Komputeralgebrai algoritmusok

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Ezek a programok csak szemléltetésre szolgálnak.

▶ 1. Történet

▶ 2. Algebrai alapok

▶ 3. Normál formák, reprezentáció

▶ 4. Aritmetika

▶ 5. Kínai maradékolás

▶ 6. Newton–iteráció, Hensel–felemelés

▶ 7. Legnagyobb közös osztó

▶ 8. Faktorizálás

▶ 9. Egyenletrendszerek

▶ 10. Gröbner–bázisok

▶ 11. Racionális törtfüggvények integrálása

▼ 12. A Risch–algoritmus.

```
> restart;
> (x*(x+1)*((x^2*exp(2*x^2)-(log(x+1))^2)^2+2*x*exp(3*x^2)*(x-(2*
  x^3+2*x^2+x+1)*log(x+1))))/((x+1)*(log(x+1))^2-(x^3+x^2)*exp(2*
  x^2))^2;
```

$$\frac{1}{\left((x+1)\ln(x+1)^2-(x^3+x^2)\,e^{2x^2}\right)^2}\left(x(x+1)\left(\left(x^2e^{2x^2}-\ln(x+1)^2\right)^2\right.\right. \qquad (12.1)$$
$$\left.\left.+2xe^{3x^2}\left(x-\left(2x^3+2x^2+x+1\right)\ln(x+1)\right)\right)\right)$$

```
> int(%,x);
```

$$x-\ln(x+1)+\frac{e^{x^2}x\ln(x+1)}{x^2e^{2x^2}-\ln(x+1)^2}+\frac{1}{2}\ln\left(e^{x^2}x+\ln(x+1)\right)-\frac{1}{2}\ln\left(-e^{x^2}x+\ln(x+1)\right) \tag{12.2}$$

```
> int(1/(1+exp(x)),x); simplify(%,symbolic);
```

$$-\ln(1+e^x)+\ln(e^x)$$

$$-\ln(1+e^x)+x \tag{12.3}$$

```
> int(x/(1+exp(x)),x);
```

$$\frac{1}{2}x^2-x\ln(1+e^x)-\mathrm{polylog}(2,-e^x) \tag{12.4}$$

```
> int(1/(1+x^2),x); convert(%,ln);
```

$$\arctan(x)$$

$$\frac{1}{2}I\left(\ln(1-Ix)-\ln(1+Ix)\right) \tag{12.5}$$

```
> int(cos(x),x); convert(%,exp);
```

$$\sin(x)$$

$$-\frac{1}{2}I\left(e^{Ix}-\frac{1}{e^{Ix}}\right) \tag{12.6}$$

```
> int(1/sqrt(1-x^2),x); convert(%,ln);
```

$$\arcsin(x)$$

$$-I\ln\left(\sqrt{1-x^2}+Ix\right) \tag{12.7}$$

```
> int(arccosh(x),x); convert(arccosh(x),ln); int(%,x);
```

$$x\,\mathrm{arccosh}(x)-\sqrt{x-1}\,\sqrt{x+1}$$

$$\ln\left(x+\sqrt{x-1}\,\sqrt{x+1}\right)$$

$$\ln\left(x+\sqrt{x-1}\,\sqrt{x+1}\right)x-\sqrt{x-1}\,\sqrt{x+1} \tag{12.8}$$

E 12.1. Példa.

```
> f:=exp(x)+exp(2*x)+exp(x/2); int(f,x);
```

$$f:=e^x+e^{2x}+e^{\frac{1}{2}x}$$

$$e^x+\frac{1}{2}e^{2x}+2e^{\frac{1}{2}x} \tag{12.1.1}$$

E 12.2. Példa.

```
> g:=sqrt(log(x^2+3*x+2)*(log(x+1)+log(x+2)));
```

(12.2.1)

$$g := \sqrt{\ln\left(x^2+3\,x+2\right)\left(\ln\left(x+1\right)+\ln\left(x+2\right)\right)} \tag{12.2.1}$$

