

Balanced reconstruction of multigraphs

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Let $n \geq 2$ be a positive integer and let $\Delta_n(\mathbf{r}, \mathbf{c})$ be the set of directed multigraphs on $n + 1$ vertices having prescribed outdegrees $\mathbf{r} = (r_1, r_2, \dots, r_n)$ and prescribed indegrees $\mathbf{c} = (c_1, c_2, \dots, c_n)$. Let the element m_{ij} of the matrix $M_{(n+1) \times (n+1)}$ denote the number of arcs directed from the vertex V_i to vertex V_j . We present an algorithm MINIMAX constructing a directed multigraph $D \in \Delta_n(\mathbf{r}, \mathbf{c})$ having the following properties:

- D has the prescribed indegrees and outdegrees;
- D has the minimal value of $r_{n+1} + c_{n+1}$;
- D has the minimal value of $\max(m_{ij} + m_{ji})$;
- D has the maximal value of $\min(m_{ij} + m_{ji})$.

This algorithm is based on the algorithms due to Landau [7], Havel [4], Hakimi [2, 3], Ryser [10], and Iványi [6]. The algorithm generalizes the results due to Landau [7], Moon [9], Havel [4], Hakimi [2, 3], Meierling and Volkman [8], Iványi [5, 6], Erdős, Miklós and Toroczkai [1], Ryser [10].

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