Számítógépes számelmélet

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Ezek a programok csak szemléltetésre szolgálnak

1. A prímek eloszlása, szitálás

```
> restart; with(numtheory);
```

[*GIgcd*, *bigomega*, *cfrac*, *cfracpol*, *cyclotomic*, *divisors*, *factorEQ*, *factorset*, *fermat*, *imagunit*, *index*, *integral_basis*, *invcfrac*, *invphi*, *issqrfree*, *jacobi*, *kronecker*, λ, *legendre*, *mcombine*, *mersenne*, *migcdex*, *minkowski*, *mipolys*, *mlog*, *mobius*, *mroot*, *msqrt*, *nearestp*, *nthconver*, *nthdenom*, *nthnumer*, *nthpow*, *order*, *pdexpand*, ϕ, π, *pprimroot*, *primroot*, *quadres*, *rootsunity*, *safeprime*, σ, *sq2factor*, *sum2sqr*, τ, *thue*] (1.1)

1.1. A prímszámtétel.

```
> [i$i=1..20]; evalf(map(i->log[2](mersenne([i])+1),%));
```

[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20]

[2., 3., 5., 7., 13., 17., 19., 31., 61., 89., 107., 127., 521., 607., 1279., 2203., 2281., 3217., 4253., 4423.] (1.1.1)

```
> primeprod:=proc(n) local p,i;
    p:=1;
    for i from 2 to n do if isprime(i) then p:=p*i fi od;
    p,log[2.](p);
  end;
```

primeprod := **proc**(*n*)
  local *p*, *i*;
  p := 1;
  for *i* **from** 2 **to** *n* **do**
    if *isprime*(*i*) **then**
      p := *p* * *i*
    end if
  end do;
  p, log[2.](*p*) (1.1.2)

end proc

```
> primeprod(16);
```

30030, 14.87411685 (1.1.3)

```
> primeprod(32);
```

200560490130, 37.54524647 (1.1.4)

```
> primeprod(64);
```

117288381359406970983270, 76.63440629 (1.1.5)

```
> primeprod(128);
```

401447693933303618909444119902604513664588524730, (1.1.6)

161.4577606

```
> primeprod(256);
```

642663309179086448723306352281067133108801865916092081142\ (1.1.7)

475868089815036788070315252520074323442023O,

334.8768726

```
> primeprod(512);
```

319640812871968885546407425302739497128055991894413730095 2\ (1.1.8)

163618243066798282807774278885302980793179820710688 5\

718293402541861369758591699412090677759187666930105 4\

914697253507189410737229349850420921542053211440 30,

702.6032796

```
> primeprod(1024);
```

208325544418697180526278559204028744572686528568890074734 0\ (1.1.9)

490078401814571872862443019158728631608857214863138 9\

379309284743016940885980871887083026597753881317772 6\

058850383316252820523111213067921935404833217036456 3\

007177616888535712671502325086556344276636618033120 0\

980711247645589424056809053468323906745795726223468 4\

834336252590008874119591973239736134883450319130587 7\

535868469057614606627687505859610023611226005494428 7\

636530, 1419.522136

```
> primeprod(2048);
```

197157095083609531143746613721408325735276689161058727949\ (1.1.10)

263330602978714496517345824099916688434418202662783\

781187284851481680525273376709196742234690129691786\

729772270727953983062634938303563818299635539849771\
064762235532892368744909654908146839074797749897425\
989730594927246559335250090143667706061137979097877\
564299462793791484227987302350969242557878943184 44\
573403623311844845130030697902418254711000674356157\
913162039526024244954324590807054406582406525020073\
975531981951185943622210800004276044070982887534516\
363128369569046341642355926574077123471904312482147\
027158943156290368578925571104571283463232579465831\
006386931780448228113180797908049326557952003152973\
039210036423877111412618605633456075203379801505959\
564202697863299536694940700241824688642545887707923\
129879948356136794865481835327104828757822436784225\
687828590551763312291578230217555752801 90, 2864.481363

> **pd16:=1/ln(2.)/16; # approximate density of primes <=16 bit**

$$pd16 := 0.09016844006 \tag{1.1.11}$$

> **p16:=2.^16/ln(2.)/16; # approximate number of primes <=16 bit**

$$p16 := 5909.278888 \tag{1.1.12}$$

> **pd32:=1/ln(2.)/32; # approximate density of primes <=32 bit**

$$pd32 := 0.04508422003 \tag{1.1.13}$$

> **p32:=2.^32/ln(2.)/32; # approximate number of primes <=32 bit**

$$p32 := 1.936352506\ 10^{8} \tag{1.1.14}$$

> **pd35:=1/ln(2.)/35; # approximate density of primes <=35 bit**

$$pd35 := 0.04121985831 \tag{1.1.15}$$

> **p35:=2.^35/ln(2.)/35; # approximate number of primes <=35 bit**

$$p35 := 1.416303547\ 10^{9} \tag{1.1.16}$$

> **pd40:=1/ln(2.)/40; # approximate density of primes <=40 bit**

$$pd40 := 0.03606737602 \tag{1.1.17}$$

> **p40:=2.^40/ln(2.)/40; # approximate number of primes <=40 bit**

$$p40 := 3.965649933\ 10^{10} \tag{1.1.18}$$

> **pd50:=1/ln(2.)/50; # approximate density of primes <=50 bit**

$$pd50 := 0.02885390082 \tag{1.1.19}$$

> **p50:=2.^50/ln(2.)/50; # approximate number of primes <=50 bit**

$$p50 := 3.248660425\ 10^{13} \tag{1.1.20}$$

```
> pd64:=1/ln(2.)/64; # approximate density of primes <=64 bit
```

$$pd64 := 0.02254211002 \tag{1.1.21}$$

```
> p64:=2.^64/ln(2.)/64; # approximate number of primes <=64 bit
```

$$p64 := 4.158285343\ 10^{17} \tag{1.1.22}$$

1.2. Kérdés: zeta gyökei.

1.3. Kérdés: π(x).

1.4. Ikerprímek.

1.5. Kérdés: π_2(x)

1.6. Kérdés: az ikerprímek reciprokainak összege.

1.7. Sejtés.

```
> #
  # This procedure approximate Cs calculating the product
  # for primes below x.
  #

  Cs:=proc(s::posint,x::posint) local P,p;
  P:=1.; p:=nextprime(s);
  while p<x do P:=P*(1-s/p)/(1-1/p)^s; p:=nextprime(p) od;
  P end;
```

$$Cs := \mathbf{proc}(s::posint, x::posint) \tag{1.7.1}$$

$$\mathbf{local}\ P, p;$$

$$P := 1.;$$

$$p := nextprime(s);$$

$$\mathbf{while}\ p < x\ \mathbf{do}$$

$$P := P*(1 - s/p)/(1 - 1/p)^s;$$

$$p := nextprime(p)$$

$$\mathbf{end\ do};$$

$$P$$

$$\mathbf{end\ proc}$$

```
> Cs(2,10); Cs(2,100); Cs(2,1000); Cs(2,10000); Cs(2,100000);
  Cs2:=Cs(2,1000000);
```

$$0.6835937498$$

0.6613770846

0.6602457447

0.6601682974

0.6601623428

$$Cs2 := 0.6601618366 \tag{1.7.2}$$

```
> Cs(3,10); Cs(3,100); Cs(3,1000); Cs(3,10000); Cs(3,100000);
  Cs3:=Cs(3,1000000);
```

0.7089120370

0.6386939650

0.6354087220

0.6351850600

0.6351678830

$$Cs3 := 0.6351664804 \tag{1.7.3}$$

```
> Cs(4,10); Cs(4,100); Cs(4,1000); Cs(4,10000); Cs(4,100000);
  Cs4:=Cs(4,1000000);
```

0.3876862703

0.3109330904

0.3077296758

0.3075129938

0.3074963629

$$Cs4 := 0.3074950120 \tag{1.7.4}$$

```
> Cs(5,10); Cs(5,100); Cs(5,1000); Cs(5,10000); Cs(5,100000);
  Cs5:=Cs(5,1000000);
```

0.6175411522

0.4175713994

0.4103968221

0.4099151272

0.4098781719

$$Cs5 := 0.4098751550 \tag{1.7.5}$$

```
> Cs(6,10); Cs(6,100); Cs(6,1000); Cs(6,10000); Cs(6,100000);
  Cs6:=Cs(6,1000000);
```

0.3602323389

0.1919159671

0.1869709920

0.1866417820

0.1866165459

$$Cs6 := 0.1866144865 \tag{1.7.6}$$

```
> Cs(7,10); Cs(7,100); Cs(7,1000); Cs(7,10000); Cs(7,100000);
  Cs7:=Cs(7,1000000);
```

1.

0.3842733486

0.3704268393

0.3695136936

0.3694437283

$$Cs7 := 0.3694380193 \tag{1.7.7}$$

```
> Cs(8,10); Cs(8,100); Cs(8,1000); Cs(8,10000); Cs(8,100000);
  Cs8:=Cs(8,1000000);
```

1.

0.2449971936

0.2332498708

0.2324832417

0.2324245538

$$Cs8 := 0.2324197707 \tag{1.7.8}$$

```
> evalf(9*exp(-2*gamma)*Cs(3,10));
  evalf(9*exp(-2*gamma)*Cs(3,100));
  evalf(9*exp(-2*gamma)*Cs(3,1000));
  evalf(9*exp(-2*gamma)*Cs(3,10000));
  evalf(9*exp(-2*gamma)*Cs(3,100000));
  evalf(9*exp(-2*gamma)*Cs(3,1000000));
```

2.011276149

1.812058297

1.802737633

1.802103075

1.802054341

1.802050362 (1.7.9)

