Numbers, Functions, Equations 2013
(NFE'13)

Dedicated to Professors

Zoltán Daróczy **Imre Kátai**

on the occasion of their 75th birthday

Hotel Visegrád, Visegrád, Hungary

June 28–30, 2013

http://compalg.inf.elte.hu/~nfe13/

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Program

June 28, Friday

12:30–14:00	Lunch
1^{ST}	**Afternoon Session** **Chairman: Zsolt Páles**
14:00–14:10	**Conference opening**
14:10–14:35	**Járai, Antal**: On regularity properties of solutions defined on positive definite matrices of a Cauchy-type functional equation
14:35–15:00	**Ger, Roman**: Alienation of additive and logarithmic equations
15:00–15:25	**Losonczi, László**: Solved and unsolved problems for Páles means
15:25–15:50	**Gselmann, Eszter**: On some classes of partial difference equations
15:50–16:15	Coffee Break
2^{ND}	**Afternoon Session** **Chairman: Karl-Heinz Indlekofer**
16:15–16:40	**Manstavičius, Eugenijus**: Additive function theory on permutations
16:40–17:05	**Matkowski, Janusz**: Invariance formula for generalized quasi-arithmetic means
17:05–17:30	**Kovács, Attila**: Some results on simultaneous number systems in the ring of integers of imaginary quadratic fields
17:30–17:55	**Tóth, Viktória**: Collision and avalanche effect in pseudorandom sequences
18:00–20:00	Dinner

Program

June 29, Saturday, Morning

Two sections: (**FE**) and (**N**)

8:00-9:00	Breakfast
1^{ST}	**Morning Session (FE) Chairman: Ludwig Reich**
9:00–9:25	**Jarczyk, Witold**: Uniqueness of solutions of simultaneous difference equations
9:25–9:50	**Maksa, Gyula**: Some remarks and problems related to real derivations
9:50–10:15	**Głazowska, Dorota**: Invariance of the quasi-arithmetic mean in the class of generalized weighted quasi-arithmetic means
10:15–10:40	**Jarczyk, Justyna and Pál Burai**: Conditional homogenity and translativity of Makó–Páles means
1^{ST}	**Morning Session (N) Chairman: Aleksandar Ivić**
9:00–9:25	**Indlekofer, Karl-Heinz**: On spaces of arithmetical functions
9:25–9:50	**De Koninck, Jean-Marie; Nicolas Doyon; Florian Luca**: Consecutive integers divisible by the square of their largest prime factors
9:50–10:15	**Kallós, Gábor**: On Rényi–Parry expansions
10:15–10:40	**Farkas, Gábor**: Sieving for large Cunningham chains of length 3 of the first kind
10:40–11:05	Coffee Break

Program

June 29, Saturday, Morning

Two sections: (**FE**) and (**N**)

2^{ND}	**Morning Session (FE) Chairman: Antal Járai**
11:05–11:30	**Reich, Ludwig**: Generalized Aczél–Jabotinsky differential equations, Briot–Bouquet equations and implicit functions
11:30–11:55	**Sablik, Maciej and Agata Nowak**: Chini's equations in actuarial mathematics
11:55–12:20	**Baron, Karol**: On some orthogonally additive functions on inner product spaces
12:20–12:45	**Lajkó, Károly; Fruzsina Mészáros; Gyula Pap**: Characterization of a bivariate distribution through a functional equation
2^{ND}	**Morning Session (N) Chairman: Eugenijus Manstavičius**
11:05–11:30	**Varbanets, Pavel and Sergey Varbanets**: Twisted exponential sums and the distribution of solutions of the congruence $f(x, y) \equiv 0 \pmod{p^m}$ over $Z[i]$
11:30–11:55	**Laurinčikas, Antanas; Kohji Matsumoto; Jörn Steuding**: Hybrid universality of certain composite functions involving Dirichlet L-functions
11:55–12:20	**Kaya, Erdener**: Sums of independent random variables on additive arithmetical semigroups
12:20–12:45	**Nagy, Gábor**: Number systems of the Gaussian integers with modified canonical digit sets
12:45–14:00	Lunch

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Program

June 29, Saturday, Afternoon

Two sections: (**FE**) and (**N**)

1^{ST}	**Afternoon Session (FE)** **Chairman: László Kozma**
14:00–14:25	**Totik, Vilmos**: A conjecture of Petruska and a conjecture of Widom
14:25–14:50	**Molnár, Lajos**: Sequential isomorphisms and endomorphisms of Hilbert space effect algebras
14:50–15:15	**Bessenyei, Mihály**: Convexity structures induced by regular pairs
15:15–15:40	**Boros, Zoltán and Noémi Nagy**: Notes on approximately convex functions
1^{ST}	**Afternoon Session (N)** **Chairman: Jean-Marie De Koninck**
14:00–14:25	**Ivić, Aleksandar**: The divisor problem in short intervals
14:25–14:50	**Misevičius, Gintautas and Audronė Rimkevičienė**: Joint limit theorems for periodic Hurwitz zeta-function II.
14:50–15:15	**Kačinskaitė, Roma**: A note on functional independence of some zeta-functions
15:15–15:40	**Fridli, Sándor and Ferenc Schipp**: Construction of rational function systems with applications
15:40–16:00	Coffee Break
2^{ND}	**Afternoon Session (Laudations)** **Chairman: Ferenc Schipp**
16:00–16:30	**De Koninck, Jean-Marie**: Laudation to Imre Kátai
16:30–17:00	**Páles, Zsolt**: Laudation to Zoltán Daróczy
18:00–21:00	Banquet

