

In order to prove the above two theorems, we need the following Lemmas.

**Lemma 5** If  $M$  is a  $k$ -partite hypertournament of order  $\sum_1^k n_i$  with score lists  $S_i = [s_{ij_i}]_{j_i=1}^{n_i}$  for each  $i$  ( $1 \leq i \leq k$ ), then

$$\sum_{i=1}^k \sum_{j_i=1}^{n_i} s_{ij_i} = \left[ \left( \sum_{i=1}^k \alpha_i \right) - 1 \right] \prod_{i=1}^k \binom{n_i}{\alpha_i}.$$

**Proof.** We have  $n_i \geq \alpha_i$  for each  $i$  ( $1 \leq i \leq k$ ). If  $r_{ij_i}$  is the losing score of  $u_{ij_i} \in U_i$ , then

$$\sum_{i=1}^k \sum_{j_i=1}^{n_i} r_{ij_i} = \prod_{i=1}^k \binom{n_i}{\alpha_i}.$$

The number of  $[\alpha_i]_1^k$  arcs containing  $u_{ij_i} \in U_i$  for each  $i$ , ( $1 \leq i \leq k$ ), and  $1 \leq j_i \leq n_i$  is

$$\frac{\alpha_i}{n_i} \prod_{t=1}^k \binom{n_t}{\alpha_t}.$$

Thus,

$$\begin{aligned} \sum_{i=1}^k \sum_{j_i=1}^{n_i} s_{ij_i} &= \sum_{i=1}^k \sum_{j_i=1}^{n_i} \binom{\alpha_i}{n_i} \prod_1^k \binom{n_t}{\alpha_t} - \binom{n_i}{\alpha_i} \\ &= \left( \sum_{i=1}^k \alpha_i \right) \prod_1^k \binom{n_t}{\alpha_t} - \prod_1^k \binom{n_i}{\alpha_i} \\ &= \left[ \left( \sum_{i=1}^k \alpha_i \right) - 1 \right] \prod_1^k \binom{n_i}{\alpha_i}. \end{aligned}$$

□

**Lemma 6** *If  $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$  ( $1 \leq i \leq k$ ) are  $k$  losing score lists of a  $k$ -partite hypertournament  $M$ , then there exists some  $h$  with  $r_{1h} < \frac{\alpha_1}{n_1} \prod_1^k \binom{n_p}{\alpha_p}$  so that  $R'_1 = [r_{11}, r_{12}, \dots, r_{1h+1}, \dots, r_{1n_1}]$ ,  $R'_s = [r_{s1}, r_{s2}, \dots, r_{st-1}, \dots, r_{sn_s}]$  ( $2 \leq s \leq k$ ) and  $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$  ( $2 \leq i \leq k$ ),  $i \neq s$  are losing score lists of some  $k$ -partite hypertournament,  $t$  is the largest integer such that  $r_{s(t-1)} < r_{st} = \dots = r_{sn_s}$ .*

**Proof.** Let  $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$  ( $1 \leq i \leq k$ ) be losing score lists of a  $k$ -partite hypertournament  $M$  with vertex sets  $U_i = \{u_{i1}, u_{i2}, \dots, u_{ij_i}\}$  so that  $d^-(u_{ij_i}) = r_{ij_i}$  for each  $i$  ( $1 \leq i \leq k$ ,  $1 \leq j_i \leq n_i$ ).

Let  $h$  be the smallest integer such that

$$r_{11} = r_{12} = \dots = r_{1h} < r_{1(h+1)} \leq \dots \leq r_{1n_1}$$

and  $t$  be the largest integer such that

$$r_{s1} \leq r_{s2} \leq \dots \leq r_{s(t-1)} < r_{st} = \dots = r_{sn_s}$$

Now, let

$$R'_1 = [r_{11}, r_{12}, \dots, r_{1h+1}, \dots, r_{1n_1}],$$

$$R'_s = [r_{s1}, r_{s2}, \dots, r_{st-1}, \dots, r_{sn_s}]$$

( $2 \leq s \leq k$ ), and  $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$  ( $2 \leq i \leq k$ ),  $i \neq s$ .