1.8. Példa.

```
> # Twin prime, old
> f1:=h->(3.+30*h)*2.^38880.-1;
  f2:=f1+2; f2(1);
  g:=h->1/ln(f1(h))/ln(f2(h));
```

$$f1 := h \to (3. + 30\,h)\,2.^{38880.} - 1$$

$$f2 := f1 + 2$$

$$3.670670111\ 10^{11705}$$

$$g := h \to \frac{1}{\ln(f1(h))\ln(f2(h))} \tag{1.8.1}$$

```
> H:=2.^27; H/6*(g(0)+4*g(H/2)+g(H));
```

$$H := 1.34217728\ 10^{8}$$

$$0.1845532660 \tag{1.8.2}$$

```
> Cf1f2:=C2*(1-1/3)^2/(1-2/3)*(1-1/5)^2/(1-2/5)/(1-1/2)^2/(1
  -1/3)^2/(1-1/5)^2;
```

$$Cf1f2 := 20\ C2 \tag{1.8.3}$$

```
> %%*20*Cs2;
```

$$2.436700461 \tag{1.8.4}$$

```
> f1:=h->(5775.+30030*h)*2.^171960.-1;
  f2:=f1+2; f2(1);
  g:=h->1/ln(f1(h))/ln(f2(h));
```

$$f1 := h \to (5775. + 30030\,h)\,2.^{1.71960\,10^{5}} - 1$$

$$f2 := f1 + 2$$

$$4.698920054\ 10^{51769}$$

$$g := h \to \frac{1}{\ln(f1(h))\ln(f2(h))} \tag{1.8.5}$$

```
> H:=2.^35; H/6*(g(0)+4*g(H/2)+g(H));
```

$$H := 3.435973837\ 10^{10}$$

$$2.417280248 \tag{1.8.6}$$

```
> C2*(1-1/3)^2/(1-2/3)*(1-1/5)^2/(1-2/5)*(1-1/7)^2/(1-2/7)*(1
  -1/11)^2/(1-2/11)*(1-1/13)^2/(1-2/13);
```

$$\frac{16384}{11011}\ C2 \tag{1.8.7}$$

```
> Cf1f2:=%/(1-1/2)^2/(1-1/3)^2/(1-1/5)^2/(1-1/7)^2/(1-1/11)^2/
  (1-1/13)^2;
```

$$Cf1f2 := \frac{364}{9}\ C2 \tag{1.8.8}$$

```
> %*%%%; subs(C2=Cs2,%);
```

$$97.76555670\ C2$$

$$64.54108947 \tag{1.8.9}$$

```
> # Cunningham chain I, length 3
```

```
> f1:=h->(3.+78*h)*2.^(273*128)-1;
  f2:=2*f1+1; f3:=2*f2+1; f3(1);
  g:=h->1/ln(f1(h))/ln(f2(h))/ln(f3(h));
```

$$f1 := h \to (3. + 78\,h)\,2.^{34944} - 1$$

$$f2 := 2\,f1 + 1$$

$$f3 := 4\,f1 + 3$$

$$5.043284780\,10^{10521}$$

$$g := h \to \frac{1}{\ln(f1(h))\ln(f2(h))\ln(f3(h))} \tag{1.8.10}$$

```
> H:=2.^40; H/6*(g(0)+4*g(H/2)+g(H));
```

$$H := 1.099511628\,10^{12}$$

$$0.07711637083 \tag{1.8.11}$$

```
> Cf1f2f3:=C3/(1-1/2)^3/(1-1/3)^3*(1-1/13)^3/(1-3/13);
```

$$Cf1f2f3 := \frac{23328}{845}\,C3 \tag{1.8.12}$$

```
> %*%%; subs(C3=Cs3,%); HCCI3:=H/%;
```

$$2.128959407\,C3$$

$$1.352243653$$

$$HCCI3 := 8.131017110\,10^{11} \tag{1.8.13}$$

```
> # Cunningham chain I, length 4
> f1:=h->(375.+390*h)*2.^(77*128)-1;
  f2:=2*f1+1; f3:=2*f2+1; f4:=2*f3+1; f4(1);
  g:=h->1/ln(f1(h))/ln(f2(h))/ln(f3(h))/ln(f4(h));
```

$$f1 := h \to (375. + 390\,h)\,2.^{9856} - 1$$

$$f2 := 2\,f1 + 1$$

$$f3 := 4\,f1 + 3$$

$$f4 := 8\,f1 + 7$$

$$5.475057522\,10^{2970}$$

$$g := h \to \frac{1}{\ln(f1(h))\ln(f2(h))\ln(f3(h))\ln(f4(h))} \tag{1.8.14}$$

```
> H:=2.^47; H/6*(g(0)+4*g(H/2)+g(H));
```

$$H := 1.407374884\,10^{14}$$

$$0.06335396657 \tag{1.8.15}$$

```
> Cf1f2f3f4:=C4*(1-1/2)/(1-1/2)^4*(1-2/3)/(1-1/3)^4*(1-3/7)/(1
  -4/7)*(1-1/5)^4/(1-4/5)*(1-1/13)^4/(1-4/13);
```

(1.8.16)

$$Cf1f2f3f4 := \frac{10616832}{274625}\ C4 \tag{1.8.16}$$

```
> %*%%; subs(C4=Cs4,%); HCCI4:=H/%;
```

$$2.449225014\ C4$$

$$0.7531244751$$

$$HCCI4 := 1.868714841\ 10^{14} \tag{1.8.17}$$

```
> # Cunningham chain I, length 5
> f1:=h->(255.+390*h)*2.^(32*128)-1;
  f2:=2*f1+1; f3:=2*f2+1; f4:=2*f3+1; f5:=2*f4+1; f5(1);
  g:=h->1/ln(f1(h))/ln(f2(h))/ln(f3(h))/ln(f4(h))/ln(f5(h));
```

$$f1 := h \to (255. + 390\,h)\ 2.^{4096} - 1$$

$$f2 := 2\ f1 + 1$$

$$f3 := 4\ f1 + 3$$

$$f4 := 8\ f1 + 7$$

$$f5 := 16\ f1 + 15$$

$$1.077809325\ 10^{1237}$$

$$g := h \to \frac{1}{\ln(f1(h))\ln(f2(h))\ln(f3(h))\ln(f4(h))\ln(f5(h))} \tag{1.8.18}$$

```
> H:=2.^54; H/6*(g(0)+4*g(H/2)+g(H));
```

$$H := 1.801439851\ 10^{16}$$

$$0.09140472270 \tag{1.8.19}$$

```
> Cf1f2f3f4f5:=C5*(1-1/2)/(1-1/2)^5*(1-2/3)/(1-1/3)^5*(1-4/5)/
  (1-1/5)^5*(1-2/7)/(1-4/7)*(1-1/13)^5/(1-5/13)*(1-7/17)/(1
  -5/17);
```

$$Cf1f2f3f4f5 := \frac{34171875}{913952}\ C5 \tag{1.8.20}$$

```
> %*%%; subs(C5=Cs5,%); HCCI5:=H/%;
```

$$3.417543546\ C5$$

$$1.400766191$$

$$HCCI5 := 1.286038928\ 10^{16} \tag{1.8.21}$$

```
> # Cunningham chain I, length 6
> f1:=h->(375.+390*h)*2.^(29*64)-1;
  f2:=2*f1+1; f3:=2*f2+1; f4:=2*f3+1; f5:=2*f4+1; f6:=2*f4+1;
  f6(1);
  g:=h->1/ln(f1(h))/ln(f2(h))/ln(f3(h))/ln(f4(h))/ln(f5(h))/ln
  (f6(h));
```

$$f1 := h \to (375. + 390\,h)\ 2.^{1856} - 1$$

$$f2 := 2\,f1 + 1$$

$$f3 := 4\,f1 + 3$$

$$f4 := 8\,f1 + 7$$

$$f5 := 16\,f1 + 15$$

$$f6 := 16\,f1 + 15$$

$$6.301636821\,10^{562}$$

$$g := h \to \frac{1}{\ln(f1(h))\ln(f2(h))\ln(f3(h))\ln(f4(h))\ln(f5(h))\ln(f6(h))} \tag{1.8.22}$$

```
> H:=2.^60; H/6*(g(0)+4*g(H/2)+g(H));
```

$$H := 1.152921505\,10^{18}$$

$$0.2105695090 \tag{1.8.23}$$

```
> Cf1f2f3f4f5f6:=C6*(1-1/2)/(1-1/2)^6*(1-2/3)/(1-1/3)^6*(1-4/5)
  /(1-1/5)^6*(1-1/7)/(1-4/7)*(1-1/13)^6/(1-6/13)*(1-14/17)/(1
  -6/17)*(1-5/31)/(1-6/31);
```

$$Cf1f2f3f4f5f6 := \frac{132860250}{2199197}\,C6 \tag{1.8.24}$$

```
> %*%%; subs(C6=Cs6,%); HCCI6:=H/%;
```

$$12.72115122\,C6$$

$$2.373951103$$

$$HCCI6 := 4.856551188\,10^{17} \tag{1.8.25}$$

```
> # Cunningham chain I, length 6
> f1:=h->(375.+2*3*5*7*11*13*17*19*23*29*31*37*41*47*53*59*61*
  67*h)*2.^(29*64)-1;
  f2:=2*f1+1; f3:=2*f2+1; f4:=2*f3+1; f5:=2*f4+1; f6:=2*f4+1;
  f6(1);
  g:=h->1/ln(f1(h))/ln(f2(h))/ln(f3(h))/ln(f4(h))/ln(f5(h))/ln
  (f6(h));
```

$$f1 := h \to (375. + 18275166397861086176463 0\,h)\,2.^{1856} - 1$$

$$f2 := 2\,f1 + 1$$

$$f3 := 4\,f1 + 3$$

$$f4 := 8\,f1 + 7$$

$$f5 := 16\,f1 + 15$$

$$f6 := 16\,f1 + 15$$

$$1.505404726\,10^{583}$$

$$g := h \to \frac{1}{\ln(f1(h))\ln(f2(h))\ln(f3(h))\ln(f4(h))\ln(f5(h))\ln(f6(h))} \tag{1.8.26}$$

```
> H:=2.^60; H/6*(g(0)+4*g(H/2)+g(H));
```

$$H := 1.152921505\ 10^{18}$$

$$0.1784120840 \tag{1.8.27}$$

```
> Cf1f2f3f4f5f6:=C6*(1-1/2)/(1-1/2)^6*(1-2/3)/(1-1/3)^6*(1-4/5)
  /(1-1/5)^6*(1-1/7)/(1-4/7)*(1-1/13)^6/(1-6/13)*(1-14/17)/(1
  -6/17)*(1-5/31)/(1-6/31);
```

$$Cf1f2f3f4f5f6 := \frac{132860250}{2199197}\ C6 \tag{1.8.28}$$

```
> %*%%; subs(C6=Cs6,%); HCCI61:=H/%;
```

$$10.77842234\ C6$$

$$2.011409750$$

$$HCCI61 := 5.731907708\ 10^{17} \tag{1.8.29}$$

```
> # Cunnigham chain II, length 2 + twinprime
> f1:=h->(5775.+30030*h)*2.^(1983*128)-1;
  f2:=f1+2; f3:=2*f2-1; f3(1);
  g:=h->1/ln(f1(h))/ln(f3(h));
```

$$f1 := h \to (5775. + 30030\ h)\ 2.^{253824} - 1$$

$$f2 := f1 + 2$$

$$f3 := 2\ f1 + 3$$

$$3.108802182\ 10^{76413}$$

$$g := h \to \frac{1}{\ln(f1(h))\ln(f3(h))} \tag{1.8.30}$$

```
> H:=2.^35; H/6*(g(0)+4*g(H/2)+g(H));
```

$$H := 3.435973837\ 10^{10}$$

$$1.109646873 \tag{1.8.31}$$

```
> C2*(1-1/3)^2/(1-2/3)*(1-1/5)^2/(1-2/5)*(1-1/7)^2/(1-2/7)*(1
  -1/11)^2/(1-2/11)*(1-1/13)^2/(1-2/13);
```

$$\frac{16384}{11011}\ C2 \tag{1.8.32}$$

```
> Cf1f3:=%/(1-1/2)^2/(1-1/3)^2/(1-1/5)^2/(1-1/7)^2/(1-1/11)^2/
  (1-1/13)^2;
```