Program

June 30, Sunday

Two sections: (**FE**) and (**N**)

8:00–9:00	Breakfast
1^{ST}	**Morning Session (FE) Chairman: Vilmos Totik**
9:00–9:25	**Székelyhidi, László**: Exponential polynomials on Abelian groups
9:25–9:50	**Burai, Pál**: Monotone operators and local-global minimum property of nonlinear optimization problems
9:50–10:15	**Lénárd, Margit**: A $(0,2)$-type Pál interpolation problem
10:15–10:40	**Daróczy, Bálint; András A. Benczúr; Lajos Rónyai**: Fisher kernels for image descriptors: a theoretical overview and experimental results
1^{ST}	**Morning Session (N) Chairman: Gediminas Stepanauskas**
9:00–9:25	**Kozma, László**: On semantic descriptions of software systems
9:25–9:50	**Iványi, Antal**: Testing of random sequences
9:50–10:15	**Fülöp, Ágnes**: Estimation of the Kolmogorov entropy in the generalized number system
10:15–10:40	**Vatai, Emil**: Speeding up distributed sieving with the inverse sieve
12:00–14:00	Lunch

Abstracts

Karol Baron
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On some orthogonally additive functions on inner product spaces

Let E be a real inner product space of dimension at least 2. If $f : E \to E$ satisfies

$$f(x+y) = f(x) + f(y) \quad \text{for all orthogonal } x, y \in E$$

and

$$f(f(x)) = x \quad \text{for } x \in E,$$

then f is additive.

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Convexity structures induced by regular pairs

The notion of (planar) convexity can be generalized using regular pairs. A regular pair is a pair of continuous functions with the property that each pair of points of the Cartesian plane (with distinct first coordinates) can be interpolated by a unique member of the linear hull of the components. Replacing classical lines by members of the linear hull, one can define a convexity structure on the plane. The aim of the talk is to determine the Radon-, Helly-, and Carathéodory-numbers of such structures and give some applications in separation problems.

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(joint work with **Noémi Nagy**)

Notes on approximately convex functions

The purpose of this note is to show that approximately convex (respectively, approximately Wright-convex) functions with locally small error terms are, in fact, convex (respectively, Wright-convex).

Let $I \subset \mathbb{R}$ be an open interval, $c \geq 0$, $p > 1$, and assume that the function $f : I \to \mathbb{R}$ fulfils the inequality

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda)f(y) + c\,(\lambda(1-\lambda)|x-y|)^p \tag{1}$$

for every $x, y \in I$ and $\lambda \in [0,1]$. Then f is convex (i.e., f satisfies inequality (1) with $c = 0$ as well).

The proof is strongly related to the usual characterization of convexity with the aid of one sided derivatives. Establishing and applying results concerning the behavior of certain generalized derivatives, it is possible to prove an analogous result for Wright-convexity. Namely, under the same assumptions on c and p, if the function $f : I \to \mathbb{R}$ fulfils the inequality

$$f(\lambda x + (1-\lambda)y) + f(\lambda y + (1-\lambda)x) \leq f(x) + f(y) + c\,(\lambda(1-\lambda)|x-y|)^p \tag{2}$$

for every $x, y \in I$ and $\lambda \in [0,1]$, then f is Wright-convex (i.e., f satisfies inequality (2) with $c = 0$ as well).

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Monotone operators and local-global minimum property of nonlinear optimization problems

Our main goal is to prove that every local minimizer of a certain nonlinear optimization problem is global. For this we use some results from the theory of monotone operators and connected functions.

At last we show applications of the main results in control theory.

References

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Fisher kernels for image descriptors: a theoretical overview and experimental results

Visual words have recently proved to be a key tool in image classification. Best performing Pascal VOC and ImageCLEF systems use Gaussian mixtures or k-means clustering to define visual words based on the content-based features of points of interest. In most cases, Gaussian Mixture Modeling (GMM) with a Fisher information based distance over the mixtures yields the most accurate classification results. In this presentation we overview the theoretical foundations of the Fisher kernel method. We indicate that it yields a natural metric over images characterized by low level content descriptors generated from a Gaussian mixture. We justify the theoretical observations by reproducing standard measurements over the Pascal VOC 2007 data. Our accuracy is comparable to the most recent best performing image classification systems.

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Consecutive integers divisible by the square of their largest prime factors

We examine the occurrence of strings of consecutive integers divisible by a given power of their largest prime factor. In particular, we count how many strings of consecutive integers, each of which is divisible by the square of its largest prime factor, are located below a given number.

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Sieving for large Cunningham chains of length 3 of the first kind

Our prospective purpose to reach new world record, namely find the largest known Cunningham chains of the first kind of length 3. This means the proof the primality of natural numbers p, $2p+1$, $4p+3$, each in the magnitude of 2^{34944} (more than 10 500 decimal digits). This would be impossible without sieving which reduces the estimated time of our project. I would like to talk about the details of the theoretical background of the sieving process.