E 12.3. Példa.

```
> h:=exp(ln(x)/2);
```

$$h := \sqrt{x} \tag{12.3.1}$$

E 12.4. Példa.

```
> g:=log(sqrt(x^2+1)+x)+log(sqrt(x^2+1)-x); theta1:=sqrt(x^2+1)
  ;
  h:=(theta1-x)^k*(theta1+x)^k1; diff(h,x)/h; simplify(%);
```

$$g := \ln\left(\sqrt{x^2+1}+x\right)+\ln\left(\sqrt{x^2+1}-x\right)$$

$$\theta 1 := \sqrt{x^2+1}$$

$$h := \left(\sqrt{x^2+1}-x\right)^k\left(\sqrt{x^2+1}+x\right)^{k1}$$

$$\frac{1}{\left(\sqrt{x^2+1}-x\right)^k\left(\sqrt{x^2+1}+x\right)^{k1}}\left(\frac{\left(\sqrt{x^2+1}-x\right)^k k\left(\frac{x}{\sqrt{x^2+1}}-1\right)\left(\sqrt{x^2+1}+x\right)^{k1}}{\sqrt{x^2+1}-x}+\frac{\left(\sqrt{x^2+1}-x\right)^k\left(\sqrt{x^2+1}+x\right)^{k1} k1\left(\frac{x}{\sqrt{x^2+1}}+1\right)}{\sqrt{x^2+1}+x}\right)$$

$$-\frac{k-k1}{\sqrt{x^2+1}} \tag{12.4.1}$$

E 12.5. Példa.

```
> exp(x^2+log(x)/2)/(2*(sqrt(x)-1))+exp(x^2+log(x)/2)/(2*(sqrt
  (x)+1));
  diff(%,x); simplify(%);
```

$$\frac{e^{x^2+\frac{1}{2}\ln(x)}}{2\sqrt{x}-2}+\frac{e^{x^2+\frac{1}{2}\ln(x)}}{2\sqrt{x}+2}$$

$$\frac{\left(2x+\frac{1}{2x}\right)e^{x^2+\frac{1}{2}\ln(x)}}{2\sqrt{x}-2}-\frac{e^{x^2+\frac{1}{2}\ln(x)}}{\left(2\sqrt{x}-2\right)^2\sqrt{x}}+\frac{\left(2x+\frac{1}{2x}\right)e^{x^2+\frac{1}{2}\ln(x)}}{2\sqrt{x}+2}-\frac{e^{x^2+\frac{1}{2}\ln(x)}}{\left(2\sqrt{x}+2\right)^2\sqrt{x}}$$
$$\frac{e^{x^2}\left(2x^3-2x^2-1\right)}{\left(\sqrt{x}-1\right)^2\left(\sqrt{x}+1\right)^2} \tag{12.5.1}$$

```
> int((2*x^3-2*x^2-1)/(x-1)^2*exp(x^2),x);
```

$$\frac{x\,e^{x^2}}{x-1} \tag{12.5.2}$$

E 12.6. Példa.

```
> int(1/ln(x),x);
```

$$-\mathrm{Ei}(1,-\ln(x)) \tag{12.6.1}$$

```
> resultant(1-z/x,theta,theta);
```

$$\frac{x-z}{x} \tag{12.6.2}$$

E 12.7. Példa.

```
> int(1/x/ln(x),x);
```

$$\ln(\ln(x)) \tag{12.7.1}$$

```
> resultant(1/x-z/x,theta,theta);
```

$$-\frac{-1+z}{x} \tag{12.7.2}$$

```
> oR:=1-z;
```

$$oR := 1-z \tag{12.7.3}$$

```
> v1:=gcd(1/x-1/x,theta);
```

$$v1 := 0 \tag{12.7.4}$$

E 12.8. Példa.

```
> (x*(x+1)*((x^2*exp(2*x^2)-(log(x+1))^2)^2+2*x*exp(3*x^2)*(x-
  (2*x^3+2*x^2+x+1)*log(x+1))))/((x+1)*(log(x+1))^2-(x^3+x^2)*
  exp(2*x^2))^2;
```

$$\frac{1}{\left((x+1)\ln(x+1)^2-\left(x^3+x^2\right)e^{2x^2}\right)^2}\left(x(x+1)\left(\left(x^2e^{2x^2}-\ln(x+1)^2\right)^2\right.\right. \tag{12.8.1}$$

