Komputeralgebrai algoritmusok

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Ezek a programok csak szemléltetésre szolgálnak.

1. Történet

```
> restart;
```

E 1.1. Példa.

```
> 33!/2^31+41^41;
```

$$1330877630632711998713399240963346255989932815022128910520\902250516 \quad (1.1.1)$$

```
> 43!/(2^43-1);
```

$$\frac{60415263063373835637355132068513997507264512000000000}{8796093022207} \quad (1.1.2)$$

```
> 48395254577457437 3476/122354323571234 mod 1000003;
```

$$887782 \quad (1.1.3)$$

```
> 10*(8+6*I)^(-1/2);
```

$$\frac{10}{\sqrt{8+6\,I}} \quad (1.1.4)$$

```
> evalc(%);
```

$$3-I \quad (1.1.5)$$

```
> sqrt(15523/3-98/2);
```

$$\frac{124}{3}\sqrt{3} \quad (1.1.6)$$

```
> a:=sin(Pi/3)*exp(2+ln(33));
```

$$a := \frac{1}{2}\sqrt{3}\,e^{2+\ln(33)} \quad (1.1.7)$$

```
> simplify(a);
```

$$\frac{33}{2}\sqrt{3}\,e^{2} \quad (1.1.8)$$

```
> evalf(a);
```

$$211.1706396 \quad (1.1.9)$$

```
> evalf(a,60);
```

$$211.170639624855418173457016949952935319763238458535271731\859 \quad (1.1.10)$$

```
> n:=193802871990921965256085980559909428418 20;
```

$$n := 193802871990921965256085980559909428418 20 \tag{1.1.11}$$

```
> isprime(n);
```

$$false \tag{1.1.12}$$

```
> ifactor(n);
```

$$(2)^2\,(3)^2\,(5)\,(19)^3\,(101)^4\,(12282045523619)^2 \tag{1.1.13}$$

```
> nextprime(n);
```

$$193802871990921965256085980559909428420 43 \tag{1.1.14}$$

```
> igcd(159903359728483469683239257887714049 85,
151636590443704897 80);
```

$$1263638253697540815 \tag{1.1.15}$$

```
> a:=(x+y)^12-(x-y)^12;
```

$$a := (x+y)^{12} - (x-y)^{12} \tag{1.1.16}$$

```
> expand(a);
```

$$24\,y\,x^{11} + 440\,y^3\,x^9 + 1584\,y^5\,x^7 + 1584\,y^7\,x^5 + 440\,y^9\,x^3 + 24\,y^{11}\,x \tag{1.1.17}$$

```
> quo(x^3*y-x^3*z+2*x^2*y^2-2*x^2*z^2+x*y^3+x*y^2*z-x*z^3,x+y+
z,x);
```

$$(y-z)\,x^2 + (y^2 - z^2)\,x + y\,z^2 \tag{1.1.18}$$

```
> gcd(x^3*y-x^3*z+2*x^2*y^2-2*x^2*z^2+x*y^3+x*y^2*z-x*z^3,x+y+
z);
```

$$1 \tag{1.1.19}$$

```
> b:=(x^4-y^4)/(x^3+y^3)-(x^5+y^5)/(x^4-y^4);
```

$$b := \frac{x^4 - y^4}{x^3 + y^3} - \frac{x^5 + y^5}{x^4 - y^4} \tag{1.1.20}$$

```
> normal(b);
```

$$-\frac{x^3\,y^3}{(x^3 - x^2\,y + x\,y^2 - y^3)\,(x^2 - x\,y + y^2)} \tag{1.1.21}$$

```
> f:=(x+y)*(x-y)^6; g:=(x^2-y^2)*(x-y)^3; f/g;
```

$$f := (x+y)\,(x-y)^6$$

$$g := (x^2 - y^2)\,(x-y)^3$$

$$\frac{(x+y)\,(x-y)^3}{x^2 - y^2} \tag{1.1.22}$$

```
> normal(f/g);
```

$$(x-y)^2 \tag{1.1.23}$$

```
> factor(x^6-x^5+x^2+1);
```

$$x^6 - x^5 + x^2 + 1 \tag{1.1.24}$$

```
> factor(5*x^4-4*x^3-48*x^2+44*x+3);
```

$$(x-1)\,(x-3)\,(5\,x^2 + 16\,x + 1) \tag{1.1.25}$$

```
> Factor(x^6-x^5+x^2+1) mod 13;
```