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Construction of rational function systems with applications

In our talk we present our latest results on rational orthogonal and biorthogonal systems that are inspired by applications including ECG processing and surface representation. We show several examples that demonstrate the benefit of such systems in various situations. Constructions in the one and also in the two dimensional cases are shown. Also the problem of discretization is addressed.

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Estimation of the Kolmogorov entropy in the generalized number system

I study the dynamical properties of the complex number systems. A map is introduced on the transition graph by analogy of statistical physics systems. I gain an estimation of the Kolmogorov entropy by the Grassberger–Procaccia algorithm for a finite approximation of B_γ.

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Alienation of additive and logarithmic equations

Let $(R, +, \cdot)$ stand for an Archimedean totally ordered unitary ring and let $(H, +)$ be an Abelian group. Denote by C the cone of all strictly positive elements in R. We study the solutions $f, g : C \longrightarrow H$ of a Pexider type functional equation

$$f(x+y) + g(xy) = f(x) + f(y) + g(x) + g(y) \tag{E}$$

resulting from summing up the additive and logarithmic equations side by side. We show that modulo an additive constant equation (E) forces f and g to split back to the system of two Cauchy equations

$$\begin{cases} f(x+y) = f(x) + f(y) \\ g(xy) = g(x) + g(y) \end{cases}$$

for every $x, y \in C$ (alienation phenomenon).

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Invariance of the quasi-arithmetic mean in the class of generalized weighted quasi-arithmetic means

We consider the following invariance equation

$$A_\varphi \circ (M_{f_1,g_1}, M_{f_2,g_2}) = A_\varphi, \tag{1}$$

where A_φ stands for the quasi-arithmetic mean and M_{f_1,g_1}, M_{f_2,g_2} are two generalized weighted quasi-arithmetic means. We solve this equation under four times continuous differentiability of the unknown functions $\varphi, f_1, f_2, g_1, g_2$.

We examine also a special case of the equation (1), i.e.

$$A_\varphi \circ (M_{f,g}, M_{g,f}) = A_\varphi.$$

We give the solution of this equation under natural assumptions on the unknown functions φ, f, g.

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On some classes of partial difference equations

In [1], J. A. Baker initiated the systematic investigation of some partial difference equations. The main purpose of my talk is the investigation of partial difference equations of the form

$$f(x_1, \ldots, x_n) = \sum_{i=1}^{m} \gamma_i(x_1, \ldots, x_n) f\left(x_1 + \rho_{1,i}h, \ldots, x_n + \rho_{i,n}h\right),$$

where $m \in \mathbb{N}$, $\rho_{j,i} \in \mathbb{R}$, $i = 1, \ldots, m$, $j = 1, \ldots, n$ are fixed as well as the functions γ_i, $i = 1, \ldots, m$.

Firstly, we present how such type of equations can be classified into elliptic, parabolic and hyperbolic subclasses, respectively. After that, we show solution methods in each of these classes. Finally, some examples will follow.

References

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On spaces of arithmetical functions

Here we consider

- almost even functions,
- limit periodic functions,
- almost multiplicative functions

and extend the results to general algebras of arithmetical functions.

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Testing of random sequences

Let ξ be a random integer vector, having uniform distribution

$$\mathbf{P}\{\xi = (i_1, i_2, \ldots, i_n) = 1/n^n\}, \quad \text{for} \quad 1 \leq i_1, \ldots, i_n \leq n.$$

A realization $(i_1, i_2, \ldots, i_n)$ of ξ is called *good*, if its elements are different. We analyze algorithms BACKWARD, FORWARD, LINEAR, GARBAGE, RANDOM, TREE, MODULAR which decide whether a given random realization is good.

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The divisor problem in short intervals

Some new results involving $\Delta(X+U)-\Delta(X)$ are presented, where the interval $[X+U,\, X]$ is "short" in the sense that one can have $U=o(X)$ as $X\to\infty$. As usual,

$$\Delta(x) := \sum_{n\leqslant x} d(n) - x\log x - (2\gamma-1)x$$

is the error term in the classical Dirichlet divisor problem, $d(n)=\sum_{\delta|n}1$ is the number of divisors of n, and $\gamma=-\Gamma'(1)=0.5772157\ldots$ is Euler's constant. The author (2009) proved, sharpening a result of M. Jutila (1984): For $1\ll U=U(T)\le \frac{1}{2}\sqrt{T}$ we have ($c_3=8\pi^{-2}$)

$$\int_T^{2T}\Bigl(\Delta(x+U)-\Delta(x)\Bigr)^2\,\mathrm{d}x = TU\sum_{j=0}^{3}c_j\log^j\Bigl(\frac{\sqrt{T}}{U}\Bigr)+O_\varepsilon(T^{1/2+\varepsilon}U^2)+O_\varepsilon(T^{1+\varepsilon}U^{1/2}),$$

so that for $T^\varepsilon\le U=U(T)\le T^{1/2-\varepsilon}$ this is a true asymptotic formula. The speaker and W. Zhai (2012) proved: Suppose $\log^2 T\ll U\leqslant T^{1/2}/2, T^{1/2}\ll H\leqslant T$, then we have

$$\int_T^{T+H}\max_{0\leqslant u\leqslant U}\Bigl|\Delta(x+u)-\Delta(x)\Bigr|^2\,\mathrm{d}x \ll HU\mathcal{L}^5+T\mathcal{L}^4\log\mathcal{L}+$$
$$+H^{1/3}T^{2/3}U^{2/3}\mathcal{L}^{10/3}(\log\mathcal{L})^{2/3}),$$

where $\mathcal{L}:=\log T$. They also proved: Suppose T,U,H are large parameters and $C>1$ is a large constant such that

$$T^{131/416+\varepsilon}\ll U\leqslant C^{-1}T^{1/2}\mathcal{L}^{-5},\quad CT^{1/4}U\mathcal{L}^5\log\mathcal{L}\leqslant H\le T.$$