$$Cf1f3 := \frac{364}{9}\ C2 \tag{1.8.33}$$

```
> %*%%%; subs(C2=Cs2,%);
```

$$44.87905131\ C2$$

$$29.62743694 \tag{1.8.34}$$

```
> # Cunningham chain II, length 3
```

```
> f1:=h->(3.+6*h)*2.^(247*128)+1;
  f2:=2*f1-1; f3:=2*f2-1; f3(1);
  g:=h->1/ln(f1(h))/ln(f2(h))/ln(f3(h));
```

$$f1 := h \to (3. + 6\,h)\,2.^{31616} + 1$$

$$f2 := 2\,f1 - 1$$

$$f3 := 4\,f1 - 3$$

$$8.330007900\,10^{9518}$$

$$g := h \to \frac{1}{\ln(f1(h))\ln(f2(h))\ln(f3(h))} \tag{1.8.35}$$

```
> H:=2.^43; H/6*(g(0)+4*g(H/2)+g(H));
```

$$H := 8.796093022\,10^{12}$$

$$0.8327300467 \tag{1.8.36}$$

```
> Cf1f2f3:=C3*(1-1/2)/(1-1/2)^3*(1-2/3)/(1-1/3)^3;
```

$$Cf1f2f3 := \frac{9}{2}\,C3 \tag{1.8.37}$$

```
> %*%%; subs(C3=Cs3,%); HCCII3:=H/%;
```

$$3.747285210\,C3$$

$$2.380149958$$

$$HCCII3 := 3.695604553\,10^{12} \tag{1.8.38}$$

```
> # Cunningham chain II, length 4
> f1:=h->(15.+30*h)*2.^(61*128)+1;
  f2:=2*f1-1; f3:=2*f2-1; f4:=2*f3-1; f4(1);
  g:=h->1/ln(f1(h))/ln(f2(h))/ln(f3(h))/ln(f4(h));
```

$$f1 := h \to (15. + 30\,h)\,2.^{7808} + 1$$

$$f2 := 2\,f1 - 1$$

$$f3 := 4\,f1 - 3$$

$$f4 := 8\,f1 - 7$$

$$9.965719176\,10^{2352}$$

$$g := h \to \frac{1}{\ln(f1(h))\ln(f2(h))\ln(f3(h))\ln(f4(h))} \tag{1.8.39}$$

```
> H:=2.^49; H/6*(g(0)+4*g(H/2)+g(H));
```

$$H := 5.629499534\,10^{14}$$

$$0.6408232407 \tag{1.8.40}$$

```
> Cf1f2f3f4:=C4*(1-1/2)/(1-1/2)^4*(1-2/3)/(1-1/3)^4*(1-4/5)/(1
  -1/5)^4*(1-3/7)/(1-4/7);
```

(1.8.41)

$$Cf1f2f3f4 := \frac{1125}{128}\ C4 \tag{1.8.41}$$

```
> %*%%; subs(C4=Cs4,%); HCCII4:=H/%;
```

$$5.632235514\ C4$$

$$1.731884327$$

$$HCCII4 := 3.250505502\ 10^{14} \tag{1.8.42}$$

```
> # Cunningham chain II, length 5
> f1:=h->(15.+30*h)*2.^(30*128)+1;
  f2:=2*f1-1; f3:=2*f2-1; f4:=2*f3-1; f5:=2*f4-1; f5(1);
  g:=h->1/ln(f1(h))/ln(f2(h))/ln(f3(h))/ln(f4(h))/ln(f5(h));
```

$$f1 := h \to (15. + 30\ h)\ 2.^{3840} + 1$$

$$f2 := 2\ f1 - 1$$

$$f3 := 4\ f1 - 3$$

$$f4 := 8\ f1 - 7$$

$$f5 := 16\ f1 - 15$$

$$6.494053261\ 10^{1158}$$

$$g := h \to \frac{1}{\ln(f1(h))\ln(f2(h))\ln(f3(h))\ln(f4(h))\ln(f5(h))} \tag{1.8.43}$$

```
> H:=2.^54; H/6*(g(0)+4*g(H/2)+g(H));
```

$$H := 1.801439851\ 10^{16}$$

$$0.1262771782 \tag{1.8.44}$$

```
> Cf1f2f3f4f5:=C5*(1-1/2)/(1-1/2)^5*(1-2/3)/(1-1/3)^5*(1-4/5)/
  (1-1/5)^5*(1-2/7)/(1-4/7);
```

$$Cf1f2f3f4f5 := \frac{84375}{2048}\ C5 \tag{1.8.45}$$

```
> %*%%; subs(C5=Cs5,%); HCCII5:=H/%;
```

$$5.202459429\ C5$$

$$2.132358865$$

$$HCCII5 := 8.448108246\ 10^{15} \tag{1.8.46}$$

```
> # Cunningham chain II, length 6
> f1:=h->(15.+30*h)*2.^(29*64)+1;
  f2:=2*f1-1; f3:=2*f2-1; f4:=2*f3-1; f5:=2*f4-1; f6:=2*f4-1;
  f6(1);
  g:=h->1/ln(f1(h))/ln(f2(h))/ln(f3(h))/ln(f4(h))/ln(f5(h))/ln
  (f6(h));
```

$$f1 := h \to (15. + 30\ h)\ 2.^{1856} + 1$$

$$f2 := 2\ f1 - 1$$

$$f3 := 4\,f1 - 3$$

$$f4 := 8\,f1 - 7$$

$$f5 := 16\,f1 - 15$$

$$f6 := 16\,f1 - 15$$

$$3.706845189\,10^{561}$$

$$g := h \to \frac{1}{\ln(f1(h))\ln(f2(h))\ln(f3(h))\ln(f4(h))\ln(f5(h))\ln(f6(h))} \tag{1.8.47}$$

```
> H:=2.^58; H/6*(g(0)+4*g(H/2)+g(H));
```

$$H := 2.882303762\,10^{17}$$

$$0.05355737752 \tag{1.8.48}$$

```
> Cf1f2f3f4f5f6:=C6*(1-1/2)/(1-1/2)^6*(1-2/3)/(1-1/3)^6*(1-4/5)
  /(1-1/5)^6*(1-1/7)/(1-4/7)*(1-5/31)/(1-6/31);
```

$$Cf1f2f3f4f5f6 := \frac{394875}{2048}\,C6 \tag{1.8.49}$$

```
> %*%%; subs(C6=Cs6,%); HCCII6:=H/%;
```

$$10.32640110\,C6$$

$$1.927056039$$

$$HCCII6 := 1.495703137\,10^{17} \tag{1.8.50}$$

▶ 1.9. Kérdés.

▼ 1.10. Eratosztenész szitája.

```
> sieve:=proc(N::posint) local n,B,i,j;
  n:=floor((N-1)/2);
  B:=Array(0..n-1);
  for j from 0 to n-1 do B[j]:=1 od;
  j:=0;
  while j<n do
    while B[j]=0 do j:=j+1 od;
    i:=2*j^2+6*j+3;
    if i>=n then break fi;
    while i<n do B[i]:=0; i:=i+2*j+3 od;
    j:=j+1;
  od; B; end;
```

$$sieve := \mathbf{proc}(N::posint) \tag{1.10.1}$$

$$\quad \mathbf{local}\ n, B, i, j;$$

$$\quad n := \mathrm{floor}(1/2*N - 1/2);$$

$$\quad B := Array(0..n-1);$$

```
    for j from 0 to n - 1 do
        B[j] := 1
    end do;
    j := 0;
    while j < n do
        while B[j] = 0 do
            j := j + 1
        end do;
        i := 2 * j^2 + 6 * j + 3;
        if n <= i then
            break
        end if;
        while i < n do
            B[i] := 0;
            i := i + 2 * j + 3
        end do;
        j := j + 1
    end do;
    B
end proc
```

```
> debug(sieve); sieve(21);
```

$$sieve$$

```
{--> enter sieve, args = 21
```

$$n := 10$$

$$B := Array(0..9, \{\}, datatype = anything, storage = rectangular, order = Fortran\_order)$$

$$B_0 := 1$$

$$B_1 := 1$$

$$B_2 := 1$$

$$B_3 := 1$$

$$B_4 := 1$$

$$B_5 := 1$$

$$B_6 := 1$$

$$B_7 := 1$$

$$B_8 := 1$$

$$B_9 := 1$$

$$j := 0$$

$$i := 3$$

$$B_3 := 0$$

$$i := 6$$

$$B_6 := 0$$

$$i := 9$$

$$B_9 := 0$$

$$i := 12$$

$$j := 1$$

$$i := 11$$

$$Array(0..9, \{0 = 1, 1 = 1, 4 = 1, 5 = 1, 7 = 1, 8 = 1, 2 = 1\},$$
$$datatype = anything, storage = rectangular, order = Fortran\_order)$$

```
<-- exit sieve (now at top level) = Array(0..9, {(1) = 1,
(2) = 1, (3) = 1, (4) = 0, (5) = 1, (6) = 1, (7) = 0, (8)
= 1, (9) = 1})}
```

$$Array(0..9, \{0 = 1, 1 = 1, 4 = 1, 5 = 1, 7 = 1, 8 = 1, 2 = 1\}, \quad (1.10.2)$$
$$datatype = anything, storage = rectangular, order = Fortran\_order)$$

```
> undebug(sieve); sieve(10000);
```

$$sieve$$

$$\begin{bmatrix} 0 .. 4998\ Array \\ Data\ Type: anything \\ Storage: rectangular \\ Order: Fortran\_order \end{bmatrix} \quad (1.10.3)$$

```
> #
  # This is a simple pretest for a number
  # using gcd. If there is a true divisor
  # below 1000 then the result is false else true.
  #

  pre:=proc(n::integer) local p;
```

```
p:=[2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,
73,79,83,89,97];
if n < 2 then false
elif has(p,n) then true
elif igcd(230556796394551842475310214733175607O,n) <> 1 then
false
elif n < 10201 then true
elif igcd(\
84969694892334181105323399091873499659260625866489327366115454
263422038932\
70769390909069477309509137509786917118668028861499333825097682
386722983737\
96296306675767413112673657893644078815718696989373063311306647
862044862494\
92573240226273954373636390387526081667586612559568346306972204
475122988482\
22228550062683786342519960225996301315945644470064720696621750
477244528915\
927867113,n) <> 1 then
  false
else
  true
fi end;
```

$$pre := \mathbf{proc}(n::integer) \tag{1.10.4}$$

```
local p;
p := [2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61,
67, 71, 73, 79, 83, 89, 97];
if n < 2 then
    false
elif has(p, n) then
    true
elif igcd(230556796394551842475310214733175607O,
n) <> 1 then
    false
elif n < 10201 then
    true
elif igcd(
    84969694892334181105323399091873499659260625866489\
    27366115454263422038932707693909090694773095091375\
    09786917118668028861499333825097682386722983737\
    96296306675767413112673657893644078815718696989373\
    06331130664786204486249\
    47512298848222285500626837863425199602259963013159\
    45644470064720696621750477244528915\
    927867113\
```

```
        306675767413112673657893644078815718696989373063311\
        306647862044862494925732402262739543736363903875260\
        816675866125595683463069722044751229884822222855006\
        268378634251996022599630131594564447006472069662175\
        0477244528915927867113, n) <> 1 then
        false
    else
        true
    end if
end proc
```

```
> #
  # This simple routine do the the simple pretest for n+id, 0<=
  i<N.
  # The result is the list of i's not find to be composite.
  #

  preseq:=proc(n,d,N) local i,L;
  L:=[];
  for i from 0 to N-1 do
    if pre(n+i*d) then L:=[op(L),i]; fi;
  od; L; end;
```

$$preseq := \mathbf{proc}(n, d, N) \tag{1.10.5}$$

```
    local i, L;
    L := [ ];
    for i from 0 to N - 1 do
        if pre(n + i* d) then
            L := [op(L), i]
        end if
    end do;
    L
end proc
```

```
> Digits:=10000; n:=ceil(Pi*10^4999): L:=preseq(n,2,30000):
```

$$Digits := 10000 \tag{1.10.6}$$

▶ 1.11. Feladat.