$$\left(x^3+10\,x^2+8\,x+11\right)\left(x^3+2\,x^2+11\,x+6\right) \tag{1.1.26}$$

```
> factor(x^12-y^12);
```

$$\left(x-y\right)\left(y^2+x^2+x\,y\right)\left(x+y\right)\left(x^2-x\,y+y^2\right)\left(x^2+y^2\right)\left(x^4-x^2\,y^2+y^4\right) \tag{1.1.27}$$

```
> restart;
> alias(a=RootOf(x^4-2));
```

$$a \tag{1.1.28}$$

```
> factor(x^12-2*x^8+4*x^4-8,a);
```

$$\left(x^4-2\,x^2+2\right)\left(x^4+2\,x^2+2\right)\left(x^2+a^2\right)\left(x+a\right)\left(x-a\right) \tag{1.1.29}$$

```
> Factor(x^6-2*x^4+4*x^2-8,a) mod 5;
```

$$\left(x+3\right)\left(x+2\right)\left(x+1\right)\left(x+4\right)\left(x+\mathrm{RootOf}\left(_Z^4+3\right)^2\right)\left(x+4\,\mathrm{RootOf}\left(_Z^4+3\right)^2\right) \tag{1.1.30}$$

```
> V:=vandermonde([x,y,z]);
```

$$V := \mathit{vandermonde}\left([x,\,y,\,z]\right) \tag{1.1.31}$$

```
> with(linalg);
```

$$[\mathit{BlockDiagonal},\ \mathit{GramSchmidt},\ \mathit{JordanBlock},\ \mathit{LUdecomp},\ \mathit{QRdecomp},\ \mathit{Wronskian},\ \mathit{addcol},\ \mathit{addrow},\ \mathit{adj},\ \mathit{adjoint},\ \mathit{angle},\ \mathit{augment},\ \mathit{backsub},\ \mathit{band},\ \mathit{basis},\ \mathit{bezout},\ \mathit{blockmatrix},\ \mathit{charmat},\ \mathit{charpoly},\ \mathit{cholesky},\ \mathit{col},\ \mathit{coldim},\ \mathit{colspace},\ \mathit{colspan},\ \mathit{companion},\ \mathit{concat},\ \mathit{cond},\ \mathit{copyinto},\ \mathit{crossprod},\ \mathit{curl},\ \mathit{definite},\ \mathit{delcols},\ \mathit{delrows},\ \mathit{det},\ \mathit{diag},\ \mathit{diverge},\ \mathit{dotprod},\ \mathit{eigenvals},\ \mathit{eigenvalues},\ \mathit{eigenvectors},\ \mathit{eigenvects},\ \mathit{entermatrix},\ \mathit{equal},\ \mathit{exponential},\ \mathit{extend},\ \mathit{ffgausselim},\ \mathit{fibonacci},\ \mathit{forwardsub},\ \mathit{frobenius},\ \mathit{gausselim},\ \mathit{gaussjord},\ \mathit{geneqns},\ \mathit{genmatrix},\ \mathit{grad},\ \mathit{hadamard},\ \mathit{hermite},\ \mathit{hessian},\ \mathit{hilbert},\ \mathit{htranspose},\ \mathit{ihermite},\ \mathit{indexfunc},\ \mathit{innerprod},\ \mathit{intbasis},\ \mathit{inverse},\ \mathit{ismith},\ \mathit{issimilar},\ \mathit{iszero},\ \mathit{jacobian},\ \mathit{jordan},\ \mathit{kernel},\ \mathit{laplacian},\ \mathit{leastsqrs},\ \mathit{linsolve},\ \mathit{matadd},\ \mathit{matrix},\ \mathit{minor},\ \mathit{minpoly},\ \mathit{mulcol},\ \mathit{mulrow},\ \mathit{multiply},\ \mathit{norm},\ \mathit{normalize},\ \mathit{nullspace},\ \mathit{orthog},\ \mathit{permanent},\ \mathit{pivot},\ \mathit{potential},\ \mathit{randmatrix},\ \mathit{randvector},\ \mathit{rank},\ \mathit{ratform},\ \mathit{row},\ \mathit{rowdim},\ \mathit{rowspace},\ \mathit{rowspan},\ \mathit{rref},\ \mathit{scalarmul},\ \mathit{singularvals},\ \mathit{smith},\ \mathit{stackmatrix},\ \mathit{submatrix},\ \mathit{subvector},\ \mathit{sumbasis},\ \mathit{swapcol},\ \mathit{swaprow},\ \mathit{sylvester},\ \mathit{toeplitz},\ \mathit{trace},\ \mathit{transpose},\ \mathit{vandermonde},\ \mathit{vecpotent},\ \mathit{vectdim},\ \mathit{vector},\ \mathit{wronskian}] \tag{1.1.32}$$

```
> V:=vandermonde([x,y,z]);
```

