Komputeralgebrai algoritmusok

Járai Antal

Ezek a programok csak szemléltetésre szolgálnak.

▶ 1. Történet

▶ 2. Algebrai alapok

▼ 3. Normál formák, reprezentáció

```
> restart;
```

▼ E 3.1. Példa.

```
> a:=(3*x^2-1)*(5*x+2)+7*x^4-5*x^3+7*x^6;
```

$$a := \left(3\,x^2-1\right)\left(5\,x+2\right)+7\,x^4-5\,x^3+7\,x^6 \tag{3.1.1}$$

```
> a:=expand(a);
```

$$a := 10\,x^3+6\,x^2-5\,x-2+7\,x^4+7\,x^6 \tag{3.1.2}$$

```
> sort(a);
```

$$7\,x^6+7\,x^4+10\,x^3+6\,x^2-5\,x-2 \tag{3.1.3}$$

▼ E 3.2. Példa.

```
> a:=(3*y^2+(-2*z^3)*y+5*z^2)*x^2+4*x+((-6*z+1)*y^3+3*y^2+
  (z^4+1));
```

$$a := \left(3\,y^2-2\,z^3\,y+5\,z^2\right)x^2+4\,x+\left(-6\,z+1\right)y^3+3\,y^2+z^4+1 \tag{3.2.1}$$

```
> b:=expand(a);
```

$$b := 3\,x^2\,y^2-2\,x^2\,z^3\,y+5\,x^2\,z^2+4\,x-6\,y^3\,z+y^3+3\,y^2+z^4+1 \tag{3.2.2}$$

```
> sort(b,[x,y,z],plex);
```

$$3\,x^2\,y^2-2\,x^2\,y\,z^3+5\,x^2\,z^2+4\,x-6\,y^3\,z+y^3+3\,y^2+z^4+1 \tag{3.2.3}$$

▼ E 3.3. Példa.

```
> a:=((x^2-x*y+x)+(x^2+3)*(x-y+1))*((y^3-3*y^2-9*y-5)+x^4*
  (y^2+2*y+1));
```

(3.3.1)

$$a := \left(x^2 - x\,y + x + \left(x^2 + 3\right)\left(x - y + 1\right)\right)\left(y^3 - 3\,y^2 - 9\,y - 5 + x^4\left(y^2 + 2\,y + 1\right)\right) \tag{3.3.1}$$

```
> b:=expand(a);
```

$$b := -15 - 10\,x^2 + 3\,x^4 - 5\,x^3 + 2\,x^6 - 20\,x - 12\,y + 18\,y^2 + 12\,y^3 + 3\,x^2\,y^2 - 31\,x\,y - 13\,x^2\,y - 3\,x^4\,y^2 + 3\,x^4\,y + 5\,x^2\,y^3 + 3\,x^6\,y - x\,y^4 + 7\,x\,y^3 - 3\,x\,y^2 - x^5\,y^3 + 2\,x^5\,y^2 + 7\,x^5\,y + x^3\,y^3 - 3\,x^3\,y^2 - 9\,x^3\,y + x^7\,y^2 + 2\,x^7\,y - x^2\,y^4 - x^6\,y^3 - 3\,x^4\,y^3 + 4\,x^5 + x^7 - 3\,y^4 \tag{3.3.2}$$

```
> sort(b,[x,y],plex);
```

$$x^7\,y^2 + 2\,x^7\,y + x^7 - x^6\,y^3 + 3\,x^6\,y + 2\,x^6 - x^5\,y^3 + 2\,x^5\,y^2 + 7\,x^5\,y + 4\,x^5 - 3\,x^4\,y^3 - 3\,x^4\,y^2 + 3\,x^4\,y + 3\,x^4 + x^3\,y^3 - 3\,x^3\,y^2 - 9\,x^3\,y - 5\,x^3 - x^2\,y^4 + 5\,x^2\,y^3 + 3\,x^2\,y^2 - 13\,x^2\,y - 10\,x^2 - x\,y^4 + 7\,x\,y^3 - 3\,x\,y^2 - 31\,x\,y - 20\,x - 3\,y^4 + 12\,y^3 + 18\,y^2 - 12\,y - 15 \tag{3.3.3}$$

```
> c:=expand(op(1,a))*expand(op(2,a));
```

$$c := \left(2\,x^2 - x\,y + 4\,x + x^3 - x^2\,y - 3\,y + 3\right)\left(y^3 - 3\,y^2 - 9\,y - 5 + x^4\,y^2 + 2\,x^4\,y + x^4\right) \tag{3.3.4}$$

```
> factor(a);
```

$$\left(x + x^2 + 3\right)\left(x - y + 1\right)\left(y + 1\right)^2\left(y + x^4 - 5\right) \tag{3.3.5}$$

E 3.4. Példa.

```
> a:=3*x^2*y^2-2*x^2*y*z^3+5*x^2*z^2+4*x-z^4+1;
```

$$a := 3\,x^2\,y^2 - 2\,x^2\,y\,z^3 + 5\,x^2\,z^2 + 4\,x - z^4 + 1 \tag{3.4.1}$$

```
> collect(a,[x,y,z],`recursive`);
```

$$\left(3\,y^2 - 2\,z^3\,y + 5\,z^2\right)x^2 + 4\,x - z^4 + 1 \tag{3.4.2}$$

E 3.5. Példa.

4. Aritmetika

5. Kínai maradékolás

6. Newton–iteráció, Hensel–felemelés

- 7. Legnagyobb közös osztó
- 8. Faktorizálás
- 9. Egyenletrendszerek
- 10. Gröbner–bázisok
- 11. Racionális törtfüggvények integrálása
- 12. A Risch–algoritmus.