$$+2\,x\,e^{3x^2}\left(x-\left(2x^3+2x^2+x+1\right)\ln(x+1)\right)\Big)\Big)$$

```
> int(%,x);
```

$$x-\ln(x+1)+\frac{e^{x^2}x\ln(x+1)}{x^2e^{2x^2}-\ln(x+1)^2}-\frac{1}{2}\ln\left(\ln(x+1)-xe^{x^2}\right)+\frac{1}{2}\ln\left(xe^{x^2}+\ln(x+1)\right) \tag{12.8.2}$$

```
> subs(exp(x^2)=theta1,%%);
```

$$\frac{1}{\left((x+1)\ln(x+1)^2-\left(x^3+x^2\right)e^{2x^2}\right)^2}\Big(x(x+1)\Big(\left(x^2e^{2x^2}-\ln(x+1)^2\right)^2+2\,xe^{3x^2}\left(x-\left(2x^3+2x^2+x+1\right)\ln(x+1)\right)\Big)\Big) \tag{12.8.3}$$

```
> subs(exp(2*x^2)=theta1^2,%);
```

$$\frac{1}{\left((x+1)\ln(x+1)^2-\left(x^3+x^2\right)\theta1^2\right)^2}\Big(x(x+1)\Big(\left(x^2\theta1^2-\ln(x+1)^2\right)^2+2\,xe^{3x^2}\left(x-\left(2x^3+2x^2+x+1\right)\ln(x+1)\right)\Big)\Big) \tag{12.8.4}$$

```
> subs(exp(3*x^2)=theta1^3,%);
```

$$\frac{1}{\left((x+1)\ln(x+1)^2-\left(x^3+x^2\right)\theta1^2\right)^2}\Big(x(x+1)\Big(\left(x^2\theta1^2-\ln(x+1)^2\right)^2+2\,x\theta1^3\left(x-\left(2x^3+2x^2+x+1\right)\ln(x+1)\right)\Big)\Big) \tag{12.8.5}$$

```
> f:=subs(ln(x+1)=theta2,%);
```

$$f:=\frac{x(x+1)\left(\left(x^2\theta1^2-\theta2^2\right)^2+2\,x\theta1^3\left(x-\left(2x^3+2x^2+x+1\right)\theta2\right)\right)}{\left((x+1)\theta2^2-\left(x^3+x^2\right)\theta1^2\right)^2} \tag{12.8.6}$$

```
> convert(f,parfrac,theta2,sqrfree);
```

$$\frac{x}{x+1}-\frac{2x^2\theta1^3\left(2\theta2x^3+2\theta2x^2+\theta2x+\theta2-x\right)}{\left(-x^2\theta1^2+\theta2^2\right)^2(x+1)} \tag{12.8.7}$$

```
> a:=(2*x^2+1)*theta1*theta2-x/(x+1)*theta1;
```

$$a:=\left(2x^2+1\right)\theta1\,\theta2-\frac{x\,\theta1}{x+1} \tag{12.8.8}$$

```
> b:=theta2^2-x^2*theta1^2;
```

$$b:=-x^2\theta1^2+\theta2^2 \tag{12.8.9}$$

```
> oR:=4*z^2-1; c1:=1/2; c2:=-1/2;
```

$$oR:=4z^2-1$$

$$c1:=\frac{1}{2}$$

$$c2:=-\frac{1}{2} \tag{12.8.10}$$

```
> v1:=theta2+x*theta1; v2:=theta2-x*theta1;
```

$$v1 := \theta 2 + x\,\theta 1$$
$$v2 := \theta 2 - x\,\theta 1 \qquad (12.8.11)$$

E 12.9. Példa.

>

E 12.10. Példa.

>

E 12.11. Példa.

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E 12.12. Példa.

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E 12.13. Példa.

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E 12.14. Példa.

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E 12.15. Példa.

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E 12.16. Példa.

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E 12.17. Példa.

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E 12.18. Példa.

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E 12.19. Példa.

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E 12.20. Példa.

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E 12.21. Példa.

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E 12.22. Példa.

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