▼ 1.12. Moduláris inverz euklidészi algoritmussal.

```
> #
  # Calculation of the greatest common
  # divisor by the Euclidean algorithm.
  #

  eucgcd:=proc(x::integer,y::integer) local u,v,r;
  u:=abs(x); v:=abs(y);
  while v<>0 do r:=irem(u,v); u:=v; v:=r od;
  u end;
```

$$eucgcd := \mathbf{proc}(x::integer, y::integer) \tag{1.12.1}$$

$$\mathbf{local}\ u, v, r;$$
$$u := \mathrm{abs}(x);$$
$$v := \mathrm{abs}(y);$$
$$\mathbf{while}\ v <> 0\ \mathbf{do}$$
$$r := irem(u, v);$$
$$u := v;$$
$$v := r$$
$$\mathbf{end\ do};$$
$$u$$
$$\mathbf{end\ proc}$$

```
> #
  # Calculation of the modular inverse by the Euclidean
  # algorithm using division.
  #

  modinvdiv:=proc(a::integer,m::integer) local x1,x2,x3,d1,d2,
  d3,q,p;
  x1:=1; d1:=a; x2:=0; d2:=m; p:=0;
  do
    if d2=0 then
      if p=0 then return [x1,d1]
      elif x1=0 then return [x1,d1]
      else return [m-x1,d1]
      fi;
    fi;
    q:=iquo(d1,d2); d3:=d1-q*d2; x3:=x1+q*x2; p:=1-p;
    x1:=x2; x2:=x3; d1:=d2; d2:=d3;
  od; end;
```

$$modinvdiv := \mathbf{proc}(a::integer, m::integer) \tag{1.12.2}$$

$$\mathbf{local}\ x1, x2, x3, d1, d2, d3, q, p;$$
$$x1 := 1;$$

```
    d1 := a;
    x2 := 0;
    d2 := m;
    p := 0;
    do
        if d2 = 0 then
            if p = 0 then
                return [x1, d1]
            elif x1 = 0 then
                return [x1, d1]
            else
                return [m - x1, d1]
            end if
        end if;
        q := iquo(d1, d2);
        d3 := d1 - q*d2;
        x3 := x1 + q*x2;
        p := 1 - p;
        x1 := x2;
        x2 := x3;
        d1 := d2;
        d2 := d3
    end do
end proc
```

```
> modinvdiv(13874,15543);
```

$$[8903, 1] \tag{1.12.3}$$

▶ 1.13. Feladat.

▼ 1.14. Moduláris inverz bináris lnko algoritmussal.

```
> #
  # Calculation of the greatest common
  # divisor by the binary algorithm.
```

```
#

bingcd:=proc(x::integer,y::integer) local u,v,k,t;
u:=abs(x); v:=abs(y);
if u=0 then RETURN(v) fi;
if v=0 then RETURN(u) fi;
k:=0;
while type(u,even) and type(v,even) do k:=k+1; u:=u/2; v:=v/2
od;
if type(u,odd) then t:=-v else t:=u fi;
while t<>0 do
  while type(t,even) do t:=t/2 od;
  if t>0 then u:=t else v:=-t fi;
  t:=u-v;
od; u*2^k end;
```

$$bingcd := \mathbf{proc}(x::integer,\ y::integer) \tag{1.14.1}$$

local $u,\ v,\ k,\ t;$

$u := \mathrm{abs}(x);$

$v := \mathrm{abs}(y);$

if $u = 0$ **then**

$RETURN(v)$

end if;

if $v = 0$ **then**

$RETURN(u)$

end if;

$k := 0;$

while $type(u,\ even)$ **and** $type(v,\ even)$ **do**

$k := k + 1;$

$u := 1 / 2 * u;$

$v := 1 / 2 * v$

end do;

if $type(u,\ odd)$ **then**

$t := -v$

else

$t := u$

end if;

while $t <> 0$ **do**

```
        while type(t, even) do
            t := 1 / 2 * t
        end do;
        if 0 < t then
            u := t
        else
            v := -t
        end if;
        t := u - v
    end do;
    u * 2 ^ k
end proc
```

1.15. Feladat.

```
> #
  # Calculation of the modular inverse with respect to an
  # odd modulus by the binary gcd algorithm.
  #
  oddmodinvbin:=proc(a::nonnegint,m::posint)
  local x1,x2,x3,d1,d2,d3,p;
  if not type(m,odd) then error "second argument have to be
  odd",m fi;
  if m=1 then return [0,1] fi;
  x1:=1; d1:=a mod m; x2:=m; d2:=m;
  if type(d1,even) then x3:=0; d3:=m; p:=1 else x3:=1; d3:=d1;
  p:=0 fi;
  while d3<>0 do
    while type(d3,even) do d3:=d3/2;
      if type(x3,even) then x3:=x3/2 else x3:=(x3+m)/2 fi;
    od;
    if p=0 then x1:=x3; d1:=d3 else x2:=m-x3; d2:=d3 fi;
    if x1>=x2 then x3:=x1-x2 else x3:=m+x1-x2 fi;
    if d1>=d2 then d3:=d1-d2; p:=0 else d3:=d2-d1; p:=1 fi;
  od; [x1,d1] end;
```

```
oddmodinvbin := proc(a::nonnegint, m::posint)                    (1.15.1)
    local x1, x2, x3, d1, d2, d3, p;
    if not type(m, odd) then
        error "second argument have to be odd", m
    end if;
```

```
if m = 1 then
    return [0, 1]
end if;
x1 := 1;
d1 := mod(a, m);
x2 := m;
d2 := m;
if type(d1, even) then
    x3 := 0;
    d3 := m;
    p := 1
else
    x3 := 1;
    d3 := d1;
    p := 0
end if;
while d3 <> 0 do
    while type(d3, even) do
        d3 := 1 / 2 * d3;
        if type(x3, even) then
            x3 := 1 / 2 * x3
        else
            x3 := 1 / 2 * x3 + 1 / 2 * m
        end if
    end do;
    if p = 0 then
        x1 := x3;
        d1 := d3
    else
        x2 := m - x3;
        d2 := d3
    end if;
```

```
        if x2 <= x1 then
            x3 := x1 - x2
        else
            x3 := m + x1 - x2
        end if;
        if d2 <= d1 then
            d3 := d1 - d2;
            p := 0
        else
            d3 := d2 - d1;
            p := 1
        end if
    end do;
    [x1, d1]
end proc
```

```
> oddmodinvbin(13874,15543);
```

$$[8903, 1] \tag{1.15.2}$$

```
> trymodinvs:=proc(n) local i,a,m,b,d;
  for i to n do a:=rand(); m:=rand();
    if type(m,even) then m:=m+1; fi;
    a:=a mod m;
    d:=modinvdiv(a,m); b:=oddmodinvbin(a,m);
    if d[1]*a-d[2]mod m<>0 or b[1]*a-b[2] mod m<>0 then print
  (a,m,d,b) fi;
  od; end;
```

$$trymodinvs := \mathbf{proc}(n) \tag{1.15.3}$$

```
    local i, a, m, b, d;
    for i to n do
        a := rand( );
        m := rand( );
        if type(m, even) then
            m := m + 1
        end if;
        a := mod(a, m);
        d := modinvdiv(a, m);
```

```
        b := oddmodinvbin(a, m);
        if mod(d[1]*a - d[2], m) <> 0 or mod(b[1]*a - b[2],
        m) <> 0 then
            print(a, m, d, b)
        end if
    end do
end proc
```

```
> trymodinvs(10);
```

1.16. Általános szita.

```
> Digits:=10;
```

$$Digits := 10 \tag{1.16.1}$$

1.17. Programozási problémák.

```
> #
  # This procedure calculate the sum of the reciprocal
  # of primes up to x and compare with ln(ln(x)).
  #

  sumprimerec:=proc(x) local s,p,i;
  s:=0.; p:=2;
  while p<x do
    s:=evalf(s+1/p); p:=nextprime(p)
  od; [s,evalf(s-ln(ln(x)))] end;
```

```
sumprimerec := proc(x)
    local s, p, i;
    s := 0.;
    p := 2;
    while p < x do
        s := evalf(s + 1 / p);
        p := nextprime(p)
    end do;
    [s, evalf(s - ln(ln(x)))]
end proc
```

(1.17.1)

```
> sumprimerec(10); sumprimerec(100); sumprimerec(1000);
  sumprimerec(10000); sumprimerec(100000); sumprimerec(1000000)
```

```
;
```

$$[1.176190476,\ 0.3421580307]$$
$$[1.802817203,\ 0.275637577]$$
$$[2.198080131,\ 0.265435397]$$
$$[2.483059958,\ 0.262733152]$$
$$[2.705272178,\ 0.261801821]$$
$$[2.887328140,\ 0.261536225] \qquad (1.17.2)$$

1.18. A szitálás dúsító hatása.

```
> #
  # This procedure calculate the factor qsAB.
  #

  qsAB:=proc(s::posint,A::posint,B::posint) local P,p;
  P:=1.; p:=nextprime(A-1);
  while p<B do P:=P*(1-s/p); p:=nextprime(p) od;
  P end;
```

$$qsAB := \mathbf{proc}(s::posint,\ A::posint,\ B::posint) \qquad (1.18.1)$$
$$\quad \mathbf{local}\ P,\ p;$$
$$\quad P := 1.;$$
$$\quad p := nextprime(A-1);$$
$$\quad \mathbf{while}\ p < B\ \mathbf{do}$$
$$\qquad P := P*(1-s/p);$$
$$\qquad p := nextprime(p)$$
$$\quad \mathbf{end\ do};$$
$$\quad P$$
$$\mathbf{end\ proc}$$

```
> qsAB(1,1,100);
```

$$0.1203172905 \qquad (1.18.2)$$

```
> B:=10: [qsAB(1,1,B),evalf(qsAB(1,1,B)-exp(-gamma)/ln(B))];
  B:=100: [qsAB(1,1,B),evalf(qsAB(1,1,B)-exp(-gamma)/ln(B))];
  B:=1000: [qsAB(1,1,B),evalf(qsAB(1,1,B)-exp(-gamma)/ln(B))];
  B:=10000: [qsAB(1,1,B),evalf(qsAB(1,1,B)-exp(-gamma)/ln(B))];
  B:=100000: [qsAB(1,1,B),evalf(qsAB(1,1,B)-exp(-gamma)/ln(B))]
  ;
  B:=1000000: [qsAB(1,1,B),evalf(qsAB(1,1,B)-exp(-gamma)/ln(B))
  ];
```

$$[0.2285714285,\ -0.0152673270]$$
$$[0.1203172905,\ -0.0016020873]$$
$$[0.08096526349,\ -0.00031432167]$$
$$[0.06088469238,\ -0.00007499650]$$
$$[0.04875291757,\ -0.00001483353]$$

```
Warning,  computation interrupted
```