$$V := \begin{bmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{bmatrix} \tag{1.1.33}$$

```
> inverse(V);
```

$$\begin{bmatrix} \frac{zy}{zy-xz+x^2-xy} & \frac{xz}{xz-xy+y^2-zy} & -\frac{xy}{xz-xy-z^2+zy} \\ -\frac{z+y}{zy-xz+x^2-xy} & -\frac{x+z}{xz-xy+y^2-zy} & \frac{x+y}{xz-xy-z^2+zy} \\ \frac{1}{zy-xz+x^2-xy} & \frac{1}{xz-xy+y^2-zy} & -\frac{1}{xz-xy-z^2+zy} \end{bmatrix} \tag{1.1.34}$$

```
> det(V);
```

$$yz^2-y^2z-xz^2+x^2z+xy^2-x^2y \tag{1.1.35}$$

```
> factor(%);
```

$$(z-y)(-y+x)(-z+x) \tag{1.1.36}$$

```
> e1:=(1-eps)*x+2*y-4*z-1=0;
```

$$e1 := (1-eps)x+2y-4z-1=0 \tag{1.1.37}$$

```
> e2:=(3/2-eps)*x+3*y-5*z-2=0;
```

$$e2 := \left(\frac{3}{2}-eps\right)x+3y-5z-2=0 \tag{1.1.38}$$

```
> e3:=(5/2+eps)*x+5*y-7*z-3=0;
```

$$e3 := \left(\frac{5}{2}+eps\right)x+5y-7z-3=0 \tag{1.1.39}$$

```
> sols:=solve([e1,e2,e3],[x,y,z]);
```

$$sols := \left[\left[x=-\frac{1}{2\,eps},\ y=\frac{1}{4}\,\frac{1+7\,eps}{eps},\ z=\frac{3}{4}\right]\right] \tag{1.1.40}$$

```
> subs(eps=10^(-20),sols);
```

$$\left[\left[x=-50000000000000000000,\ y=\frac{100000000000000000007}{4},\ z=\frac{3}{4}\right]\right] \tag{1.1.41}$$

```
> f:=x^2*y*(1-x-y)^3;
```

$$f := x^2y(1-x-y)^3 \tag{1.1.42}$$

```
> e1:=diff(f,x); e2:=diff(f,y);
```

$$e1 := 2xy(1-x-y)^3-3x^2y(1-x-y)^2$$

$$e2 := x^2(1-x-y)^3-3x^2y(1-x-y)^2 \tag{1.1.43}$$

```
> solve([e1,e2],[x,y]);
```

$$\left[[x=0,\ y=y],\left[x=\frac{1}{3},\ y=\frac{1}{6}\right],[x=1-y,\ y=y],[x=1-y,\ y=y]\right] \tag{1.1.44}$$

```
> limit(tan(x)/x,x=0);
```

$$1 \tag{1.1.45}$$

```
> diff(ln(sec(x)),x);
```

$$\tan(x) \tag{1.1.46}$$

```
> series(tan(sinh(x))-sinh(tan(x)),x=0,15);
```

$$\frac{1}{90}x^7+\frac{13}{756}x^9+\frac{1451}{75600}x^{11}+\frac{6043}{332640}x^{13}+O\left(x^{15}\right) \tag{1.1.47}$$

```
> series(BesselJ(0,x)/BesselJ(1,x),x,12);
```

$$2x^{-1}-\frac{1}{4}x-\frac{1}{96}x^3-\frac{1}{1536}x^5-\frac{1}{23040}x^7-\frac{13}{4423680}x^9+O\left(x^{10}\right) \tag{1.1.48}$$

```
> int(((3*x^2-7*x+15)*exp(x)+3*x^2-14)/(x-exp(x))^2,x);
```

$$\frac{14+3x^2-e^x}{x-e^x} \tag{1.1.49}$$

```
> int((3*x^3-x+14)/(x^2+4*x-4),x);
```

$$\frac{3}{2}x^2-12x+\frac{59}{2}\ln\left(x^2+4x-4\right)+38\sqrt{2}\,\operatorname{arctanh}\left(\frac{1}{8}(2x+4)\sqrt{2}\right) \tag{1.1.50}$$

```
> int(x*exp(x^3),x);
```

