On the speed of computers with paged and interleaved memory

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Abstract: A performance measure (the speed) of computer mathematical models is defined. This measure is given as a function of hardware and program behaviour parameters for Bélády's computer model with paged memory and Vulihman's model with interleaved memory.

KEY WORDS AND PHRASES: computer system performance, demand paging, interleaved memory, program behaviour.

1. Introduction

Computer performance is investigated by empirical, simulation and analytical methods [1].

The analytical method is based on the analysis of mathematical models reflecting the essence of processes by "exact" methods /e.g. queueing or Markov-chain theory, combinatorics etc./.

Due to the inaccuracy of models the analytical method usually gives only a rough estimate, but the results are general and convenient for computer planning or development.

In this lecture we recommend an analytical method, based on Bélády's [2], Coffman's [3] and Kogan's [4] methods and give some concrete formulas derived by this method.

2. Definition of the speed

The set $N = \{\nu_1, \ldots, \nu_n\}$ $(1 < n < \infty)$ is called a program, and the sequence $\omega_T = r_1 \ldots r_T$ $(1 \leq T \leq \infty,\ r_t \in N,\ t=1,\ldots, T)$ consisting of elements of N /T-element permutations with repetition/ is called a program realization of length T. Denote N^T the set of all possible sequences ω_T. Denote $\tau[\omega_T]$ the processing time of a sequence ω_T on given computer model. The distribution of the elements of N in the sequences ω_T is called program behaviour [5]. This behaviour is given by the set $D = \{D_1, \ldots, D_T\}$ of distribution-functions $D_1, \ldots, D_T$ where $D_T[\omega_T]$ gives the probability of ω_T in the space of events N^T, that is

/2.1/ $$\forall \omega_T \qquad 0 \leq D_T[\omega_T] \leq 1$$

and

/2.2/ $$\forall T \qquad \sum_{\omega_T \in N^T} D_T[\omega_T] = 1.$$

Further we suppose

/2.3/ $$\sum_{i=1}^{n} D_{T+1}[\omega_T \nu_i] = D_T[\omega_T].$$

Instead of $D_T[\omega_T]$ we use the marking $D[\omega_T]$. Denote the set of D's satisfying the conditions /2.1/, /2.2/ and /2.3/ by $\mathcal{D}$.

In this lecture we use 6 simple bahaviour model: homogeneous [6], cyclical [6], random [2], random with step [3], random with repetition [7] and independent [5] ones. Let HOM, CYCL, RAN, $STEP_p$, REP_p and $IND_{p_1,\ldots,p_n}$ denote **them.**

According to the homogeneous model the references are equivalent, that is

/2.4/ $P\{r_1 = \nu_i\} = \frac{1}{n}$ and $r_t = r_1$ $(t=2,3,\dots;\ i=1,\dots,n)$.

This formula is equivalent to the following definition:

/2.5/ $$\mathrm{HOM}[\omega_k] = \begin{cases} \frac{1}{n}, & \text{if in } \omega_k\ r_1=r_2=\dots=r_k \\ 0, & \text{otherwise.} \end{cases} \quad (k=1,2,\dots).$$

According to the cyclical model the step ν_i, ν_{i+1} $(\nu_{n+1} \equiv \nu_1)$ has a probability 1, that is

/2.6/ $$P\{r_1 = \nu_i\} = \frac{1}{n} \quad \text{and} \quad P\{r_{t+1} = \nu_{i+1}\} = \begin{cases} 1, & \text{if } r_t = \nu_i, \\ 0, & \text{if } r_t \neq \nu_i \end{cases}$$

$(t=1,2,\dots;\ i=1,\dots,n)$.

This formula is equivalent to the following definition:

/2.7/ $$\mathrm{CYCL}[\omega_k] = \begin{cases} \frac{1}{n}, & \text{if in } \omega_k \text{ from } r_t=\nu_i, r_{t+1}=\nu_j \text{ follows } j \equiv i+1 \pmod{n} \\ 0, & \text{otherwise.} \end{cases}$$

$(t=1,2,\dots)$.

According to the random model the references occur randomly, that is

/2.8/ $P\{r_t = \nu_i\} = \frac{1}{n}$ $(t=1,2,\dots;\ i=1,\dots,n)$.

