Komputeralgebrai algoritmusok

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Ezek a programok csak szemléltetésre szolgálnak.

▶ 1. Történet

▶ 2. Algebrai alapok

▶ 3. Normál formák, reprezentáció

▶ 4. Aritmetika

▶ 5. Kínai maradékolás

▶ 6. Newton–iteráció, Hensel–felemelés

▶ 7. Legnagyobb közös osztó

▶ 8. Faktorizálás

▶ 9. Egyenletrendszerek

▼ 10. Gröbner–bázisok

▼ E 10.1. Lexikografikus rendezés.


```
> p:=x+y+z+x^2+z^3+x*y+y^2;
```

$$p := x + y + z + x^2 + z^3 + x\,y + y^2 \tag{10.1.1}$$

```
> sort(p,[x,y,z],plex);
```

$$x^2 + x\,y + x + y^2 + y + z^3 + z \tag{10.1.2}$$

```
> sort(p,[z,y,x],plex);
```

$$z^3 + z + y^2 + y\,x + y + x^2 + x \tag{10.1.3}$$

▼ E 10.2. Fokszám rendezés.

```
> sort(p,[x,y,z]);
```

$$z^3 + x^2 + x\,y + y^2 + x + y + z \tag{10.2.1}$$

E 10.3. Példa.

```
> p:=-2*x^2*y*z+x^2*y^2+x^2*z^2+x^2*y+2*x*y^2*z^2-3*x*y*z^3-x*
  y+y*z+z^2+5;
```

$$p := -2\,x^2\,y\,z + x^2\,y^2 + x^2\,z^2 + x^2\,y + 2\,x\,y^2\,z^2 - 3\,x\,y\,z^3 - x\,y + y\,z + z^2 + 5 \tag{10.3.1}$$

```
> sort(p,[x,y,z]);
```

$$2\,x\,y^2\,z^2 - 3\,x\,y\,z^3 + x^2\,y^2 - 2\,x^2\,y\,z + x^2\,z^2 + x^2\,y - x\,y + y\,z + z^2 + 5 \tag{10.3.2}$$

```
> op(1,p); lcoeff(%,[x,y,z],'t'); t;
```

$$2\,x\,y^2\,z^2$$

$$2$$

$$x\,y^2\,z^2 \tag{10.3.3}$$

```
> collect(p,[x,y]);
```

$$\left(y^2 + \left(-2\,z + 1\right)y + z^2\right)x^2 + \left(2\,y^2\,z^2 + \left(-1 - 3\,z^3\right)y\right)x + z^2 + 5 + y\,z \tag{10.3.4}$$

```
> pp:=collect(p,[x,y],`distributed`);
```

$$pp := z^2 + 5 + 2\,x\,y^2\,z^2 + \left(-1 - 3\,z^3\right)x\,y + x^2\,y^2 + \left(-2\,z + 1\right)x^2\,y + y\,z + x^2\,z^2 \tag{10.3.5}$$

```
> sort(pp,[x,y]);
```

$$x^2\,y^2 + \left(-2\,z + 1\right)x^2\,y + 2\,z^2\,x\,y^2 + z^2\,x^2 + \left(-1 - 3\,z^3\right)x\,y + z\,y + z^2 + 5 \tag{10.3.6}$$

```
> op(1,p); lcoeff(%,[x,y],'t'); t;
```

$$2\,z^2\,x\,y^2$$

$$2\,z^2$$

$$y^2\,x \tag{10.3.7}$$

```
> with(Groebner);
```

[*Basis*, *FGLM*, *HilbertDimension*, *HilbertPolynomial*, *HilbertSeries*, *InterReduce*, *IsProper*, *IsZeroDimensional*, *LeadingCoefficient*, *LeadingMonomial*, *LeadingTerm*, *MonomialOrder*, *MultiplicationMatrix*, *NormalForm*, *NormalSet*, *Reduce*, *RememberBasis*, *SPolynomial*, *Solve*, *TestOrder*, *ToricIdealBasis*, *UnivariatePolynomial*, *Walk*, *fglm_algo*] (10.3.8)

```
> LeadingTerm(p,tdeg(x,y,z)); %[1]*%[2];
```

$$2,\ x\,y^2\,z^2$$

$$2\,x\,y^2\,z^2 \tag{10.3.9}$$

```
> LeadingMonomial(p,tdeg(x,y,z));
```

$$x\,y^2\,z^2 \tag{10.3.10}$$

```
> LeadingCoefficient(p,tdeg(x,y,z));
```

$$2 \tag{10.3.11}$$

E 10.4. Példa.

```
> p:=6*x^4+13*x^3-6*x+1; q:=3*x^2+5*x-1;
```

$$p := 6\,x^4 + 13\,x^3 - 6\,x + 1$$

$$q := 3\,x^2 + 5\,x - 1 \tag{10.4.1}$$

```
> p-2*x^2*q;
```

$$6\,x^4 + 13\,x^3 - 6\,x + 1 - 2\,x^2\left(3\,x^2 + 5\,x - 1\right) \tag{10.4.2}$$

```
> expand(p-2*x^2*q);
```

