CEJOR manuscript No.
(will be inserted by the editor)

Parallel Erdős-Gallai algorithm

Antal Iványi · Lóránd Lucz · Péter Sótér · Shariefuddin Pirzada

Received: date / Accepted: date

Abstract Havel in 1956 [7], Erdős and Gallai in 1960 [4], Hakimi in 1962 [6], Ruskey, Cohen, Eades and Scott in 1994 [20], Barnes and Savage in 1997 [1], Tripathi, Venugopalan and West in 2010 [23] proposed a method to decide, whether a sequence of nonnegative integers can be the degree sequence of a simple graph (such sequences are called graphical). The running time of their algorithms in worst case is $\Omega(n^2)$. In [10] the authors proposed a new algorithm called EGL (Erdös-Gallai Linear algorithm), whose worst running time is $O(n)$. As an application of this linear time algorithm we describe a quick parallel algorithm EGQ (Erdős-Gallai Parallel algorithm) and enumerate the different degree sequences of simple graphs for 24, ..., 29 vertices [21].

Keywords Simple graphs · Score sequences · Parallel algorithms

Mathematics Subject Classification (2000) 05C20, 05C85, 68R10

1 Introduction

In the practice an often appearing problem is the ranking of different objects as hardware or software products, cars, economical decisions, persons etc. A typical method of the ranking is the pairwise comparison of the objects, assignment of points to the objests and sorting the objects according to the sums of the numbers of the received points.

Antal Iványi
Eötvös Loránd University, Dept. of Computer Algebra
Tel.: 36-1-2016-421
E-mail: tony@compalg.inf.elte.hu

Loránd Lucz and Péter Sótér
Eötvös Loránd University, Dept. of Computer Algebra
E-mail: lorand.lucz@gmail.com and mapoleon@freemail.hu

For example Landau [15] references to biological, Hakimi [6] to chemical, Kim et al. [14], Newman and Barabási [18] to net-centric, Bozóki, Fülöp, Poesz, Rónyai and Temesi to economical [2,3,22], Liljeros et al. [16] to human applications, while Iványi, Lucz, Sótér and Pirzada [8,9,12,10] to applications in sports.

From several popular possibilities we follow the terminology and notations used by Pál Erdős and Tibor Gallai [4].

2 Linear Erdős-Gallai algorithm

Text with citations [2] and [4] and [10] and [11] and [22] and [13].

```
Erdős-Gallai-Linear(n, b, L)
01 H_0 = 0                              ▷ Line 01: initialization
02 for i = 1 to n        ▷ Lines 02–03: computation of the elemnts of H
03      H_i = H_{i-1} + b_i
04 if H_n odd                      ▷ Lines 04–06: test of the parity
05    L = False
06    return L
07 b_0 = n - 1           ▷ Line 07: initialization of a working variable
08 for i = 1 to n          ▷ Lines 08–12: computation of the indices
09      if b_i < b_{i-1}
10         for j = b_{i-1} downto b_i + 1
11              m_j = i - 1
12         m_{b_i} = i
13 for j = b_n downto 1                   ▷ Lines 13–14: large indices
14      m_j = n
15 for i = 1 to n            ▷ Lines 15–23: test of the elements of b
16      if i ≤ m_i          ▷ Lines 16–19: test of indices for large m_i's
17         if H_i > i(i - 1) + i(m_i - i) + H_n - H_{m_i}
18            L = False
19            return L
20      if i > m_i          ▷ Lines 20–23: test of indices for small m_i's
21         if H_i > i(i - 1) + H_n - H_i
22            L = False
23            return L
24 L = True    ▷ Lines 24–25: the program ends with the value True
25      return L
```

3 Quick Erdős-Gallai algorithm

[10,24]

Table 1 Please write your table caption here

first	second	third
number	number	number
number	number	number

4 Parallel Erdős-Gallai algorithm

5 Application: the number of score sequences

[17,19]

5.1 Subsection title

Paragraph headings Use paragraph headings as needed.

$$a^2 + b^2 = c^2 \tag{1}$$

Acknowledgements The authors are indebted to

References

1. Barnes TM, Savage CD, Efficient generation of graphical partitions. Discrete Appl. Math. 78:17–26
2. Bozóki S, Fülöp J, Poesz A (2011) On pairwise comparison matrices that can be made consistent by the modification of a few elements. Centr Eur J Oper Res 19:157–175
3. Bozóki S, Fülóp J, Rónyai L (2010) On optimal completion of incomplete pairwise comparison matrices. Math Comput Modelling 52:318–333
4. Erdős P, Gallai T (1960) Graphs with prescribed degree sequence (Hungarian). Mat. Lapok, 11:264–274
5. Frank A (2011) Connections in Combinatorial Optimization. Oxford University Press, Oxford
6. Hakimi S (1962) On the realizability of a set of integers as degrees of the vertices of a simple graph. J. SIAM Appl. Math. 10:496–506
7. Havel V (1956) A remark on the existence of finite graphs (Czech). Časopis Pěst. Mat. 80:477–480
8. Iványi A (2009), Reconstruction of complete interval tournaments. Acta Univ. Sapientiae, Informatica 1:71–88
9. Iványi A (2010), Reconstruction of complete interval tournaments. II. Acta Univ. Sapientiae, Math, 2:47–71
10. Iványi A, Lucz L., Sótér P. (2011) Quick Erdős-Gallai algorithms, Acta Univ. Sapientiae, Informatica 3, to appear
11. Iványi A, Lucz L., Sótér P., Linear Erdős-Gallai test (Hungarian), Alkalmazott Matematikai Lapok, submitted
12. Iványi A, Pirzada S (2011) Comparison based ranking. In: Algorithms of Informatics (electronic book, ed. by A. Iványi), AnTonCom, Budapest, 2011. 1410–1450
13. Kéri G. (2011) On qualitatively consistent, transitive and contradictory judgment matrices emerging from multiattribute decision procedures. Centr Eur J Oper Res 19:215–224

14. Kim H, Toroczkai Z, Miklós I, Erdős PL, Székely LA (2009) Degree-based graph construction. J. Physics: Math. Theor. A 42:392–401.
15. Landau HG (1953) On dominance relations and the structure of animal societies. III. The condition for a score sequence. Bull. Math. Biophys 15:143–148
16. Liljeros F, Edling CR, Amaral A, Stanley H, Áberg Y (2001), The web of human sexual contacts. Nature 411:907–908
17. Narayana TV, Bent DH (1964) Computation of the number of score sequences in round-robin tournaments. Canad. Math. Bull. 7:133–136
18. Newman MEJ, Barabási AL (2006) The Structure and Dynamics of Networks. Princeton University Press, Princeton, NJ. 2006.
19. Pécsy G, Szűcs L (2000) Parallel verification and enumeration of tournaments, Studia Univ. Babes-Bolyai Inform. XLV:11–26
20. Ruskey F, Cohen R, Eades P, Scott A (1994) Alley CAT's in search of good homes. Congressus Numerantium, 102:97–110
21. Sloane NJA (2011) The number of degree-vectors for simple graphs. In: The On-line Encyclopedia of the Integer Sequences (ed. by NJA Sloane). http://oeis.org/A004251
22. Temesi J (2011) Pairwise comparison matrices and the error-free property of the decision maker. Centr Eur J Oper Res 19:239–249
23. Tripathi A, Venugopalanb S, West DB (2010) A short constructive proof of the Erdős-Gallai characterization of graphic lists. Discrete Math. 310:833–834
24. Tripathi A, Vijay S (2003) A note on a theorem of Erdős & Gallai. Discrete Math. 265:417–420