This formula is equivalent to the following definition:

/2.9/ $\mathrm{RAN}[\omega_k] = \frac{1}{n^k}$ $(k=1,2,\dots,\ \omega_k \in N^k)$.

According to the random model with repetition the repetition has a probability p, and other references have a probability $\frac{1-p}{n-1}$:

$$/2.10/\quad P\{r_1=\nu_i\} = \frac{1}{n};\quad P\{r_t=\nu_i\}=\begin{cases} p, & \text{if } r_t=\nu_i, \\ \frac{1-p}{n-1} & \text{if } r_t\neq\nu_i, \end{cases}$$

$$(t=2,3,\ldots;\ i=1,\ldots,n)\ .$$

This formula is equivalent to the following definition:

$$/2.11/\quad REP_p\left[\omega_k\right]=\frac{1}{n}\cdot p^f\left(\frac{1-p}{n-1}\right)^{k-f-1}\quad (k=1,2,\ldots)\ ,$$

where f is the number of the repetitions in ω_k.

According to the random model with step [3] the step ν_i, ν_{i+1} $(\nu_{n+1}\equiv\nu_1$ in ω_k has a probability p, and other references have a probability $\frac{1-p}{n-1}$:

$$/2.12/\quad P\{r_1=\nu_i\} = \frac{1}{n};\quad P\{r_t= {}_{i+1}\}=\begin{cases} p, & \text{if } r_{t-1}=\nu_i, \\ \frac{1-p}{n-1}, & \text{if } r_{t-1}=\nu_i, \end{cases}$$

$$(t=2,3,\ldots;i=1,\ldots,n).$$

This formula is equivalent to the following definition:

$$/2.13/\quad STEP_p\left[\omega_1\right]=\frac{1}{n};\quad STEP\left[\omega_k\right]=\frac{1}{n}\cdot p^f\left(\frac{1-p}{n-1}\right)^{k-f-1}(k=1,2,\ldots),$$

where f is the number of the steps in ω_k.

According to the independent model [5] the reference to the page ν_i has a probability p_i, that is

$$/2.14/\quad P\{r_t=\nu_i\}=p_i\qquad (t=1,2,\ldots)\ .$$

This formula is equivalent to the following definition:

$$/2.15/\quad IND_{p_1,\ldots,p_n}\left[\omega_k\right]=\prod_{i=1}^{n}\left(p_i\right)^{f_i},$$

where f_i is the number of the references to the page ν_i.

Computer performance is characterized by the number of operations in a time unit: V.V is called the speed of the computer model and is determined by the formula

$$V \overset{def}{=} \liminf_{k \to \infty} \frac{1}{\sum_{\omega_k \in N^k} D[\omega_k] \frac{\tau[\omega_k]}{k}}. \qquad /2.16/$$

If in /2.16/ we have existance of the lim in addition to the lim inf, then this limit is denoted by V' .

Our aim is to determine the speed for various computer and program behaviour models.

3. The mathematical model of computers with paged memory

For the investigation of computers with paged memory we use the well-known model proposed by Bélády [2] in 1966.

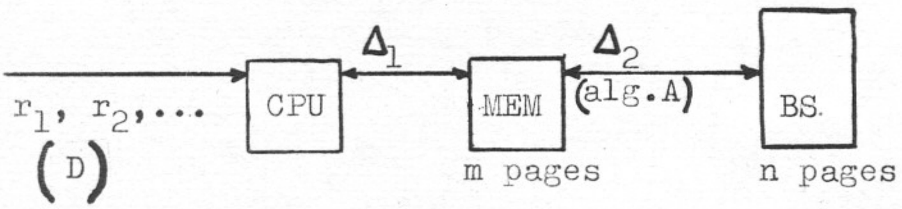

Fig 1. The scheme of a computer with 2 level paged memory.

The computer consists of a central processor unit /CPU/, an m-paged main memory /MEM/ and an n-paged backing store /BS/. The CPU has direct access to MEM-access time Δ_1 - while an indirect access to BS-access time $\Delta_1 + \Delta_2$. The paging is controlled by a demand paging algorithm. The set of demand paging algorithms is denoted by $\mathcal{A}$.