Clearly,  $R'_1$  and  $R'_s$  are both in non-decreasing order.

Since  $r_{1h} < \frac{\alpha_1}{n_1} \prod_1^k \binom{n_p}{\alpha_p}$ , there is at least one  $[\alpha_i]_1^k$ -arc  $e$  containing both  $u_{1h}$  and  $u_{st}$  with  $u_{st}$  as the last element in  $e$ , let  $e' = (u_{1h}, u_{st})$ . Clearly,  $R'_1$ ,  $R'_s$

and  $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$  for each  $i$  ( $2 \leq i \leq k$ ),  $i \neq s$  are the  $k$  losing score lists of  $M' = (M - e) \cup e'$ .  $\square$

The next observation follows from Lemma 6, and the proof can be easily established.

**Lemma 7** *Let  $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$ , ( $1 \leq i \leq k$ ) be  $k$  non-decreasing sequences of non-negative integers satisfying (1). If  $r_{1n_1} < \frac{\alpha_1}{n_1} \prod_1^k \binom{n_t}{\alpha_t}$ , then there exists  $s$  and  $t$  ( $2 \leq s \leq k$ ),  $1 \leq t \leq n_s$  such that  $R'_1 = [r_{11}, r_{12}, \dots, r_{1h} + 1, \dots, r_{1n_1}]$ ,  $R'_s = [r_{s1}, r_{s2}, \dots, r_{st} - 1, \dots, r_{sn_s}]$  and  $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$ , ( $2 \leq i \leq k$ ),  $i \neq s$  satisfy (1).*

**Proof of Theorem 3. Necessity.** Let  $R_i$ , ( $1 \leq i \leq k$ ) be the  $k$  losing score lists of a  $k$ -partite hypertournament  $M(\mathbf{U}_i, 1 \leq i \leq k)$ . For any  $p_i$  with  $\alpha_i \leq p_i \leq n_i$ , let  $\mathbf{U}'_i = \{\mathbf{u}_{ij_i}\}_{j_i=1}^{p_i}$  ( $1 \leq i \leq k$ ) be the sets of vertices such that  $d^-(\mathbf{u}_{ij_i}) = r_{ij_i}$  for each  $1 \leq j_i \leq p_i$ ,  $1 \leq i \leq k$ . Let  $M'$  be the  $k$ -partite hypertournament formed by  $\mathbf{U}'_i$  for each  $i$  ( $1 \leq i \leq k$ ).

Then,

$$\begin{aligned} \sum_{i=1}^k \sum_{j_i=1}^{p_i} r_{ij_i} &\geq \sum_{i=1}^k \sum_{j_i=1}^{p_i} d_{M'}^-(\mathbf{u}_{ij_i}) \\ &= \prod_1^k \binom{p_t}{\alpha_t}. \end{aligned}$$

**Sufficiency.** We induct on  $n_1$ , keeping  $n_2, \dots, n_k$  fixed. For  $n_1 = \alpha_1$ , the result is obviously true. So, let  $n_1 > \alpha_1$ , and similarly  $n_2 > \alpha_2, \dots, n_k > \alpha_k$ . Now,

$$\begin{aligned} r_{1n_1} &= \sum_{i=1}^k \sum_{j_i=1}^{n_i} r_{ij_i} - \left( \sum_{j_1=1}^{n_1-1} r_{1j_1} + \sum_{i=2}^k \sum_{j_i=1}^{n_i} r_{ij_i} \right) \\ &\leq \prod_1^k \binom{n_t}{\alpha_t} - \binom{n_1-1}{\alpha_1} \prod_2^k \binom{n_t}{\alpha_t} \\ &= \left[ \binom{n_1}{\alpha_1} - \binom{n_1-1}{\alpha_1} \right] \prod_2^k \binom{n_t}{\alpha_t} \\ &= \binom{n_1-1}{\alpha_1-1} \prod_2^k \binom{n_t}{\alpha_t}. \end{aligned}$$

We consider the following two cases.