Then in the interval $[T,T+H]$ there are $\gg HU^{-1}$ subintervals of length $\gg U$ such that on each subinterval one has $\pm\Delta(x)\ge c_\pm T^{1/4}$ for some $c_\pm>0$.

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On regularity properties of solutions defined on positive definite matrices of a Cauchy-type functional equation

For the real-valued solution of the functional equation

$$f\bigl(x^{1/2}yx^{1/2}\bigr) = f(x)f(y)$$

satisfied for all positive definite matrices x, y it is proved that all measureble solutions or solutions having the property of Baire are infinitely many times differentiable. More general equations are also treated.

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Conditional homogenity and translativity of Makó–Páles means

We characterize the homogeneous and translative members of the class of Makó-Páles means (see [1]). This is a common generalization of the classes of weighted quasi-arithmetic means and Lagrangian means. So, as an application we get the description of homogeneous and translative means also within these classes.

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Uniqueness of solutions of simultaneous difference equations

Given a set T of positive numbers, a function $c : T \to \mathbb{R}$ and a real number p we deal with the simultaneous linear difference equations

$$\varphi(tx) = \varphi(x) + c(t)x^p, \tag{E_t}$$

where t runs through T. We present uniqueness result in the case when individual equations (E_t) usually have a lot of solutions in the considered class of functions.

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A note on functional independence of some zeta-functions

In 1887, O. Hölder proved [1] the algebraic-differential independence for the gamma function $\Gamma(s)$, which means that there exists no polynomial $P \not\equiv 0$ such that, for $n \in \mathbb{N} \cup \{0\}$ and all $s \in \mathbb{C}$,

$$P(s, \Gamma(s), ..., \Gamma^{(n)}(s)) = 0.$$

In 1973, S. M. Voronin proved [2] the functional independence of the Riemann zeta-function $\zeta(s)$ showing that $\zeta(s)$ does not satisfy any differential equation of the form

$$s^m F_m(y(s), y'(s), ..., y^{(n-1)}(s)) + ... + F_0(y(s), y'(s), ..., y^{(n-1)}(s)) = 0,$$

where $F_0, ..., F_m$ are continuous functions not all identically zero.

In the talk, we present the generalization of Voronin's result for some zeta-functions. In 2011, A. Laurinčikas and the author considered [3] the functional independence of periodic zeta-function $\zeta(s; \mathfrak{a})$ and periodic Hurwitz zeta-function $\zeta(s, \alpha; \mathfrak{b})$. One more generalization of the Voronin theorem for Dirichlet L-function $L(s, \chi)$ and the function $\zeta(s, \alpha; \mathfrak{b})$ by the author is obtained in [4].

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On Rényi–Parry expansions

The properties of the expansions of real numbers in noninteger bases have been systematically investigated since the late 50s, when the seminal works by Alfréd Rényi and William Parry were published. The original idea was to choose the digits in "greedy" way, but the analysis was extended soon to general β- or q-expansions. In this talk we give a brief review about the results in this field.

The central questions in the presentation are as follows:

- What kind of expansions can be defined?
- How many possible expansions can exist?
- Which parts can occur in general and special expansions?
- What type of regularities can be found in expansions?
- Especially, what are the 1-expansions like?

Jointly, pay a tribute to activities of Professors Zoltán Daróczy and Imre Kátai, some beautiful parts of their results will be presented; namely the field of the univoque numbers and new normal number-family constructions.

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Sums of independent random variables on additive arithmetical semigroups

Let (G, ∂) be an additive arithmetical semigroup if G, generated by a countable set P of prime elements, is a commutative semigroup with identity element 1_G, and ∂ is an integer valued degree mapping $\partial : G \to \mathbb{N}_0$ [5]. Assume that the generating function Z of G is an element of the $exp - log$ *class* $\mathcal{F}$ (the class of Indlekofer [3]).

In this paper, we embed the additive arithmetical semigroup in a probability space $\Omega := (\beta G, \sigma(\bar{\mathcal{A}}), \bar{\delta})$ where βG denote the Stone-Čech compactification of G [2, 4]. We show that every additive function g on G, $g(a) = \sum_{p^k \| a} g(p^k)$ $(a \in G)$, can be identified with a sum $\bar{g} = \sum_{p} \bar{X}_p$ of independent random variables on Ω. The main result will be that the existence of the limit distribution of a real-valued additive function g is equivalent to the a.e. convergence of $\bar{g}$. And finally, we characterize the essentially distributed additive functions in case that the generating function Z of G belongs to a subclass of the $exp - log$ *class* $\mathcal{F}$ [1].