For this model the speed V_p is [8]

/3.1/ $$V_p = \frac{1}{\Delta_1 + \Delta_2 \cdot C},$$

where C is the average cost of a reference, that is the page fault probability [5]. By definition

/3.2/ $$C = C(m,n,A,D) = \limsup_{k \to \infty} \sum_{\omega_k \in N^k} \left(D[\omega_k] \frac{\sum_{i=1}^{k} \delta_i}{k} \right),$$

where $A \in \mathcal{A}$, $D \in \mathcal{D}$,

/3.3/ $$\delta_i = \delta(i,m,n,\omega_T,A) = \begin{cases} 0, & \text{if } r_i \in S_t, \\ 1, & \text{if } r_i \notin S_t, \end{cases}$$

and S_t is the set of pages in MEM at time t. S_t is called the memory state. If in /3.2/ there exist a limit, then it is denoted by C'.

4. General assertions on the speed of computers with paged memory

Lemma 1. ([7]). If $\Delta_1 > 0$, and $1 \leq m < n < \infty$, then

/4.1/ $$0 = C(m,n,A,HOM) \leq C(m,n,A,D) \leq C(m,n,LRU,CYCL) = 1,$$

that is for the speed

/4.2/ $$\frac{1}{\Delta_1 + \Delta_2} = V_p(\Delta_1, \Delta_2, m, n, LRU, CYCL) \leq V_p(\Delta_1, \Delta_2, m, n, A, D) \leq V_p(\Delta_1, \Delta_2, m, n, A, HOM) =$$

$$= \frac{1}{\Delta_1}$$

holds.

Definition 1. ([9]) The demand paging algorithms, for which

/4.3/ $$\forall T_1, \forall T_2 \quad \sum_{i=1}^{T_1} \delta(i,m,n,\omega_{T_1},A) = \sum_{i=1}^{T_1} \delta(i,m,n,\omega_{T_1}\omega_{T_2},A)$$

are called sequential [6]. The set of the sequential algorithms is denoted by $\mathcal{B}$.

Lemma 2. If $1 \leq m < n < \infty$, then for every $B \in \mathcal{B}$ and for every $D \in \mathcal{D}$

/4.4/ $$C_{inf} = \liminf_{k \to \infty} \sum_{\omega_k \in N^k} D[\omega_k]\delta_k \leq C(m,n,B,D) \leq$$
$$\leq \limsup_{k \to \infty} \sum_{\omega_k \in N^k} D[\omega_k]\delta_k .$$

Definition 2. Let $D \in \mathcal{D}$ and N_+^k $(k=1,2,\ldots;\ N_+^k \subseteq N^k)$ be given. Denote a_k the sum $\sum_{\omega_k \in N_+^k} D[\omega_k]$. If

/4.5/ $$\lim_{k \to \infty} a_k = 0,$$

then we shall say, that the sequence N_+^k has zero limit-density in N^k.

Lemma 3. ([7]). If for a given D there exist an m-tuple of pages $\mu_1,\ldots,\mu_m$ and $\varepsilon > 0$, for which

/4.6/ $$\forall \omega_k \qquad D[\omega_k \nu_i] \geq \varepsilon \cdot D[\omega_k]$$ holds, then the sequence N_+^k has zero limit-density in N^k, where N_+^k is the set of $\omega_k \in N^k$, for which $|S_t| = |S_t(m,\omega_k,B)| < m$.

Definition 3. Let ω_T be given. The sequences of length /T+f/ /f=0,1,.../ identical to ω_T up to the T-th element, are called the bundle $\Pi_f[\omega_T]$ with root ω_T and length f .

Definition 4. The average cost of a references C^{Π} in a given bundle $\Pi_f[\omega_T]$ is by definition

/4.7/ $$C = C^{\Pi}/m,n,A,D,\omega_T/ = \lim_{k \to \infty} \sum_{\omega_k \in \Pi_{k-T}[\omega_T]} D[\omega_k]\delta_k ,$$

where $D^{\pi}[\omega_k]$ is the probability of sequence ω_k $\left(\omega_k \in \Pi_{k-T}[\omega_T]\right)$ within the bundle, that is

$$/4.8/ \quad D^{\pi}[\omega_k] = \frac{D[\omega_k]}{\sum_{\omega_k \in \Pi_{k-T}[\omega_T]} D[\omega_k]} = \frac{D[\omega_k]}{D[\omega_T]} .$$

Lemma 4. ([7]). Let N_+^k denote the set of ω_k'-s not belonging to any bundle, which has a cost C^{π}. If the sequence N_+^k /k=1,2,.../ has a zero limit density in N^k, then $C(m,n,B,D)$ exists and is C^{π}.