**Case 1.**  $r_{1n_1} = \binom{n_1-1}{\alpha_1-1} \prod_2^k \binom{n_t}{\alpha_t}$ . Then,

$$\begin{aligned} \sum_{j_1=1}^{n_1-1} r_{1j_1} + \sum_{i=2}^k \sum_{j_i=1}^{n_i} r_{ij_i} &= \sum_{i=1}^k \sum_{j_i=1}^{n_i} r_{ij_i} - r_{1n_1} \\ &= \prod_1^k \binom{n_t}{\alpha_t} - \binom{n_1-1}{\alpha_1-1} \prod_2^k \binom{n_t}{\alpha_t} \\ &= \left[ \binom{n_1}{\alpha_1} - \binom{n_1-1}{\alpha_1-1} \right] \prod_2^k \binom{n_t}{\alpha_t} \\ &= \binom{n_1-1}{\alpha_1} \prod_2^k \binom{n_t}{\alpha_t}. \end{aligned}$$

By induction hypothesis  $[r_{11}, r_{12}, \dots, r_{1(n_1-1)}]$ ,  $R_2, \dots, R_k$  are losing score lists of a  $k$ -partite hypertournament  $M'(\mathcal{U}'_1, \mathcal{U}_2, \dots, \mathcal{U}_k)$  of order  $\left(\sum_{i=1}^k n_i\right) - 1$ . Construct a  $k$ -partite hypertournament  $M$  of order  $\sum_{i=1}^k n_i$  as follows. In  $M'$ , let  $\mathcal{U}'_1 = \{u_{11}, u_{12}, \dots, u_{1(n_1-1)}\}$ ,  $\mathcal{U}_i = \{u_{ij_i}\}_{j_i=1}^{n_i}$  for each  $i$ , ( $2 \leq i \leq k$ ). Adding a new vertex  $u_{1n_1}$  to  $\mathcal{U}'_1$ , for each  $\left(\sum_{i=1}^k \alpha_i\right)$ -tuple containing  $u_{1n_1}$ , arrange  $u_{1n_1}$  on the last entry. Denote  $E_1$  to be the set of all these  $\binom{n_1-1}{\alpha_1-1} \prod_2^k \binom{n_t}{\alpha_t}$   $\left(\sum_{i=1}^k \alpha_i\right)$ -tuples. Let  $E(M) = E(M') \cup E_1$ . Clearly,  $R_i$  for each  $i$ , ( $1 \leq i \leq k$ ) are the  $k$  losing score lists of  $M$ .

**Case 2.**  $r_{1n_1} < \binom{n_1-1}{\alpha_1-1} \prod_2^k \binom{n_t}{\alpha_t}$ .

Applying Lemma 7 repeatedly on  $R_1$  and keeping each  $R_i$ , ( $2 \leq i \leq k$ ) fixed until we get a new non-decreasing list  $R'_1 = [r'_{11}, r'_{12}, \dots, r'_{1n_1}]$  in which now  $r'_{1n_1} = \binom{n_1-1}{\alpha_1-1} \prod_2^k \binom{n_t}{\alpha_t}$ . By Case 1,  $R'_1, R_i$  ( $2 \leq i \leq k$ ) are the losing score lists of a  $k$ -partite hypertournament. Now, apply Lemma 6 on  $R'_1, R_i$  ( $2 \leq i \leq k$ ) repeatedly until we obtain the initial non-decreasing lists  $R_i$  for each  $i$  ( $1 \leq i \leq k$ ). Then by Lemma 6,  $R_i$  for each  $i$  ( $1 \leq i \leq k$ ) are the losing score lists of a  $k$ -partite hypertournament.  $\square$