$$3\,x^3 - 6\,x + 1 + 2\,x^2 \tag{10.4.3}$$

```
> expand(p-13/3*x*q);
```

$$6\,x^4 - \frac{5}{3}\,x + 1 - \frac{65}{3}\,x^2 \tag{10.4.4}$$

E 10.5. Példa.

```
> p:=2*y^2*z-x*z^2; q1:=7*y^2+y*z-4; q2:=2*y*z-3*x+1;
```

$$p := 2\,y^2\,z - x\,z^2$$

$$q1 := 7\,y^2 + y\,z - 4$$

$$q2 := 2\,y\,z - 3\,x + 1 \tag{10.5.1}$$

```
> sort(p,[x,y,z]); sort(q1,[x,y,z]); sort(q2,[x,y,z]);
```

$$-x\,z^2 + 2\,y^2\,z$$

$$7\,y^2 + y\,z - 4$$

$$2\,y\,z - 3\,x + 1 \tag{10.5.2}$$

```
> expand(p-2/7*z*q1); expand(p-y*q2);
```

$$-x\,z^2 - \frac{2}{7}\,y\,z^2 + \frac{8}{7}\,z$$

$$-x\,z^2 + 3\,x\,y - y \tag{10.5.3}$$

```
> expand(%+1/7*q2);
```

$$-x\,z^2 + 3\,x\,y - y + \frac{2}{7}\,y\,z - \frac{3}{7}\,x + \frac{1}{7} \tag{10.5.4}$$

A 10.1. Algoritmus.

```
> reducers:=proc(p,Q,ord) local phmonom,qhmonom,q,R;
```

```
    R:=[]; phmonom:=LeadingMonomial(p,ord);
    for q in Q do
      if q<>0 then
        qhmonom:=LeadingMonomial(q,ord);
        if divide(phmonom,qhmonom) then R:=[op(R),q]; fi;
      fi;
    od; R;
  end;
```

```
reducers := proc(p, Q, ord)                                    (10.6.1)
    local phmonom, qhmonom, q, R;
    R := [ ];
    phmonom := Groebner:-LeadingMonomial(p, ord);
    for q in Q do
        if q <> 0 then
            qhmonom := Groebner:-LeadingMonomial(q, ord);
            if divide(phmonom, qhmonom) then
                R := [op(R), q]
            end if
        end if
    end do;
    R
end proc
```

```
> Q:=[q1,q2]; QQ:=reducers(p,Q,tdeg(x,y,z));
```

$$Q := \left[7\,y^2 + y\,z - 4,\ 2\,y\,z - 3\,x + 1\right]$$

$$QQ := \left[7\,y^2 + y\,z - 4,\ 2\,y\,z - 3\,x + 1\right] \qquad (10.6.2)$$

```
> reducer:=proc(p,Q,ord) Q[1] end;
```

$$reducer := \mathbf{proc}(p, Q, ord)\ Q[1]\ \mathbf{end\ proc} \qquad (10.6.3)$$

```
> reducer(p,QQ,tdeg(x,y,z));
```

$$7\,y^2 + y\,z - 4 \qquad (10.6.4)$$

```
> myreduce:=proc(p,Q,ord) local QQ,q,pp,ppp,lpp,lq,t;
    pp:=p; ppp:=0;
    while pp<>0 do
      QQ:=reducers(pp,Q,ord);
      if QQ<>[] then
        q:=reducer(p,QQ,ord);
        t:=LeadingTerm(pp,ord); lpp:=t[1]*t[2];
        t:=LeadingTerm(q,ord); lq:=t[1]*t[2];
        pp:=expand(pp-lpp*q/lq);
      else
        LeadingTerm(pp,ord); lpp:=%[1]*%[2];
        ppp:=ppp+lpp;
```

```
        pp:=pp-lpp;
      fi;
    od; ppp;
  end;
```