5. Theorems on the speed of computers with paged memory

Theorem A /Bélády, 1966/ [2]. If L is a nonlookahead demand paging algorithm, then

$$/5.1/ \quad C(m,m,L,RAN) = \frac{n-m}{n} .$$

Theorem B /Aho, Denning, Ullman, 1971/([5]). If $1 \leq m < n < \infty$, then

$$/5.2/ \quad C'(m,n,OPT,IND) = \sum_{i=m}^{n} p_i - \frac{\sum_{i=m}^{n} p_i^2}{\sum_{i=m}^{n} p_i} ,$$

where OPT is the optimal paging algorithm, always replacing the page of S_t with minimal p_i [5].

Theorem C. /Stoyan , 1975/([8]). If $1 \leq m < n < \infty$, then

$$/5.3/ \quad C'(m,n,REF_a, RAN) = \frac{n-m}{n+a} \quad /a=0,1,\ldots,m-1/,$$

where REF_a is a lookahead algorithm, which knows a references ahead, and holds required pages in the memory if possible, and chooses randomly among the others.

Theorem 1 ([7]). If $1 \leq m < n < \infty$ and $0 \leq a \leq m$, then

/5.4 /
$$C'/m,n,REF_a,REP_p/ = \frac{/n-m/\ /1-p/}{n-1+\left[\min/a,m-1/\right]/1-p/+\left[\max/0,a-m+1/\right]\left(1-m!\frac{/1-p/^{m-1}}{/n-1/_{m-1}}\right)/1-p/}.$$

Theorems A and C follow from theorem 1 /in cases $a=0$, $p=\frac{1}{n}$ and $0 \leq a \leq m-1$, $p=\frac{1}{n}$ /.

Theorem 2. ([7]). If $1 \leq m < n < \infty$, then

/5.5 /
$$C'\left(m,n,PP_b,RAN\right) = \frac{n-m}{n}\left(\frac{n-1}{n}\right)^b \quad /b=0,1,/,$$

where PP_b is a lookahead algorithm, which knows at time t the next b references, differing from r_t and each other, and hold these pages if possible, in the memory, and chooses randomly among the others.

In case $b=1$, $m=2$, $n=3$, it follows from theorem 2 the partial resolution of the problem, investigated by Bélády in 1966, namely $C'\left(2,3,MIN,RAN\right) = \frac{2}{9}$.

Theorem 3 ([7]). If $1 \leq m < n < \infty$ and $a \geq 0$, then

/5.6 /
$$C'/2,3,REF_a,REP_p/ = \frac{1-p}{2+\left[\min/a,1/\right]/1-p/+\mathrm{sign}\left[\max/0,a-1/\right]p/1-p^{a-1}/}.$$

From this theorem it follows /in the case $a \to \infty$, when

$REF_a \to MIN$/, that

/5.7 / $C'/2,3,MIN,REP_p/ = \frac{1-p}{3}$.

6. Mathematical model of computers with interleaved memory

We investigate the following model of computers with interleaved memory due to V.E.Vulihman [10]:

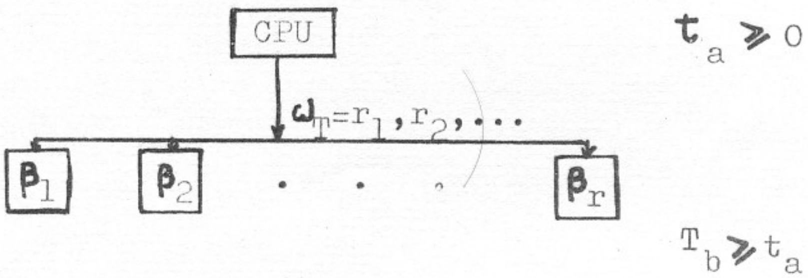

Fig.2. Scheme of computers with interleaved memory

The computer consists of a central processor unit /CPU/ and modules of memory $\beta_1, \dots, \beta_r$. The set of modules is denoted by B. The elements of ω_T are generated by the CPU requireing $t_a \geqslant 0$ time per element. A request to a module reserve that module for a time $T_b > t_a$ during this time other request can't be served by this module. If the module being requested is occupied, then the generation of ω_T will be suspended until the module is free.