**Proof of Theorem 4.** Let  $S_i = [s_{ij_i}]_{j_i=1}^{n_i}$  ( $1 \leq i \leq k$ ) be the  $k$  score lists of a  $k$ -partite hypertournament  $M(\mathcal{U}_i, 1 \leq i \leq k)$ , where  $\mathcal{U}_i = \{u_{ij_i}\}_{j_i=1}^{n_i}$  with

$d_M^+(\mathbf{u}_{ij_i}) = s_{ij_i}$ , for each  $i$ , ( $1 \leq i \leq k$ ). Clearly,

$$d^+(\mathbf{u}_{ij_i}) + d^-(\mathbf{u}_{ij_i}) = \frac{\alpha_i}{n_i} \prod_1^k \binom{n_t}{\alpha_t}, \quad (1 \leq i \leq k, 1 \leq j_i \leq n_i).$$

Let  $r_{i(n_i+1-j_i)} = d^-(\mathbf{u}_{ij_i})$ , ( $1 \leq i \leq k, 1 \leq j_i \leq n_i$ ).

Then  $R_i = [r_{ij_i}]_{j_i=1}^{n_i}$  ( $i = 1, 2, \dots, k$ ) are the  $k$  losing score lists of  $M$ . Conversely, if  $R_i$  for each  $i$  ( $1 \leq i \leq k$ ) are the losing score lists of  $M$ , then  $S_i$  for each  $i$ , ( $1 \leq i \leq k$ ) are the score lists of  $M$ . Thus, it is enough to show that conditions (1) and (2) are equivalent provided  $s_{ij_i} + r_{i(n_i+1-j_i)} =$

$$\left(\frac{\alpha_i}{n_i}\right) \prod_1^k \binom{n_t}{\alpha_t}, \quad \text{for each } i \text{ } (1 \leq i \leq k \text{ and } 1 \leq j_i \leq n_i).$$

First assume (2) holds. Then,

$$\begin{aligned} \sum_{i=1}^k \sum_{j_i=1}^{p_i} r_{ij_i} &= \sum_{i=1}^k \sum_{j_i=1}^{p_i} \left(\frac{\alpha_i}{n_i}\right) \left(\prod_1^k \binom{n_t}{\alpha_t}\right) - \sum_{i=1}^k \sum_{j_i=1}^{p_i} s_{i(n_i+1-j_i)} \\ &= \sum_{i=1}^k \sum_{j_i=1}^{p_i} \left(\frac{\alpha_i}{n_i}\right) \left(\prod_1^k \binom{n_t}{\alpha_t}\right) - \left[ \sum_{i=1}^k \sum_{j_i=1}^{n_i} r_{ij_i} - \sum_{i=1}^k \sum_{j_i=1}^{n_i-p_i} s_{ij_i} \right] \\ &\geq \left[ \sum_{i=1}^k \sum_{j_i=1}^{p_i} \left(\frac{\alpha_i}{n_i}\right) \left(\prod_1^k \binom{n_t}{\alpha_t}\right) \right] \\ &\quad - \left[ \left( \left( \sum_1^k \alpha_i \right) - 1 \right) \prod_1^k \binom{n_i}{\alpha_i} \right] \\ &\quad + \sum_{i=1}^k (n_i - p_i) \left(\frac{\alpha_i}{n_i}\right) \prod_1^k \binom{n_t}{\alpha_t} \\ &\quad + \prod_1^k \binom{n_i - (n_i - p_i)}{\alpha_i} - \prod_1^k \binom{n_i}{\alpha_i} \\ &= \prod_1^k \binom{n_i}{\alpha_i}, \end{aligned}$$

with equality when  $p_i = n_i$  for each  $i$  ( $1 \leq i \leq k$ ). Thus (1) holds.

Now, when (1) holds, using a similar argument as above, we can show that (2) holds. This completes the proof.  $\square$



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