1.19. Példa.

```
> qsAB(2,7,1000000);
```

```
Warning,  computation interrupted
```

```
> %*(ln(1000000.)/ln(44000.*2^25))^2;
```

$$0.005300634160 \tag{1.19.1}$$

```
> # Cunningham chain I, length 3
> f:=qsAB(3,16,64);
```

$$f := 0.3083691608 \tag{1.19.2}$$

```
> f:=f*(1-3/5)*(1-3/7)*(1-3/11);
```

$$f := 0.05126136699 \tag{1.19.3}$$

```
> H:=HCCI3; H/8; H*f; 2*%;
```

$$H := 8.131017110\ 10^{11}$$
$$1.016377139\ 10^{11}$$
$$4.168070521\ 10^{10}$$
$$8.336141042\ 10^{10} \tag{1.19.4}$$

```
> P:=20; f0:=qsAB(3,16,P); f0:=f0*(1-3/5)*(1-3/7)*(1-3/11);
  HCCI30:=H*f0; %/8; 2*%%;
  log[2.](H*f0); primeprod(P); op(2,[%])+%%;
```

$$P := 20$$
$$f0 := 0.6934984520$$
$$f0 := 0.1152828596$$
$$HCCI30 := 9.396724965\ 10^{9}$$
$$1.174590621\ 10^{9}$$
$$1.879344993\ 10^{10}$$
$$33.12951088$$
$$9699690,\ 23.20950721$$
$$56.33901809 \tag{1.19.5}$$

```
> P:=72; f1:=qsAB(3,16,P); f1:=f1*(1-3/5)*(1-3/7)*(1-3/11);
  HCCI31:=H*f1; %/8; 2*%%;
  log[2.](H*f1); primeprod(P); op(2,[%])+%%;
```

$$P := 72$$

$$f1 := 0.2821153222$$

$$f1 := 0.04689709252$$

$$HCCI31 := 3.813210617\ 10^{10}$$

$$4.766513271\ 10^{9}$$

$$7.626421234\ 10^{10}$$

$$35.15028717$$

$$5579408301266989609674153 90,\ 88.85024260$$

$$124.0005298 \qquad (1.19.6)$$

```
> P:=125; f:=qsAB(3,16,P); f:=f*(1-3/5)*(1-3/7)*(1-3/11); H*f;
  %/8; 2*%;
  log[2.](H*f); primeprod(P); op(2,[%])+%%;
```

$$P := 125$$

$$f := 0.2035989160$$

$$f := 0.03384501461$$

$$2.751943929\ 10^{10}$$

$$3.439929911\ 10^{9}$$

$$6.879859822\ 10^{9}$$

$$34.67973202$$

$$316100546404176077881452062915436624932746869 90,$$

$$154.4690759$$

$$189.1488079 \qquad (1.19.7)$$

```
> P:=165; f2:=qsAB(3,16,P); f2:=f2*(1-3/5)*(1-3/7)*(1-3/11);
  HCCI32:=H*f2; %/8; 2*%;
  log[2.](H*f2); primeprod(P); op(2,[%])+%%;
```

$$P := 165$$

$$f2 := 0.1718878746$$

$$f2 := 0.02857356876$$

$$HCCI32 := 2.323321765\ 10^{10}$$

$$2.904152206\ 10^{9}$$

$$5.808304412\ 10^{9}$$

34.43546992

576615221997595165902363003533613430656538401560606631985\

6068810, 211.8090789

246.2445488 (1.19.8)

```
> f10:=qsAB(3,16,2^16); f10:=f10*(1-3/5)*(1-3/7)*(1-3/11); H*
  f10; 2*%;
```

$$f10 := 0.01738880779$$

$$f10 := 0.002890607009$$

$$2.350357505\ 10^{9}$$

$$4.700715010\ 10^{9} \quad (1.19.9)$$

```
> f13:=f10*(16/19)^3; H*f13; 2*%;
```

$$f13 := 0.001726188411$$

$$1.403566750\ 10^{9}$$

$$2.807133500\ 10^{9} \quad (1.19.10)$$

```
> f15:=f13*(19/21)^3; H*f15; 2*%;
```

$$f15 := 0.001278471689$$

$$1.039527518\ 10^{9}$$

$$2.079055036\ 10^{9} \quad (1.19.11)$$

```
> f18:=f15*(21/24)^3; H*f18; 2*%;
```

$$f18 := 0.0008564761510$$

$$6.964022238\ 10^{8}$$

$$1.392804448\ 10^{9} \quad (1.19.12)$$

```
> f26:=f18*(24/32)^3; H*f26; 2*%;
```

$$f26 := 0.0003613258762$$

$$2.937946882\ 10^{8}$$

$$5.875893764\ 10^{8} \quad (1.19.13)$$

```
> f29:=f26*(32/35)^3; H*f26; 2*%;
```

$$f29 := 0.0002761498848$$

$$2.937946882\ 10^{8}$$

$$5.875893764\ 10^{8} \quad (1.19.14)$$

```
> f34:=f29*(35/40)^3; H*f34; 2*%;
```

$$f34 := 0.0001849988486$$

$$1.504228803\ 10^{8}$$

$$3.008457606\,10^{8} \tag{1.19.15}$$

```
> f44:=f34*(40/50)^3; H*f44; 2*%;
```

$$f44 := 0.00009471941048$$

$$7.701651473\,10^{7}$$

$$1.540330295\,10^{8} \tag{1.19.16}$$

```
> #64 bit multiplier
  H:=HCCI30; %/8; 2*%%; P:=20.;
```

$$H := 9.396724965\,10^{9}$$

$$1.174590621\,10^{9}$$

$$1.879344993\,10^{10}$$

$$P := 20. \tag{1.19.17}$$

```
> 150*p50/3.2/10^9; # 64 bit reciprocal modulo primes <= 50
  bit, sec
```

$$1.522809574\,10^{6} \tag{1.19.18}$$

```
> # density and number after sieve with <=16 bit primes
  f10/f0; %*H;
```

$$0.02507403979$$

$$2.356138557\,10^{8} \tag{1.19.19}$$

```
> # sieve events for <=16 bit primes
  3*H*ln(16./ln(P));
```

$$4.722981963\,10^{10} \tag{1.19.20}$$

```
> # density and number after sieve with <=32 bit primes
  f26/f0; %*H;
```

$$0.003134254975$$

$$2.945173197\,10^{7} \tag{1.19.21}$$

```
> # sieve events for <=32 bit primes
  3*H*ln(32./ln(P));
```

$$6.676975986\,10^{10} \tag{1.19.22}$$

```
> # sieve events for <=50 bit primes
  3*H*ln(50./ln(P));
```

$$7.935067137\,10^{10} \tag{1.19.23}$$

```
> %/0.8/10^9; # total sieve time in sec
```

$$99.18833921 \tag{1.19.24}$$

```
> #128 bit multiplier
  H:=HCCI31; %/8; 2*%%; P:=72.;
```

$$H := 3.813210617\,10^{10}$$

$$4.766513271\ 10^{9}$$

$$7.626421234\ 10^{10}$$

$$P := 72. \tag{1.19.25}$$

```
> 200*p50/3.2/10^9; # 128 bit reciprocal modulo primes <=50
  bit, sec
```

$$2.030412766\ 10^{6} \tag{1.19.26}$$

```
> # density and number after sieve with <=16 bit primes
  f10/f1; %*H;
```

$$0.06163723279$$

$$2.350357505\ 10^{9} \tag{1.19.27}$$

```
> # sieve events for <=16 bit primes
  3*H*ln(16./ln(P));
```

$$1.509362140\ 10^{11} \tag{1.19.28}$$

```
> # density and number after sieve with <=32 bit primes
  f26/f1; %*H;
```

$$0.007704654101$$

$$2.937946882\ 10^{8} \tag{1.19.29}$$

```
> # sieve events for <=32 bit primes
  3*H*ln(32./ln(P));
```

$$2.302296997\ 10^{11} \tag{1.19.30}$$

```
> # sieve events for <=50 bit primes
  3*H*ln(50./ln(P));
```

$$2.812833013\ 10^{11} \tag{1.19.31}$$

```
> %/0.8/10^9; # total sieve time in sec
```

$$351.6041266 \tag{1.19.32}$$

```
> #256 bit multiplier
  H:=HCCI32; %/8; 2*%%; P:=165.;
```

$$H := 2.323321765\ 10^{10}$$

$$2.904152206\ 10^{9}$$

$$4.646643530\ 10^{10}$$

$$P := 165. \tag{1.19.33}$$

```
> 300*p50/3.2/10^9; # 256 bit reciprocal modulo primes <=50
  bit, sec
```

$$3.045619148\ 10^{6} \tag{1.19.34}$$

```
> # density and number after sieve with <=16 bit primes
  f10/f2; %*H;
```

$$0.1011636675$$

$$2.350357505\ 10^{9} \tag{1.19.35}$$

```
> # sieve events for <=16 bit primes
  3*H*ln(16./ln(P));
```

$$7.960976442\ 10^{10} \tag{1.19.36}$$

```
> # density and number after sieve with <=32 bit primes
  f26/f2; %*H;
```

$$0.01264545844$$

$$2.937946882\ 10^{8} \tag{1.19.37}$$

```
> # sieve events for <=32 bit primes
  3*H*ln(32./ln(P));
```

$$1.279218824\ 10^{11} \tag{1.19.38}$$

```
> # sieve events for <=50 bit primes
  3*H*ln(50./ln(P));
```

$$1.590279386\ 10^{11} \tag{1.19.39}$$

```
> %/0.8/10^9; # total sieve time in sec
```

$$198.7849232 \tag{1.19.40}$$

```
> H*f44/f2*300/8^2/1.352243653; # SPU time in sec
```

$$2.669747512\ 10^{8} \tag{1.19.41}$$

```
> %/24/3600/420; # time for all SPU in days
```

$$7.357108444 \tag{1.19.42}$$

```
> # Cunningham chain I, length 4
> f:=qsAB(4,16,64);
```

$$f := 0.2014070537 \tag{1.19.43}$$

```
> f:=f*(1-3/7)*(1-4/11);
```

$$f := 0.07323892862 \tag{1.19.44}$$

```
> H:=HCCI4; H/8; H*f; 2*%;
```

$$H := 1.868714841\ 10^{14}$$

$$2.335893551\ 10^{13}$$

$$1.368626729\ 10^{13}$$

$$2.737253458\ 10^{13} \tag{1.19.45}$$

```
> P:=16; f0:=qsAB(4,16,P); f0:=f0*(1-3/7)*(1-4/11);
  HCCI40:=H*f0; %/8; 2*%%;
  log[2.](H*f0); primeprod(P); op(2,[%])+%%;
```