$$-\frac{1}{3}(-1)^{1/3}\left(\frac{x^2(-1)^{2/3}\Gamma\left(\frac{2}{3}\right)}{\left(-x^3\right)^{2/3}}-\frac{x^2(-1)^{2/3}\Gamma\left(\frac{2}{3},-x^3\right)}{\left(-x^3\right)^{2/3}}\right) \tag{1.1.51}$$

```
> diff_eqn:=diff(y(x),x$2)+t*diff(y(x),x)-2*t^2*y(x)=0;
```

$$diff_eqn := \frac{d^2}{dx^2}y(x)+t\left(\frac{d}{dx}y(x)\right)-2t^2y(x)=0 \tag{1.1.52}$$

```
> init_conds:=y(0)=t,D(y)(0)=2*t^2;
```

$$init_conds := y(0)=t,\ D(y)(0)=2t^2 \tag{1.1.53}$$

```
> dsolve({diff_eqn,init_conds},y(x));
```

$$y(x)=\frac{4}{3}te^{tx}-\frac{1}{3}te^{-2tx} \tag{1.1.54}$$

```
> Cheby:=proc(n,x) local T,k;
    T[0]:=1; T[1]:=x;
    for k from 2 to n do
      T[k]:=expand(2*x*T[k-1]-T[k-2]);
    od; T[n];
  end;
```

```
Cheby := proc(n, x)                                              (1.1.55)
  local T, k;
  T[0] := 1;
  T[1] := x;
  for k from 2 to n do
    T[k] := expand(2*x*T[k-1]-T[k-2])
  end do;
  T[n]
end proc
```

```
> Cheby(7,x);
```

$$64\,x^7 - 112\,x^5 + 56\,x^3 - 7\,x \tag{1.1.56}$$

Ex1.3. Feladat.

```
> int(x/(1+exp(x)),x);
  int(exp(x^2),x);
  int(sqrt((x^2-1)*(x^2-4)),x);
  int(sqrt((x-1)*(x-4)),x);
  int(sqrt((1+x)/(1-x)),x);
  int(log(x^2-5*x+4),x);
  int(log(x)/(1+x),x);
  int(1/log(x),x);
```

$$\frac{1}{2}\,x^2 - x\ln\left(1+\mathrm{e}^x\right) - \mathrm{polylog}\left(2, -\mathrm{e}^x\right)$$

$$-\frac{1}{2}\,\mathrm{I}\sqrt{\pi}\,\mathrm{erf}\left(\mathrm{I}\,x\right)$$

$$\frac{1}{3}\,x\sqrt{4+x^4-5\,x^2} + \frac{4}{3}\,\frac{\sqrt{1-x^2}\,\sqrt{4-x^2}\,\mathrm{EllipticF}\left(x, \frac{1}{2}\right)}{\sqrt{4+x^4-5\,x^2}}$$

$$-\frac{10}{3}\,\frac{\sqrt{1-x^2}\,\sqrt{4-x^2}\left(\mathrm{EllipticF}\left(x, \frac{1}{2}\right) - \mathrm{EllipticE}\left(x, \frac{1}{2}\right)\right)}{\sqrt{4+x^4-5\,x^2}}$$

$$\frac{1}{4}\left(2\,x-5\right)\sqrt{4+x^2-5\,x} - \frac{9}{8}\ln\left(-\frac{5}{2}+x+\sqrt{4+x^2-5\,x}\right)$$

$$\frac{\sqrt{-\frac{x+1}{x-1}}\,\left(x-1\right)\left(\sqrt{1-x^2} - \arcsin\left(x\right)\right)}{\sqrt{-\left(x-1\right)\left(x+1\right)}}$$

$$x\ln\left(4+x^2-5\,x\right) - 2\,x - \ln\left(x-1\right) - 4\ln\left(x-4\right)$$

$$\mathrm{dilog}\left(x+1\right) + \ln\left(x\right)\ln\left(x+1\right)$$

$$-\mathrm{Ei}\left(1, -\ln\left(x\right)\right) \tag{1.2.1}$$

2. Algebrai alapok

3. Normál formák, reprezentáció

4. Aritmetika

- 5. Kínai maradékolás
- 6. Newton–iteráció, Hensel–felemelés
- 7. Legnagyobb közös osztó
- 8. Faktorizálás
- 9. Egyenletrendszerek
- 10. Gröbner–bázisok
- 11. Racionális törtfüggvények integrálása
- 12. A Risch–algoritmus.