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Some results on simultaneous number systems in the ring of integers of imaginary quadratic fields

Let Λ be a lattice in $\mathbb{R}^n$, $M : \Lambda \to \Lambda$ be a linear operator such that $\det(M) \neq 0$, and let D be a finite subset of Λ containing 0. The triple (Λ, M, D) is called a *generalized number system* (GNS) if every element x of Λ has a unique, finite representation of the form $x = \sum_{i=0}^{l} M^i d_i$, where $d_i \in D$, $l \in \mathbb{N}$ and $d_l \neq 0$.

If (Λ, M, D) is a number system then (1) D must be a full residue system modulo M, (2) M must be expansive and (3) $\det(I - M) \neq \pm 1$. If a system fulfils these conditions then it is a *radix system* and the operator M is called a *radix base*.

In [2] block diagonal systems were introduced and investigated. Let the radix systems (Λ_i, M_i, D_i) be given $(1 \leq i \leq k)$. Let $\Lambda = \otimes \Lambda_i$ the direct product of the lattices, $M = \oplus_{i=1}^{k} M_i$ the direct sum of the bases and $D_h = \{(d_1^T \| d_2^T \| \cdots \| d_k^T)^T \ : \ d_i \in \in D_i\}$ the homomorphic digit set. Here d^T is a row vector and $\|$ means the concatenation operator. **Kátai** et. al. [1] introduced the notion of simultaneous radix systems when the blocks $N_1, N_2, \dots, N_k$ are mutual coprime integers (none of them is $0, \pm 1$) and $D = \{\delta e\}$ where $e = (1, 1, \dots, 1)^T$, $\delta = 1, 2, \dots, |N_1 N_2 \cdots N_k| - 1$. The computation of the expansions are based on Chinese Remaindering. **Kátai** et al. showed that the system $(\mathbb{Z}^2, N_1 \oplus N_2, D)$ is GNS if and only if $N_1 < N_2 \leq -2$ and $N_2 = N_1 + 1$.

The simultaneous radix expansions were also investigated in the lattice of Gaussian integers [4, 2, 3]. In this talk we present the newest results achieved in the research of simultaneous number expansions in the ring of integers of imaginary quadratic fields.

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On semantic descriptions of software systems

Formal methods are essential for giving precise descriptions of software systems. In our talk we analysed some approaches to conventional semantics and to action semantics. As a result we suggest to use action semantics for describing semantic properties of software systems including programming and MDE languages.

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Characterization of a bivariate distribution through a functional equation

We deal with a special characterization problem of a conditionally specified absolutely continuous bivariate distribution by the help of a functional equation satisfied almost everywhere on its domain for the unknown density functions. We consider the case of conditionals in location-scale families with specified moments (we have linear regressions and conditional standard deviations).

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Hybrid universality of certain composite functions involving Dirichlet L-functions

We generalize a hybrid joint universality theorem for Dirichlet L-functions $L(s,\chi)$ obtained in [1].

Let $D = \left\{s \in \mathbb{C} : \frac{1}{2} < \sigma < 1\right\}$. Denote by $H(D)$ the space of analytic functions on D equipped with the topology of uniform convergence on compacta. Moreover, denote by $\mathcal{K}$ the class of compact subsets of D with connected complements, and let $H(K)$, $K \in \mathcal{K}$, stand for the class of continuous functions on K which are analytic in the interior of K.

Let $\beta_1 > 0, \dots, \beta_r > 0$, and $\beta = \min\limits_{1 \le j \le r} \beta_j$. We say that the operator $F : H^r(D) \to H(D)$ belongs to the Lipschitz class $Lip(\beta_1, \dots, \beta_r)$ if the following hypotheses are satisfied:

1° For each polynomial $p = p(s)$, and any $K \in \mathcal{K}$, there exists an element $(g_1, \dots, g_r) \in F^{-1}\{p\} \subset H^r(D)$ such that $g_j(s) \neq 0$ on K, $j = 1, \dots, r$;

2° For any $K \in \mathcal{K}$, there exist a constant $c > 0$, and a set $\hat{K} \in \mathcal{K}$ such that

$$\sup_{s \in K} |F(g_{11}(s), \dots, g_{1r}(s)) - F(g_{21}(s), \dots, g_{2r}(s))| \le c \sup_{1 \le j \le r} \sup_{s \in \hat{K}} |g_{1j}(s) - g_{2j}(s)|^{\beta_j}$$

for all $(g_{k1}, \dots g_{kr}) \in H^r(D)$, $k = 1, 2$.

In the report, we will present the following theorem.

Theorem 1. *Suppose that $\chi_1, \dots, \chi_r$ are pairwise non-equivalent Dirichlet characters, and that $F \in Lip(\beta_1, \dots, \beta_r)$. Let $K \in \mathcal{K}$ and $f(s) \in H(K)$. Moreover, let $\{\alpha_j : j = 1, \dots, m\}$ be any sequence of real numbers linearly independent over $\mathbb{Q}$ and $\{\theta_j : j = 1, \dots, m\}$ be any sequence of real numbers. Then, for every $\varepsilon > 0$,*

$$\liminf_{T \to \infty} \frac{1}{T} \operatorname{meas} \left\{ \tau \in [0, T] : \sup_{s \in K} |F(L(s + i\tau, \chi_1), \dots, L(s + i\tau, \chi_r)) - f(s)| < \varepsilon, \right.$$
$$\left. \max_{1 \le j \le m} \|\tau\alpha_j - \theta_j\| < \left(\frac{\varepsilon}{2}\right)^{\frac{1}{\beta}} \right\} > 0.$$

