# Score sets of $(a, b)$-tournaments

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Let $a$ and $b \geq a$ be nonnegative integers and let $m$ and $n \geq m$ be positive integers. An $(a, b, n)$-tournament $T_n(a, b)$ is a loopless directed graph on $n$ vertices, in which every pair of different vertices is connected at least $a$ and at most $b$ arcs [2, 3].

The *score sequence* $d = [d_1, d_2, \ldots, d_n]$ of $T_n(a, b)$ is the increasing ordered sequence of the out-degrees of $T_n(a, b)$. The *score set* $\mathcal{S} = \{s_1, s_2, \ldots, s_m\}$ is the set of the different out-degrees of $T_n(a, b)$.

According to the conjecture of K. B. Reid [8] any set of nonnegative integers is the score set of some $T_n(a, b)$. The conjecture was partially proved by Hager [1] and Reid [8]. Yao gave a full existence proof of the conjecture in 1989 [9]. Constructive proof of the theorem is not known.

In the talk we extend this Reid-Yao theorem to $(a, b)$-tournaments [5, 7] and investigate the question of the unicity of score sequences of tournaments with prescribed score sets [4, 6].

## References

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