$$P := 16$$

$$f0 := 1.$$

$$f0 := 0.3636363636$$

$$HCCI40 := 6.795326694\ 10^{13}$$

$$8.494158368\ 10^{12}$$

$$1.359065339\ 10^{14}$$

$$45.94960815$$

$$30030,\ 14.87411685$$

$$60.82372500 \qquad (1.19.46)$$

```
> P:=65; f1:=qsAB(4,16,P); f1:=f1*(1-3/7)*(1-4/11);
  HCCI41:=H*f1; %/8; 2*%%;
  log[2.](H*f1); primeprod(P); op(2,[%])+%%;
```

$$P := 65$$

$$f1 := 0.2014070537$$

$$f1 := 0.07323892862$$

$$HCCI41 := 1.368626729\ 10^{13}$$

$$1.710783411\ 10^{12}$$

$$2.737253458\ 10^{13}$$

$$43.63779426$$

$$11728838135940697098327 0,\ 76.63440629$$

$$120.2722006 \qquad (1.19.47)$$

```
> P:=112; f:=qsAB(4,16,P); f:=f*(1-3/7)*(1-4/11); H*f; %/8; 2*
  %%;
  log[2.](H*f); primeprod(P); op(2,[%])+%%;
```

$$P := 112$$

$$f := 0.1196385085$$

$$f := 0.04350491218$$

$$8.129827505\ 10^{12}$$

$$1.016228438\ 10^{12}$$

$$1.625965501\ 10^{13}$$

$$42.88636188$$

$$27973499681785493617827616187206780967499723 0,\ 147.6488969$$

$$190.5352588 \qquad (1.19.48)$$

```
> P:=162; f2:=qsAB(4,16,P); f2:=f2*(1-3/7)*(1-4/11);
  HCCI42:=H*f2; %/8; 2*%%;
  log[2.](H*f2); primeprod(P); op(2,[%])+%%;
```

$$P := 162$$

$$f2 := 0.09432282555$$

$$f2 := 0.03429920929$$

$$HCCI42 := 6.409544143\ 10^{12}$$

$$8.011930179\ 10^{11}$$

$$1.281908829\ 10^{13}$$

$$42.54335889$$

$$353751669937174948406357670879517442120575706478899774 22\backslash$$

$$429870,\ 204.4603507$$

$$247.0037096 \qquad (1.19.49)$$

```
> P:=352; f3:=qsAB(4,16,P); f3:=f3*(1-3/7)*(1-4/11);
  HCCI43:=H*f3; %/8; 2*%%;
  log[2.](H*f3); primeprod(P); op(2,[%])+%%;
```

$$P := 352$$

$$f3 := 0.05406366911$$

$$f3 := 0.01965951604$$

$$HCCI43 := 3.673802939\ 10^{12}$$

$$4.592253674\ 10^{11}$$

$$7.347605878\ 10^{12}$$

$$41.74041138$$

$$261546705642188677527611215060743478325487449257450867 55\backslash$$

$$984554020808257968770469622308791221292930453128 62\backslash$$

$$982002442929235116570748721133303 70,\ 466.4570019$$

$$508.1974133 \qquad (1.19.50)$$

```
> f10:=qsAB(4,16,2^16);
```

$$f10 := 0.004328381635 \qquad (1.19.51)$$

```
> f10:=f10*(1-3/7)*(1-4/11); H*f10; 2*%;
```

$$f10 := 0.001573956958$$

$$2.941276727\ 10^{11}$$

$$5.882553454\ 10^{11} \qquad (1.19.52)$$

```
> f13:=f10*(16/19)^4; H*f13; 2*%;
```

$$f13 := 0.0007915135949$$

$$1.479113202\ 10^{11}$$

$$2.958226404\ 10^{11} \tag{1.19.53}$$

```
> f15:=f13*(19/21)^4; H*f15; 2*%;
```

$$f15 := 0.0005303903374$$

$$9.911482950\ 10^{10}$$

$$1.982296590\ 10^{11} \tag{1.19.54}$$

```
> f18:=f18*(21/24)^4; H*f18; 2*%;
```

$$f18 := 0.0005020505953$$

$$9.381893984\ 10^{10}$$

$$1.876378797\ 10^{11} \tag{1.19.55}$$

```
> f26:=f18*(24/32)^4; H*f26; 2*%;
```

$$f26 := 0.0001588519462$$

$$2.968489894\ 10^{10}$$

$$5.936979788\ 10^{10} \tag{1.19.56}$$

```
> f29:=f26*(24/35)^4; H*f29; 2*%;
```

$$f29 := 0.00003512087517$$

$$6.563090066\ 10^{9}$$

$$1.312618013\ 10^{10} \tag{1.19.57}$$

```
> f34:=f29*(35/40)^4; H*f34; 2*%;
```

$$f34 := 0.00002058721223$$

$$3.847162903\ 10^{9}$$

$$7.694325806\ 10^{9} \tag{1.19.58}$$

```
> f44:=f34*(40/50)^4; H*f44; 2*%;
```

$$f44 := 0.000008432522129$$

$$1.575797925\ 10^{9}$$

$$3.151595850\ 10^{9} \tag{1.19.59}$$

```
> #64 bit multiplier
  H:=HCCI40; %/8; 2*%%; P:=16.;
```

$$H := 6.795326694\ 10^{13}$$

$$8.494158368\ 10^{12}$$

$$1.359065339\ 10^{14}$$

$$P := 16. \tag{1.19.60}$$

```
> 150*p50/3.2/10^9; # 64 bit reciprocal modulo primes <= 50
  bit, sec
```

(1.19.61)

$$1.522809574\,10^{6} \tag{1.19.61}$$

```
> # density and number after sieve with <=16 bit primes
  f10/f0; %*H;
```

$$0.004328381635$$

$$2.941276727\,10^{11} \tag{1.19.62}$$

```
> # sieve events for <=16 bit primes
  4*H*ln(16./ln(P));
```

$$4.764359244\,10^{14} \tag{1.19.63}$$

```
> # density and number after sieve with <=32 bit primes
  f26/f0; %*H;
```

$$0.0004368428521$$

$$2.968489894\,10^{10} \tag{1.19.64}$$

```
> # sieve events for <=32 bit primes
  4*H*ln(32./ln(P));
```

$$6.648423860\,10^{14} \tag{1.19.65}$$

```
> # sieve events for <=50 bit primes
  4*H*ln(50./ln(P));
```

$$7.861490524\,10^{14} \tag{1.19.66}$$

```
> %/0.8/10^9; # total sieve time in sec
```

$$9.826863155\,10^{5} \tag{1.19.67}$$

```
> #128 bit multiplier
  H:=HCCI41; %/8; 2*%%; P:=65.;
```

$$H := 1.368626729\,10^{13}$$

$$1.710783411\,10^{12}$$

$$2.737253458\,10^{13}$$

$$P := 65. \tag{1.19.68}$$

```
> 200*p50/3.2/10^9; # 128 bit reciprocal modulo primes <=50
  bit, sec
```

$$2.030412766\,10^{6} \tag{1.19.69}$$

```
> # density and number after sieve with <=16 bit primes
  f10/f1; %*H;
```

$$0.02149071522$$

$$2.941276728\,10^{11} \tag{1.19.70}$$

```
> # sieve events for <=16 bit primes
  4*H*ln(16./ln(P));
```

(1.19.71)

$$7.355663200\,10^{13} \tag{1.19.71}$$

```
> # density and number after sieve with <=32 bit primes
  f26/f1; %*H;
```

$$0.002168955079$$

$$2.968489895\,10^{10} \tag{1.19.72}$$

```
> # sieve events for <=32 bit primes
  4*H*ln(32./ln(P));
```

$$1.115030224\,10^{14} \tag{1.19.73}$$

```
> # sieve events for <=50 bit primes
  4*H*ln(50./ln(P));
```

$$1.359350406\,10^{14} \tag{1.19.74}$$

```
> %/0.8/10^9; # total sieve time in sec
```

$$1.699188008\,10^{5} \tag{1.19.75}$$

```
> #256 bit multiplier
  H:=HCCI42; %/8; 2*%%; P:=162.;
```

$$H := 6.409544143\,10^{12}$$

$$8.011930179\,10^{11}$$

$$1.281908829\,10^{13}$$

$$P := 162. \tag{1.19.76}$$

```
> 300*p50/3.2/10^9; # 256 bit reciprocal modulo primes <=50
  bit, sec
```

$$3.045619148\,10^{6} \tag{1.19.77}$$

```
> # density and number after sieve with <=16 bit primes
  f10/f2; %*H;
```

$$0.04588901583$$

$$2.941276726\,10^{11} \tag{1.19.78}$$

```
> # sieve events for <=16 bit primes
  4*H*ln(16./ln(P));
```

$$2.937579292\,10^{13} \tag{1.19.79}$$

```
> # density and number after sieve with <=32 bit primes
  f26/f2; %*H;
```

$$0.004631358841$$

$$2.968489893\,10^{10} \tag{1.19.80}$$

```
> # sieve events for <=32 bit primes
  4*H*ln(32./ln(P));
```

$$4.714682272\,10^{13} \tag{1.19.81}$$

```
> # sieve events for <=50 bit primes
  4*H*ln(50./ln(P));
```

$$5.858881024\,10^{13} \tag{1.19.82}$$

```
> %/0.8/10^9; # total sieve time in sec
```

$$73236.01280 \tag{1.19.83}$$

```
> #512 bit multiplier
  H:=HCCI43; %/8; 2*%%; P:=352.;
```

$$H := 3.673802939\,10^{12}$$

$$4.592253674\,10^{11}$$

$$7.347605878\,10^{12}$$

$$P := 352. \tag{1.19.84}$$

```
> 500*p50/3.2/10^9; # 512 bit reciprocal modulo primes <=50
  bit, sec
```

$$5.076031914\,10^{6} \tag{1.19.85}$$

```
> # density and number after sieve with <=16 bit primes
  f10/f3; %*H;
```

$$0.08006081914$$

$$2.941276727\,10^{11} \tag{1.19.86}$$

```
> # sieve events for <=16 bit primes
  4*H*ln(16./ln(P));
```

$$1.475134241\,10^{13} \tag{1.19.87}$$

```
> # density and number after sieve with <=32 bit primes
  f26/f3; %*H;
```

$$0.008080155477$$

$$2.968489894\,10^{10} \tag{1.19.88}$$

```
> # sieve events for <=32 bit primes
  4*H*ln(32./ln(P));
```

$$2.493728700\,10^{13} \tag{1.19.89}$$

```
> # sieve events for <=50 bit primes
  4*H*ln(50./ln(P));
```

$$3.149557048\,10^{13} \tag{1.19.90}$$

```
> %/0.8/10^9; # total sieve time in sec
```

$$39369.46310 \tag{1.19.91}$$

```
> H*f44/f3*300/32^2/1.902321417; # SPU time in sec
```

$$2.426822010\,10^{8} \tag{1.19.92}$$

```
> %/24/3600/420; # time for all SPU in days
```