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A (0,2)-type Pál interpolation problem

In 1975 L. Pál introduced a modification of Hermite-Fejér interpolation in which the function values and the first derivatives were prescribed on two sets of nodes $\{x_i\}$ and $\{x_i^*\}$, respectively. In [2] he investigated the case when $p_n(x_i) = 0$ for $i = 1, \ldots, n$ and $p_n'(x_i^*) = 0$ for $i = 1, \ldots, n-1$, where p_n is a polynomial of degree n with distinct zeros. In 1983 L. Szili [3] studied the inverse problem when the roles of p_n and p_n' are interchanged, namely, the first derivatives are interpolated at the zeros of p_n and the function values are interpolated at the zeros of p_n'. Several authors studied these two problems, the Pál-type (0;1)- and (1;0)-interpolation using different choices of nodes.

We consider the Pál interpolation with other kind of interpolation conditions. Instead of the Lagrange condition (only the function values are given) we have Hermite conditions (consecutive derivatives up to a higher order are prescribed). The lacunary condition we substitute with the weighted (0,1,...,r-2,r)-interpolation ($r \geq 2$), the generalization of P. Turán's problem [4], introduced by J. Balázs [1]. We study the regularity of the problem and the explicit representation of the fundamental polynomials of interpolation. We give some examples on the zeros of the classical orthogonal polynomials.

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Solved and unsolved problems for Páles means

Given two continuous functions $f, g : I \to \mathbb{R}$ on the nonempty open interval I such that g is positive and f/g is strictly monotone and a probability measure μ on the Borel subsets of $[0,1]$, the two variable mean $M_{f,g;\mu} : I^2 \to I$, called Páles mean, is defined by

$$M_{f,g;\mu}(x,y) := \left(\frac{f}{g}\right)^{-1} \left(\frac{\int_0^1 f\big(tx+(1-t)y\big)\,d\mu(t)}{\int_0^1 g\big(tx+(1-t)y\big)\,d\mu(t)} \right) \qquad (x, y \in I).$$

The aim of our talk is to report on solved and unsolved problems (among others equality, homogeneity, comparison problems) of these means.

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Some remarks and problems related to real derivations

A real derivation is a function $d : \mathbb{R} \to \mathbb{R}$ (the set of all real numbers) for which the functional equations

$$d(x+y) = d(x) + d(y) \qquad (x, y \in \mathbb{R}) \tag{1}$$

and

$$d(xy) = xd(y) + yd(x) \qquad (x, y \in \mathbb{R}) \tag{2}$$

hold simultaneously for all $x, y \in \mathbb{R}$. The solutions of equation (1) are called additive functions and it is also known (see [2]) that there is non-zero additive function d which fulfils (2), too. This makes possible to construct surprising counter-examples by using real derivations.

In this talk, firstly we show such a counter-example (see [1]) and then we present some characterizations of real derivations, finally we give sufficient conditions for additive functions to obtain that they are real derivations. Open problems will also be formulated.

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Additive function theory on permutations

Influenced by probabilistic number theory (in particular, by the works of the hero of the day Professor Imre Kátai and other hungarian mathematicians), we are dealing with the value distribution of additive functions defined on permutations. By definition, a real two-dimensional array $\{h_j(s)\}$, where $j \geq 1$, $s \geq 0$, and $h_j(0) \equiv 0$, via expression

$$h(\sigma) := \sum_{j \leq n} h_j\big(k_j(\sigma)\big),$$

determines an additive function $h : \mathbb{S}_n \to \mathbb{R}$. Here $\mathbb{S}_n$ is the symmetric group of order $n \in \mathbb{N}$ and $k_j(\sigma) \in \mathbb{Z}_+$ denotes the number of cycles of length j in the permutation $\sigma \in \mathbb{S}_n$. Assume that σ is taken at random with a probability

$$\nu_n^{(\theta)}\big(\{\sigma\}\big) := \Lambda_n^{-1} \prod_{j=1}^{n} \theta_j^{k_j(\sigma)}, \quad \sigma \in \mathbb{S}_n,\ 0^0 := 1,$$

where $\theta_j \geq 0$ if $j \leq n$ and Λ_n is an appropriate normalizing sequence. The main objective is to find general conditions under which there exist sequences $\alpha(n) \in \mathbb{R}$ and $\beta(n) > 0$ such that $\nu_n^{(\theta)}\big(h(\sigma) - \alpha(n) < x\beta(n)\big)$ weakly converges to a non-degenerate distribution function as $n \to \infty$. The case $\theta_j \equiv \theta > 0$ corresponds to the Ewens probability measure on $\mathbb{S}_n$. The interest to deal with it and more general weighted measures has been raised by some recent papers in physics.

By now, we have advanced in applying a probabilistic approach (see [1]) and a method of factorial moments (see [2]). This will be discussed in the talk.

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Invariance formula for generalized quasi-arithmetic means

A invariance formula in the class of generalized p-variable quasiarithmetic mean and an effective form of the limit of sequence of iterates of the mean-type mappings of this type will be given. Some examples and application in solving a functional equation will be presented.

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Joint limit theorems for periodic Hurwitz zeta-function II.