The speed of this model is denoted by V_i .

7. General assertions on the spead of computers with interleaved memory

Hellerman in his book [6], Bokova and Tzaturyan in their paper [11] proved the following assertions.

Lemma 5 ([6]). If $T_b > t_a \geq 0$, then for every $D \in \mathcal{D}$ and for every $r \geq 1$

/7.1/ $\frac{1}{T_b} = V_i'/t_a, T_b, r, HOM/ \leq V_i/t_a, T_b, r, D/ \leq V_i'/t_a, T_b, r, CYCL/ =$

$$= \frac{1}{T_b} \min /r, \frac{T_b}{t_a}/ ,$$

where HOM and CYCL are the homogeneous and cyclical behaviour model.

Definition 5. Let φ_i $/\varphi_o=0/$ denote the processing time of ω_T up to i-th element. Then the increment due to the i-th element is $\delta_i = \varphi_i - \varphi_{i-1}$.

Example. For every $\omega_2 \in N^2$

/7.2 / $$\delta_1 = T_b + t_a, \delta_2 = \begin{cases} T_b, & \text{if } r_2 = r_1 \\ t_a, & \text{if } r_2 \neq r_1 \end{cases} .$$

Lemma 6. ([11]). If $T_b > t_a \geq 0$, then

/7. 3/ $$\liminf_{k \to \infty} \frac{1}{\sum_{\omega_k \in N^k} D[\omega_k] \delta_k} \leq V_i/t_a, T_b, r, D/$$

$$\leq \limsup_{k \to \infty} \frac{1}{\sum_{\omega_k \in N^k} D[\omega_k] \delta_k}$$

Lemma 7. ([11]). If $T_b > t_a' > t_a'' \geq 0$, then

/7.4 / $$V_i\,(t_a', T_b, r, D) \leq V_i\,(t_a'', T_b, r, D).$$

Lemma 8. ([11]). If $T_b > t_a \geq 0$, and $r' > r''$, then

/7.5/ $V_i/t_a, T_b, r', D/ \geq V_i/t_a, T_b, r'', D/$.

8. Theorems on the speed of computers with interleaved memory

Theorem 4. ([7]). If $T_b > t_a \geq 0$, then for $r \geq 1$

$$V_i'/t_a, T_b, r, RAN/ \leq$$

/7.6/
$$\leq \frac{1}{t_a \sum_{i=1}^{r} \sum_{j=2}^{i} p_{i,j} + \left(\sum_{i=1}^{r} p_{i,1}\right)\left[\sum_{k=0}^{r-1}\left(\max\left(t_a, T_b - k\cdot t_a\right) \sum_{j=k+1}^{r} \frac{p_j}{j}\right)\right]}$$

and the equality holds

a/ if $t_a=0$, then for $r \geq 1$;

b/ if $\frac{T_b}{2} \leq t_a \leq T_b$, then for $r \geq 1$;

c/ if $0 < t_a < \frac{T_b}{2}$, then for $r=1,2$.

In the formula /7.6/

/7.7/ $$p_i = \frac{r/r-1/ \ldots /r-i+1/}{r^{i+1}} \, i \quad /1 \leq i \leq r/,$$

and

/7.8/ $$p_{i,j} = \frac{p_i}{\sum_{i=1}^{r} i \cdot p_i} \qquad /1 \leq i \leq r, \ 1 \leq j \leq i/.$$

The following corollaries follow from theorem 4 as special cases.

Corollary 1 /Hellerman, 1967/ ([6]). If $t_a = 0$ and $r \geq 1$, then

/7.9 / $$V_i' /0,T_b,r,RAN/ = \frac{1}{T_b} \sum_{i=1}^{r} i\cdot p_i .$$

Burnett, Coffman [3] and Stone [12] proved a more general assertion.