(1.19.93)

$$6.687670883 \tag{1.19.93}$$

```
> # Cunningham chain I, length 5
> f:=qsAB(5,18,64);
```

$$f := 0.1826597175 \tag{1.19.94}$$

```
> f:=f*(1-3/7)*(1-5/11)*(1-11/17);
```

$$f := 0.02009396434 \tag{1.19.95}$$

```
> H:=HCCI5; H/8; H*f; 2*%;
```

$$H := 1.286038928\ 10^{16}$$

$$1.607548660\ 10^{15}$$

$$2.584162036\ 10^{14}$$

$$5.168324072\ 10^{14} \tag{1.19.96}$$

```
> P:=60; f1:=qsAB(5,18,P); f1:=f1*(1-3/7)*(1-5/11)*(1-11/17);
  HCCI51:=H*f1; %/8; 2*%%;
  log[2.](H*f1); primeprod(P); op(2,[%])+%%;
```

$$P := 60$$

$$f1 := 0.1989686209$$

$$f1 := 0.02188806830$$

$$HCCI51 := 2.814890789\ 10^{14}$$

$$3.518613486\ 10^{13}$$

$$5.629781578\ 10^{14}$$

$$48.00007228$$

$$19227603501542126390 70, 70.70366895$$

$$118.7037412 \tag{1.19.97}$$

```
> P:=155; f2:=qsAB(5,18,P); f2:=f2*(1-3/7)*(1-5/11)*(1-11/17);
  HCCI52:=H*f2; %/8; 2*%%;
  log[2.](H*f2); primeprod(P); op(2,[%])+%%;
```

$$P := 155$$

$$f2 := 0.07271364745$$

$$f2 := 0.007999056710$$

$$HCCI52 := 1.028709832\ 10^{14}$$

$$1.285887290\ 10^{13}$$

$$2.057419664\ 10^{14}$$

$$46.54782943$$

$$225319534991831177328890236228992001350685163362356544 09\backslash$$

1910, 197.1657300

$$243.7135594 \tag{1.19.98}$$

```
> P:=348; f3:=qsAB(5,18,P); f3:=f3*(1-3/7)*(1-5/11)*(1-11/17);
  HCCI53:=H*f3; %/8; 2*%%;
  log[2.](H*f3); primeprod(P); op(2,[%])+%%;
```

$$P := 348$$

$$f3 := 0.03556400477$$

$$f3 := 0.003912312213$$

$$HCCI53 := 5.031385804\ 10^{13}$$

$$6.289232255\ 10^{12}$$

$$1.006277161\ 10^{14}$$

$$45.51602105$$

74941749467675838833126422653508159978649698927636351736\
34542699371993687326782126736043329883361161354908\
25788665595769374375572699465130, 458.0099187

$$503.5259398 \tag{1.19.99}$$

```
> P:=708; f4:=qsAB(5,18,P); f4:=f4*(1-3/7)*(1-5/11)*(1-11/17);
  HCCI54:=H*f4; %/8; 2*%%;
  log[2.](H*f4); primeprod(P); op(2,[%])+%%;
```

$$P := 708$$

$$f4 := 0.02007426457$$

$$f4 := 0.002208322458$$

$$HCCI54 := 2.839988647\ 10^{13}$$

$$3.549985809\ 10^{12}$$

$$5.679977294\ 10^{13}$$

$$44.69095040$$

1946777307011537734322150959962392523645975127818041588\
5837207534756855405403128279156705968461708578168\
6383270320345426848649201358189870448101413110086\
5589801520722077251521209385072554100321305456018\
5603695585660265284153421684796257245143362498012\
760214539505870197264858636122745485373430,
970.9640916

$$1015.655042 \tag{1.19.100}$$

```
> P:=1445; f5:=qsAB(5,18,P); f5:=f5*(1-3/7)*(1-5/11)*(1-11/17);
  HCCI55:=H*f5; %/8; 2*%%;
  log[2.](H*f5); primeprod(P); op(2,[%])+%%;
```

$$P := 1445$$

$$f5 := 0.01217022260$$

$$f5 := 0.001338817459$$

$$HCCI55 := 1.721771370\ 10^{13}$$

$$2.152214212\ 10^{12}$$

$$3.443542740\ 10^{13}$$

$$43.96895882$$

$$1403519010219287761549105830431220115755908413698112402\backslash$$
$$7484188803158848225968792931953530456906851740414\backslash$$
$$9298579923675395778118840207998057331610110266680\backslash$$
$$7053847184258236821659375569589235746486543818085\backslash$$
$$8609020387880316022271748111366650878400961675339\backslash$$
$$9464756155776230686191428602918437322026056816515\backslash$$
$$1582350802890385875943883713925628048069319405160\backslash$$
$$6928514707000049636241882348300313352955939810905\backslash$$
$$9884412563724780753344331151914116928826935340052\backslash$$
$$5081573843336158246625830959364476287275278180827\backslash$$
$$8779430325854057375727867974818602431761923534469\backslash$$
$$5160234778007078922368835190417752091176541624110\backslash$$
$$3624390, 1993.645906$$

$$2037.614865 \qquad (1.19.101)$$

```
> P:=2855; f6:=qsAB(5,18,P); f6:=f6*(1-3/7)*(1-5/11)*(1-11/17);
  HCCI56:=H*f6; %/8; 2*%%;
  log[2.](H*f6); primeprod(P); op(2,[%])+%%;
```

$$P := 2855$$

$$f6 := 0.007739639658$$

$$f6 := 0.0008514194887$$

$$HCCI56 := 1.094958607\ 10^{13}$$

$$1.368698259\ 10^{12}$$

$$2.189917214\ 10^{13}$$

$$43.31594157$$

$$1126871518000720857833670128223087548546764231553812251\backslash$$
$$98583951103071090527645902595112597140686374427 68\backslash$$
$$84531119745342945157132200570158492111805639694 88\backslash$$
$$95219570622727636453072291579647684843101574817 98\backslash$$
$$86011880385279787186620269503027324312397275463 83\backslash$$
$$01989152013115841283757375841064732036412054776 55\backslash$$
$$62304899190350075341672759060328760413736837332 65\backslash$$
$$46684748706922606287028188684590566303876095545 27\backslash$$
$$48662205502600446154410654373580725791507681560 46\backslash$$
$$24752098289534933566017605601544632709593557814 01\backslash$$
$$46229682943330103684989319910804663711091797001 22\backslash$$
$$97537905706333566370102511580877173916952874254 23\backslash$$
$$73520635121018323989117791333975095522433486402 64\backslash$$
$$96082695579953832538460251349713500114047540848 53\backslash$$
$$95122083314325713643167116602365064646394043464 02\backslash$$
$$47653470962458294063001320233277802427369132645 93\backslash$$
$$04676704982762300017976709826130649372069501711 59\backslash$$
$$67109546484242304767453951105108614131714518962 20\backslash$$
$$43480817417597528110322788445619639110470544581 60\backslash$$
$$91980063936987775973418933343376075734063402157 28\backslash$$
$$98142598136318934163434365912852519493735893948 21\backslash$$
$$17100041035454115082507068750798761012929146922 01\backslash$$
$$06796938062163101518997028826862282958060600048 16\backslash$$
$$49771968738354954374608044060883490271865794837 75\backslash$$
$$54558461418711317510355946406572038 10, 4046.280743$$

$$4089.596685 \tag{1.19.102}$$

```
> f10:=qsAB(5,18,2^16);
```

$$f10 := 0.001491299950 \tag{1.19.103}$$

```
> f10:=f10*(1-3/7)*(1-5/11)*(1-11/17); H*f10; 2*%;
```

$$f10 := 0.0001640543872$$

$$2.109803282\ 10^{12}$$

$$4.219606564\ 10^{12} \tag{1.19.104}$$

```
> f13:=f10*(16/19)^5; H*f13; 2*%;
```

$$f13 := 0.00006947359258$$

$$8.934574453\ 10^{11}$$

$$1.786914891\ 10^{12} \quad (1.19.105)$$

```
> f15:=f13*(19/21)^5; H*f15; 2*%;
```

$$f15 := 0.00004212028378$$

$$5.416832460\ 10^{11}$$

$$1.083366492\ 10^{12} \quad (1.19.106)$$

```
> f18:=f15*(21/24)^5; H*f18; 2*%;
```

$$f18 := 0.00002160386992$$

$$2.778341771\ 10^{11}$$

$$5.556683542\ 10^{11} \quad (1.19.107)$$

```
> f26:=f18*(24/32)^5; H*f26; 2*%;
```

$$f26 := 0.000005126699600$$

$$6.593135258\ 10^{10}$$

$$1.318627052\ 10^{11} \quad (1.19.108)$$

```
> f29:=f26*(32/35)^5; H*f29; 2*%;
```

$$f29 := 0.000003275273267$$

$$4.212128921\ 10^{10}$$

$$8.424257842\ 10^{10} \quad (1.19.109)$$

```
> f34:=f29*(35/40)^5; H*f34; 2*%;
```

$$f34 := 0.000001679916925$$

$$2.160438561\ 10^{10}$$

$$4.320877122\ 10^{10} \quad (1.19.110)$$

```
> f44:=f34*(40/50)^5; H*f44; 2*%;
```

$$f44 := 5.504751780\ 10^{-7}$$

$$7.079325078\ 10^{9}$$

$$1.415865016\ 10^{10} \quad (1.19.111)$$

```
> #128 bit multiplier
  H:=HCCI51; %/8; 2*%%; P:=60.;
```

$$H := 2.814890789\ 10^{14}$$

$$3.518613486\ 10^{13}$$

$$5.629781578\ 10^{14}$$

$$P := 60. \quad (1.19.112)$$

```
> 200*p50/3.2/10^9; # reciprocal modulo primes <=50 bit, sec
```

$$2.030412766\ 10^{6} \tag{1.19.113}$$

```
> # density and number after sieve with <=16 bit primes
  f10/f1; %*H;
```

$$0.007495151466$$

$$2.109803282\ 10^{12} \tag{1.19.114}$$

```
> # sieve events for <=16 bit primes
  5*H*ln(16./ln(P));
```

$$1.918322846\ 10^{15} \tag{1.19.115}$$

```
> # density and number after sieve with <=32 bit primes
  f26/f1; %*H;
```

$$0.0002342234833$$

$$6.593135257\ 10^{10} \tag{1.19.116}$$

```
> # sieve events for <=32 bit primes
  5*H*ln(32./ln(P));
```

$$2.893889652\ 10^{15} \tag{1.19.117}$$

```
> # sieve events for <=50 bit primes
  5*H*ln(50./ln(P));
```

$$3.522014381\ 10^{15} \tag{1.19.118}$$

```
> %/0.8/10^9; # total sieve time in sec
```

$$4.402517976\ 10^{6} \tag{1.19.119}$$

```
> #256 bit multiplier
  H:=HCCI52; %/8; 2*%%; P:=155.;
```

$$H := 1.028709832\ 10^{14}$$

$$1.285887290\ 10^{13}$$

$$2.057419664\ 10^{14}$$

$$P := 155. \tag{1.19.120}$$

```
> 300*p50/3.2/10^9; # reciprocal modulo primes <=50 bit, sec
```

$$3.045619148\ 10^{6} \tag{1.19.121}$$

```
> # density and number after sieve with <=16 bit primes
  f10/f2; %*H;
```

$$0.02050921667$$

$$2.109803284\ 10^{12} \tag{1.19.122}$$

```
> # sieve events for <=16 bit primes
  5*H*ln(16./ln(P));
```