Let $s = \sigma + it$ be a complex variable, α, $0 < \alpha \leqslant 1$, be a fixed parameter, and $\mathfrak{a} = \{a_m : m \in \mathbb{N} = \mathbb{N} \cup \{0\}$ be a periodic sequence of complex numbers with minimal period $k \in \mathbb{N}$. The periodic Hurwitz zeta-function $\zeta(s, \alpha; \mathfrak{a})$ is defined, for $\sigma > 1$, by the series

$$\zeta(s, \alpha, \mathfrak{a}) = \sum_{m=0}^{\infty} \frac{a_m}{(m + \alpha)^s},$$

and continues analytically to the whole complex plane, except, maybe, for a simple pole at the point $s = 1$.

In [1], two joint limit theorems on the weak convergence of probability measures on the complex plane for periodic Hurwitz zeta-functions were proved. For $j = 1, \dots, r$, let $\zeta(s, \alpha_j, \mathfrak{a}_j)$ be a periodic Hurwitz zeta-function with parameter α_j, $0 < \alpha_j \leqslant 1$, and periodic sequence of complex numbers $\mathfrak{a}_j = \{a_{mj} : m \in \mathbb{N}_0\}$ with minimal period $k_j \in \mathbb{N}$. For brevity, we use the notation $\underline{\sigma} = (\sigma_1, \dots, \sigma_r)$, $\underline{\sigma} + it = (\sigma_1 + it, \dots, \sigma_r + it)$, $\underline{\alpha} = (\alpha_1, \dots, \alpha_r)$, $\underline{\mathfrak{a}} = (\mathfrak{a}_1, \dots, \mathfrak{a}_r)$ and $\zeta(s, \underline{\alpha}; \mathfrak{a}) = \big(\zeta(s, \alpha_1, ; \mathfrak{a}_1), \dots \zeta(s, \alpha_r; \mathfrak{a}_r)\big)$. Denote by $\mathcal{B}(S)$ the class of Borel sets of the space S, and by meas$A$ the Lebesgue measure of a measurable set $A \subset \mathbb{R}$. Then in [5], the weak convergence as $T \to \infty$ of the probability measure

$$\widehat{P}_T(A) \overset{def}{=} \frac{1}{T}\text{meas}\big\{t \in [0, T] : \underline{\zeta}(\underline{\sigma} + it, \underline{\alpha}; \underline{\mathfrak{a}}) \in A\big\}, \quad A \in \mathcal{B}(\mathbb{C}^r)),$$

was discussed. The cases of algebraically independent and rational parameters $\alpha_1, \dots, \alpha_r$ were considered.

The aim of this note is to consider the weak convergence of the probability measure

$$P_T(A) = \frac{1}{T}\text{meas}\big\{t \in [0, T] : \underline{\underline{\zeta}}(\underline{\underline{\sigma}} + it, \underline{\underline{\alpha}}, \underline{\underline{\mathfrak{a}}}) \in A\big\}, \quad A \in \mathcal{B}(\mathbb{C}^{r+r_1}).$$

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Sequential isomorphisms and endomorphisms of Hilbert space effect algebras

Let $\mathcal{A}$ be a C^*-algebra. The effect algebra $E(\mathcal{A})$ of $\mathcal{A}$ consists of the self-adjoint elements a for which $0 \leq a \leq 1$. Effects play important role in the mathematical descriptions of the theory of quantum measurements. The sequential product on $E(\mathcal{A})$ is defined by $a \circ b = \sqrt{a}b\sqrt{a}$.

In this talk we first recall a former result of ours on the structure of sequential isomorphisms between effect algebras in the setting of von Neumann algebras. We next present a recent result that gives the general form of all (continuous) sequential endomorphisms of finite dimensional Hilbert space effect algebras.

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Number systems of the Gaussian integers with modified canonical digit sets

I. Kátai and J. Szabó determined in [1] all possible bases of canonical number systems of the Gaussan integers. Namely they proved that $\alpha \in \mathbb{Z}[i]$ is the base of a canonical number system if and only if $Re(\alpha) < 0$ and $Im(\alpha) = \pm 1$. The aim of the talk is to show how we can get number systems with the modification of the canonical digit set.

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Generalized Aczél–Jabotinsky differential equations, Briot–Bouquet equations and implicit functions

We consider the differential equation

$$H(X) \cdot \frac{\mathrm{d}\phi}{\mathrm{d}X}(X) = G(\phi(X)) \qquad \text{(AJ,(H,G))}$$

where $H(X)$ and $G(X)$ are given formal power series of the form $H(X) = h_k X^k + \ldots$, $G(X) = g_k X^k + \ldots$, with $h_k, g_k \neq 0$, $k \geq 1$, and the solution ϕ is an invertible power series $\phi(X) = \rho X + \ldots$.
We discuss the origin of (AJ,(H,G)) in functional equation problems from iteration theory, the theory of families of commuting power series, and from the theory of reversibility. Then we present necessary and sufficient conditions for the existence of solutions $\phi \neq 0$ and give a theorem an the structure of the set of solutions of (AJ,(H,G)) by transforming (AJ,(H,G)) to a so called Briot-Bouquet differential equation in the non-generic case. The cases $k = 1$ and $k \geq 2$ require different approaches. From now on we assume $k = 1$. Then we solve (AJ,(H,G)) also by transforming it to an implicit function problem. From this we easily deduce the dependence of the solutions on the internal parameter ρ and show that the solutions are local analytic functions of (ρ, X), if H and G are both convergent. If $H = G$ we obtain the so called standard form $\phi(\rho, X) = S^{-1}(\rho S(X))$ of the solution of the translation equation, the case $G(X) = h_k X$ yields the transformation of (AJ,(H,H)) to its normal form.
Our research is motivated by joint work with H. Fripertinger and W. Jabloński.