Theorem D /Burnett, Coffman, Stone, 1974/ [3,12].

If $t_a = 0$ and $r \geq 1$, then

$V_i' /0,T_b,r,STEP_p/ =$

/7.10 / $$= \frac{1}{T_b} \sum_{k=1}^{r} \sum_{j=0}^{k-1} \binom{k-1}{j} p^j \left(\frac{1-}{n-1}\right)^{k-j-1} \cdot C_{n-j,k-j},$$

where

/7.11/ $$C_{n,k} = \sum_{j=0}^{k-1} \left[(-1)^j \binom{k-1}{j} /n-j-1/ /n-j-2/ \dots /n-k+1/ \right] .$$

Corollary 2. If $\frac{T_b}{2} \leq t_a \leq T_b$ and $r \geq 1$, then

/7.12 / $$V_i'/t_a,T_b,r,RAN/ = \frac{1}{\frac{1}{r} T_b + \left(1-\frac{1}{r}\right) t_a}$$

Corollary 3. If $T_b > t_a \geq 0$, then

/7.13/ $$V_b^?/t_a,T_b,2,RAN/ = \frac{1}{\frac{1}{3} t_a + \frac{1}{2} T_b + \frac{1}{6} \max /t_a, T_b - t_a/}$$

Corollary 4. If $T_b > t_a \geq 0$ and $r \geq 1$, then

/7.14 / $$V_i' /t_a,T_b,r,RAN/ \leq \frac{1}{\frac{1}{r} T_b + \left(1-\frac{1}{r}\right) t_a} .$$

On the base of the formula /7.9 / it is not easy to estimate the order of $V_i'/0,T_b,r,RAN/$, therefore the following theorems are interesting.

Theorem E. /Hellerman, 1967/ ([6]). If $1 \leq r \leq 45$, then

$$/7.15/ \quad 0,96.r^{0,56} \leq V_i^{!}/0,T_b,r,RAN/ \leq 1,04 . r^{0,56} .$$

Theorem F. /Vulihman, 1972/ ([10]). If $r \geq 1$, then

$$/7.16/ \quad V_i^{!}/0,T_b,r,RAN/ \leq \left(\sqrt{2\pi r}\right) \frac{1}{T_b} .$$

We proved the following more general theorem.

Theorem 5. ([13]). If $t_a=0$ and $r \geq 1$, then

$$/7.17/ \quad \frac{1}{T_b}\left(\sqrt{\frac{\pi r}{2}} -1\right) < V_i^{!}/0,T_b,r,RAN/ = \frac{r! \sum_{k=0}^{r-1} \frac{r^k}{k!}}{T_b . r^r} \quad \frac{1}{T_b}\left(\sqrt{\frac{\pi r}{2}} +1\right) .$$

In our paper [7] we used a simple direct proof. Using a result due to G. Szegő [14] we can proof a formula with a smaller additive constant, which is exact.

Theorem G. /Szegő, 1928/ ([14]). If q is a nonnegative integer number, then

$$/7.18/ \quad \frac{1}{2} e^q = 1 + \frac{q}{1!} + \frac{q^2}{2!} + \dots + \frac{q^q}{q!} \vartheta_q ,$$

where $\vartheta_o = \frac{1}{2}$ and ϑ_q tends monotonically to $\frac{1}{3}$ as $q \to \infty$.

Theorem 6. If $t_a=0$ and $r \geq 1$, then

$$/7.19/ \quad V_i^{!}/0,T_b,r,RAN/ = \frac{1}{T_b}\left(\sqrt{\frac{\pi r}{2}} - \frac{1}{3} + \rho_r\right) ,$$

where ρ_r tends monotonically to zero as $r \to \infty$ and

$$/7.20/ \quad \rho_1 = \frac{4}{3} - \sqrt{\frac{\pi}{2}} \approx 0,08 \quad \text{and} \quad \rho_2 = \frac{11}{6} - \sqrt{\pi} \approx 0,06 .$$

It seems a hard but resolvable problem to estimate the order of expression in Coffman's theorem, as a function of p.

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