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Additive function theory on permutations

Influenced by probabilistic number theory (in particular, by the works of the hero of the day Professor Imre Kátai and other hungarian mathematicians), we are dealing with the value distribution of additive functions defined on permutations. By definition, a real two-dimensional array $\{h_j(s)\}$, where $j \geq 1$, $s \geq 0$, and $h_j(0) \equiv 0$, via expression

$$h(\sigma) := \sum_{j \leq n} h_j\big(k_j(\sigma)\big),$$

determines an additive function $h : \mathbb{S}_n \to \mathbb{R}$. Here $\mathbb{S}_n$ is the symmetric group of order $n \in \mathbb{N}$ and $k_j(\sigma) \in \mathbb{Z}_+$ denotes the number of cycles of length j in the permutation $\sigma \in \mathbb{S}_n$. Assume that σ is taken at random with a probability

$$\nu_n^{(\theta)}\big(\{\sigma\}\big) := \Lambda_n^{-1} \prod_{j=1}^{n} \theta_j^{k_j(\sigma)}, \quad \sigma \in \mathbb{S}_n,\ 0^0 := 1,$$

where $\theta_j \geq 0$ if $j \leq n$ and Λ_n is an appropriate normalizing sequence. The main objective is to find general conditions under which there exist sequences $\alpha(n) \in \mathbb{R}$ and $\beta(n) > 0$ such that $\nu_n^{(\theta)}\big(h(\sigma) - \alpha(n) < x\beta(n)\big)$ weakly converges to a non-degenerate distribution function as $n \to \infty$. The case $\theta_j \equiv \theta > 0$ corresponds to the Ewens probability measure on $\mathbb{S}_n$. The interest to deal with it and more general weighted measures has been raised by some recent papers in physics.

By now, we have advanced in applying a probabilistic approach (see [1]) and a method of factorial moments (see [2]). This will be discussed in the talk.

References

[1] **Manstavičius, E.,** On total variation distance for random assemblies, *Discrete Math. and Theor. Computer Sci.*, AofA'12 Proc., 97–108 (2012).

[2] **Bakšajeva, T. and E. Manstavičius,** On statistics of permutations chosen from the Ewens distribution (submitted; *arXiv*:0677678, 2013).