$$myreduce := \mathbf{proc}(p, Q, ord) \tag{10.6.5}$$

```
  local QQ, q, pp, ppp, lpp, lq, t;
  pp := p;
  ppp := 0;
  while pp <> 0 do
    QQ := reducers(pp, Q, ord);
    if QQ <> [ ] then
      q := reducer(p, QQ, ord);
      t := Groebner:-LeadingTerm(pp, ord);
      lpp := t[1] * t[2];
      t := Groebner:-LeadingTerm(q, ord);
      lq := t[1] * t[2];
      pp := expand(pp - lpp * q / lq)
    else
      Groebner:-LeadingTerm(pp, ord);
      lpp := %[1] * %[2];
      ppp := ppp + lpp;
      pp := pp - lpp
    end if
  end do;
  ppp
end proc
```

```
> myreduce(p,Q,tdeg(x,y,z));
```

$$-x\,z^2 - \frac{3}{7}\,z\,x + \frac{9}{7}\,z \tag{10.6.6}$$

E 10.6. Példa.

```
> p:=3*x^3*y+2*x^2*y^2-3*x*y+5*x; q1:=x^2*y+5*x^2+y^2; q2:=7*x*
  y^2-2*y^3+1;
```

$$p := 3\,x^3\,y + 2\,x^2\,y^2 - 3\,x\,y + 5\,x$$

$$q1 := x^2\,y + 5\,x^2 + y^2$$

$$q2 := 7\,x\,y^2 - 2\,y^3 + 1 \tag{10.7.1}$$

```
> expand(p-3*x*q1);
```

(10.7.2)

$$2\,x^2\,y^2-3\,x\,y+5\,x-15\,x^3-3\,x\,y^2 \tag{10.7.2}$$

```
> expand(%-2*y*q1);
```

$$-3\,x\,y+5\,x-15\,x^3-3\,x\,y^2-10\,x^2\,y-2\,y^3 \tag{10.7.3}$$

```
> expand(%+10*q1);
```

$$-3\,x\,y+5\,x-15\,x^3-3\,x\,y^2-2\,y^3+50\,x^2+10\,y^2 \tag{10.7.4}$$

```
> expand(%+3/7*q2);
```

$$-3\,x\,y+5\,x-15\,x^3-\frac{20}{7}\,y^3+50\,x^2+10\,y^2+\frac{3}{7} \tag{10.7.5}$$

E 10.7. Példa.

```
> q1:=x^3*y*z-x*z^2; q2:=x*y^2*z-x*y*z; q3:=x^2*y^2-z^2;
  p1:=x^2*y^2*z-z^3; p2:=-x^2*y^2*z+x^2*y*z;
```

$$q1:=x^3\,y\,z-x\,z^2$$
$$q2:=x\,y^2\,z-x\,y\,z$$
$$q3:=x^2\,y^2-z^2$$
$$p1:=x^2\,y^2\,z-z^3$$
$$p2:=-x^2\,y^2\,z+x^2\,y\,z \tag{10.8.1}$$

```
> expand(p1-z*q3); expand(p2+x*q2); p1+p2;
```

$$0$$
$$0$$
$$-z^3+x^2\,y\,z \tag{10.8.2}$$

E 10.8. Példa.

```
> G:=[q1,q2,q3,x^2*y*z-z^3,x*z^3-x*z^2,y*z^3-z^3,x*y*z^2-x*z^2,
  x^2*z^2-z^4,
  z^5-z^4];
```

$$G:=\left[x^3\,y\,z-x\,z^2,\ x\,y^2\,z-x\,y\,z,\ x^2\,y^2-z^2,\ -z^3\right.$$
$$\left.+x^2\,y\,z,\ x\,z^3-x\,z^2,\ y\,z^3-z^3,\ x\,y\,z^2-x\,z^2,\ x^2\,z^2-z^4,\ z^5-z^4\right] \tag{10.9.1}$$

```
> myreduce(p1,G,tdeg(x,y,z)); myreduce(p2,G,tdeg(x,y,z));
  myreduce(p1-p2,G,tdeg(x,y,z));
```

$$0$$
$$0$$
$$0 \tag{10.9.2}$$

▶ E 10.9. Példa.
▶ A 10.2. Algoritmus.
▶ E 10.10. Példa.
▶ A 10.3. Algoritmus.
▶ E 10.11. Példa.
▶ A 10.4. Algoritmus.
▶ E 10.12. Példa.
▶ E 10.13. Példa.
▶ E 10.14. Példa.
▶ E 10.15. Példa.
▶ E 10.16. Példa.
▶ A 10.5. Algoritmus.
▶ E 10.17. Példa.
▶ A 10.6. Algoritmus.
▶ E 10.18. Példa.
▶ A 10.7. Algoritmus.
▶ E 10.19. Példa.
▶ E 10.20. Példa.
▶ E 10.21. Példa.

▶ 11. Racionális törtfüggvények integrálása

▶ 12. A Risch–algoritmus.

▶ 13. ő