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(joint work with **Agata Nowak**)

Chini's equations in actuarial mathematics

We deal with an equation mentioned by M. Chini in [1] and recalled by A. Guerraggio in [2]. This is a multiplicative form of the equation previously studied by T. Riedel, P. K. Sahoo and M. Sablik (cf. [3]). We solve it completely in the case where sets of zeros of the unknown functions are nonempty, under very mild assumptions. In the remaining case, we reduce the problem to the one considered in [3].

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Exponential polynomials on Abelian groups

Exponential monomials and polynomials on Abelian groups play a basic role in the theory of functional equations and spectral synthesis. In this talk we characterize these functions by the properties of the annihilator ideal of the variety generated by them. In particular, we obtain new characterization of polynomial functions on Abelian groups.

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Collision and avalanche effect in pseudorandom sequences

Recently a constructive theory of pseudorandomness of binary sequences has been developed and many constructions for binary sequences with strong pseudorandom properties have been given. Motivated by applications, Mauduit and Sárközy in [1] generalized and extended this theory from the binary case to k-ary sequences, i.e., to k symbols. They constructed a large family of k-ary sequences with strong pseudorandom properties. I adapted the notions of collision and avalanche effect to study these pseudorandom properties of families of binary and k-ary sequences. Two of the most important binary constructions were tested in [2] for these pseudorandom properties, and it turned out that one of them is ideal from this point of view as well, while the other construction does not possess these pseudorandom properties. In another paper [3] it is shown that this weakness of the second construction can be corrected: one can take a subfamily of the given family which is just slightly smaller and collision free. The study of the pseudorandom properties mentioned above was extended to k-ary sequences in [4]. The aim of this paper is twofold. First the definitions of collision and avalanche effect were extended to k-ary sequences, and then these related properties were studied in a large family of pseudorandom k-ary sequences with "small" pseudorandom measures.

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A conjecture of Petruska and a conjecture of Widom

The talk will discuss with some historical background a conjecture of Petruska about nonconstant solutions of integral equations and a conjecture of Widom on the behavior of Chebyshev numbers. Both conjectures turn out to be false.

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(joint work with **Sergey Varbanets**)

Twisted exponential sums and the distribution of solutions of the congruence $f(x, y) \equiv 0 \pmod{p^m}$ *over* $\mathbb{Z}[i]$

Let $G = \mathbb{Z}[i]$ be the ring of Gaussian integer numbers. For $\gamma \in G$ we state G_γ(respectively, G_γ^*) for residue (reduced residue) system modulo γ. Let χ be an arbitrary multiplicative character modulo γ. We will obtain estimations of the twisted exponential sums

$$S_\chi(f, \gamma) = \sum_{x \in G_\gamma^*} \chi(x) e^{2\pi i \Re\left(\frac{f(x)}{\gamma}\right)},$$

where f is a rational function over G.

In particular, we give estimations for the twisted Kloosterman sum

$$K_\chi(\alpha, \beta; \gamma^m) := \sum_{\substack{x,y \in G \\ xy \equiv 1 \pmod{\gamma}}} \chi(x) e^{2\pi i \Re\left(\frac{\alpha x + \beta y}{\gamma}\right)}.$$

Also we study the norm Kloosterman sum

$$K(\alpha_1, \alpha_2, \alpha_3; q) := \sum_{\substack{x_1,x_2,x_3 \in G_q^* \\ N(x_1 x_2 x_3) \equiv 1 \pmod{q}}} e^{2\pi i \Re\left(\frac{\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3}{q}\right)}$$

and the character sum over norm group

$$\sum_{\substack{u \in G_{p^n} \\ N(u) \equiv 1 \pmod{p^n}}} \chi(\Phi(u)) e_{p^n}(\Psi(u)),$$

where p is a rational prime number, $\Phi(u), \Psi(u) \in G_{p^n}[u]$.

There is an applications of the estimations of such sums in problem on distribution of solutions of the special congruences over G are given. In particular, we have obtained an asymptotic formula for the number of solutions of the congruence $y^\ell \equiv f(x) \pmod{\mathfrak{p}^m}$ in the incomplete residue system, where $\ell, m \in \mathbb{N}$, $\mathfrak{p}$ is a Gaussian prime number, $f(x) \in G[x]$.

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Speeding up distributed sieving with the inverse sieve

The efficiency of sieving algorithms depends on memory management, especially when the sieve table is very large. Furthermore, a naive parallel or distributed approach does not help, because the operations are pretty simple, and one does not gain anything by distributing the workload among processing units, the memory bus is always congested.

To reduce inter processor communication, the sieve is compressed and an appropriate sieving method is used to reduce the bulk of the work. This approach can be thought of as a kind of inverse sieve, because it removes the larger sieving primes instead of the candidates in the sieve table.

Notes