# Testing and enumeration of football sequences

**Loránd Lucz, Antal Iványi, Péter Sótér**

Department of Computer Algebra, Eötvös Loránd University

tony@compalg.inf.elte.hu lorand.lucz@gmail.com mapoleon@freemail.hu

In his recent monography [2, page 60] András Frank formulated the following open problem.

**Research problem 2.3.1 (Iványi)** *Decide if a sequence of n integers can be the final score of a football tournament of n teams. The winner of a game gets 3 points, the loser no point, while both teams get 1 point for a draw.*

Working on the solution of this problem we have found that the following simpler problem plays central role in the solution.

**Problem 1.** *Decide if a sequence of n integers can be the final draw sequence of a football tournament of n teams.*

A sequence of integer numbers $q = (q_1, \ldots, q_n)$ is called *graphical* if there exists a simple graph $G$ such, that $q$ is its degree sequence.

Problem 1 is equivalent with the following Problem 2.

**Problem 2.** *Decide if a sequence of n integers can be graphical.*

There are several known answers for this question. The most popular answers are Erdős-Gallai theorem [1] and Havel-Hakimi theorem [3, 4]. Sierksma and Hoogeveen [8] gathered further conditions. The algorithms based on these results require in worst case $\Omega(n^2)$ time. Recently we have found new algorithms Linear-Erdős-Gallai and Quick-Erdős-Gallai working in all cases in $O(n)$ time [5, 6].

In the talk we present new approximate algorithms to solve the above formulated research problem and determine the number of football score sequences for new values of $n$ [7]. Our algorithms are based on the new quick versions [5, 6] of Erdős-Gallai method [1].

## References

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