(1.19.123)

$$5.938244395\,10^{14} \tag{1.19.123}$$

```
> # density and number after sieve with <=32 bit primes
  f26/f2; %*H;
```

$$0.0006409130209$$

$$6.593135261\,10^{10} \tag{1.19.124}$$

```
> # sieve events for <=32 bit primes
  5*H*ln(32./ln(P));
```

$$9.503480995\,10^{14} \tag{1.19.125}$$

```
> # sieve events for <=50 bit primes
  5*H*ln(50./ln(P));
```

$$1.179898065\,10^{15} \tag{1.19.126}$$

```
> %/0.8/10^9; # total sieve time in sec
```

$$1.474872581\,10^{6} \tag{1.19.127}$$

```
> #512 bit multiplier
  H:=HCCI53; %/8; 2*%%; P:=348.;
```

$$H := 5.031385804\,10^{13}$$

$$6.289232255\,10^{12}$$

$$1.006277161\,10^{14}$$

$$P := 348. \tag{1.19.128}$$

```
> 500*p50/3.2/10^9; # reciprocal modulo primes <=50 bit, sec
```

$$5.076031914\,10^{6} \tag{1.19.129}$$

```
> # density and number after sieve with <=16 bit primes
  f10/f3; %*H;
```

$$0.04193284643$$

$$2.109803282\,10^{12} \tag{1.19.130}$$

```
> # sieve events for <=16 bit primes
  5*H*ln(16./ln(P));
```

$$2.530210060\,10^{14} \tag{1.19.131}$$

```
> # density and number after sieve with <=32 bit primes
  f26/f3; %*H;
```

$$0.001310401451$$

$$6.593135258\,10^{10} \tag{1.19.132}$$

```
> # sieve events for <=32 bit primes
  5*H*ln(32./ln(P));
```

$$4.273955501\,10^{14} \tag{1.19.133}$$

```
> # sieve events for <=50 bit primes
  5*H*ln(50./ln(P));
```

$$5.396676800\ 10^{14} \tag{1.19.134}$$

```
> %/0.8/10^9; # total sieve time in sec
```

$$6.745846000\ 10^{5} \tag{1.19.135}$$

```
> #1024 bit multiplier
  H:=HCCI54; %/8; 2*%%; P:=708.;
```

$$H := 2.839988647\ 10^{13}$$

$$3.549985809\ 10^{12}$$

$$5.679977294\ 10^{13}$$

$$P := 708. \tag{1.19.136}$$

```
> 900*p50/3.2/10^9; # reciprocal modulo primes <=50 bit, sec
```

$$9.136857445\ 10^{6} \tag{1.19.137}$$

```
> # density and number after sieve with <=16 bit primes
  f10/f4; %*H;
```

$$0.07428914496$$

$$2.109803283\ 10^{12} \tag{1.19.138}$$

```
> # sieve events for <=16 bit primes
  5*H*ln(16./ln(P));
```

$$1.265535312\ 10^{14} \tag{1.19.139}$$

```
> # density and number after sieve with <=32 bit primes
  f26/f4; %*H;
```

$$0.002321535780$$

$$6.593135259\ 10^{10} \tag{1.19.140}$$

```
> # sieve events for <=32 bit primes
  5*H*ln(32./ln(P));
```

$$2.249800374\ 10^{14} \tag{1.19.141}$$

```
> # sieve events for <=50 bit primes
  5*H*ln(50./ln(P));
```

$$2.883525526\ 10^{14} \tag{1.19.142}$$

```
> %/0.8/10^9; # total sieve time in sec
```

$$3.604406908\ 10^{5} \tag{1.19.143}$$

```
> #2048 bit multiplier
  H:=HCCI55; %/8; 2*%%; P:=1445.;
```

$$H := 1.721771370\ 10^{13}$$

$$2.152214212\ 10^{12}$$

$$3.443542740\,10^{13}$$

$$P := 1445. \tag{1.19.144}$$

```
> 1700*p50/3.2/10^9; # reciprocal modulo primes <=50 bit, sec
```

$$1.725850851\,10^{7} \tag{1.19.145}$$

```
> # density and number after sieve with <=16 bit primes
  f10/f5; %*H;
```

$$0.1225367851$$

$$2.109803284\,10^{12} \tag{1.19.146}$$

```
> # sieve events for <=16 bit primes
  5*H*ln(16./ln(P));
```

$$6.784003640\,10^{13} \tag{1.19.147}$$

```
> # density and number after sieve with <=32 bit primes
  f26/f5; %*H;
```

$$0.003829274533$$

$$6.593135259\,10^{10} \tag{1.19.148}$$

```
> # sieve events for <=32 bit primes
  5*H*ln(32./ln(P));
```

$$1.275120850\,10^{14} \tag{1.19.149}$$

```
> # sieve events for <=50 bit primes
  5*H*ln(50./ln(P));
```

$$1.659323027\,10^{14} \tag{1.19.150}$$

```
> %/0.8/10^9; # total sieve time in sec
```

$$2.074153784\,10^{5} \tag{1.19.151}$$

```
> #4096 bit multiplier
  H:=HCCI56; %/8; 2*%%; P:=2855.;
```

$$H := 1.094958607\,10^{13}$$

$$1.368698259\,10^{12}$$

$$2.189917214\,10^{13}$$

$$P := 2855. \tag{1.19.152}$$

```
> 3300*p50/3.2/10^9; # reciprocal modulo primes <=50 bit, sec
```

$$3.350181063\,10^{7} \tag{1.19.153}$$

```
> # density and number after sieve with <=16 bit primes
  f10/f6; %*H;
```

$$0.1926833827$$

$$2.109803283\,10^{12} \tag{1.19.154}$$

```
> # sieve events for <=16 bit primes
  5*H*ln(16./ln(P));
```

$$3.824462687\ 10^{13} \tag{1.19.155}$$

```
> # density and number after sieve with <=32 bit primes
  f26/f6; %*H;
```

$$0.006021355710$$

$$6.593135260\ 10^{10} \tag{1.19.156}$$

```
> # sieve events for <=32 bit primes
  5*H*ln(32./ln(P));
```

$$7.619300045\ 10^{13} \tag{1.19.157}$$

```
> # sieve events for <=50 bit primes
  5*H*ln(50./ln(P));
```

$$1.006262956\ 10^{14} \tag{1.19.158}$$

```
> %/0.8/10^9; # total sieve time in sec
```

$$1.257828695\ 10^{5} \tag{1.19.159}$$

```
> H*f44/f6*300/64^2/1.925834177; # SPU time in sec
```

$$2.692367085\ 10^{8} \tag{1.19.160}$$

```
> %/24/3600/420; # time for all SPU in days
```

$$7.419441923 \tag{1.19.161}$$

```
> # Cunningham chain I, length 6
> f:=qsAB(6,18,64);
```

$$f := 0.1247271834 \tag{1.19.162}$$

```
> f:=f*(1-3/7)*(1-6/11)*(1-14/17)/(1-6/17)*(1-5/31)/(1-6/31);
  H:=HCCI6; H/8; H*f; 2*%;
```

$$f := 0.009188873960$$

$$H := 4.856551188\ 10^{17}$$

$$6.070688985\ 10^{16}$$

$$4.462623675\ 10^{15}$$

$$8.925247350\ 10^{15} \tag{1.19.163}$$

```
> P:=18; f:=qsAB(6,18,P);
  f:=f*(1-3/7)*(1-6/11)*(1-14/17)/(1-6/17)*(1-5/31)/(1-6/31);
  H*f; %/8; 2*%%;
  log[2.](H*f); primeprod(P); op(2,[%])+%%;
```

$$P := 18$$

$$f := 1.$$

$$f := 0.07367178276$$

$$3.577907841\ 10^{16}$$

$$4.472384801\ 10^{15}$$

$$7.155815682\ 10^{16}$$

$$54.98996575$$

$$510510,\ 18.96157970$$

$$73.95154545 \tag{1.19.164}$$

```
> P:=58; f1:=qsAB(6,18,P);
  f1:=f1*(1-3/7)*(1-6/11)*(1-14/17)/(1-6/17)*(1-5/31)/(1-6/31);
  HCCI61:=H*f1; %/8; 2*%%;
  log[2.](H*f1); primeprod(P); op(2,[%])+%%;
```

$$P := 58$$

$$f1 := 0.1539942137$$

$$f1 := 0.01134502826$$

$$HCCI61 := 5.509771047\ 10^{15}$$

$$6.887213809\ 10^{14}$$

$$1.101954209\ 10^{16}$$

$$52.29091379$$

$$32589158477190044730,\ 64.82102590$$

$$117.1119397 \tag{1.19.165}$$

```
> P:=155; f2:=qsAB(6,18,P);
  f2:=f2*(1-3/7)*(1-6/11)*(1-14/17)/(1-6/17)*(1-5/31)/(1-6/31);
  HCCI62:=H*f2; %/8; 2*%%;
  log[2.](H*f2); primeprod(P); op(2,[%])+%%;
```

$$P := 155$$

$$f2 := 0.04104198920$$

$$f2 := 0.003023636513$$

$$HCCI62 := 1.468444550\ 10^{15}$$

$$1.835555688\ 10^{14}$$

$$2.936889100\ 10^{15}$$

$$50.38321021$$

$$225319534991831177328890236228992001350685163362356544\backslash$$

$$91910,\ 197.1657300$$

$$247.5489402 \tag{1.19.166}$$

```
> P:=345; f3:=qsAB(6,18,P);
  f3:=f3*(1-3/7)*(1-6/11)*(1-14/17)/(1-6/17)*(1-5/31)/(1-6/31);
  HCCI63:=H*f3; %/8; 2*%%;
```

```
log[2.](H*f3); primeprod(P); op(2,[%])+%%;
```

$$P := 345$$

$$f3 := 0.01766912462$$

$$f3 := 0.001301715911$$

$$HCCI63 := 6.321849954\ 10^{14}$$

$$7.902312442\ 10^{13}$$

$$1.264369991\ 10^{15}$$

$$49.16734012$$

$$2159704595610254721415747050533376368260798239989520222\backslash$$
$$9494359364184419848203983074167271844044268476527\backslash$$
$$1131351200459875900396418645379 0, 449.5711268$$

$$498.7384669 \qquad (1.19.167)$$

```
> P:=700; f4:=qsAB(6,18,P);
  f4:=f4*(1-3/7)*(1-6/11)*(1-14/17)/(1-6/17)*(1-5/31)/(1-6/31);
  HCCI64:=H*f4; %/8; 2*%%;
  log[2.](H*f); primeprod(P); op(2,[%])+%%;
```

$$P := 700$$

$$f4 := 0.008810775177$$

$$f4 := 0.0006491055148$$

$$HCCI64 := 3.152414159\ 10^{14}$$

$$3.940517699\ 10^{13}$$

$$6.304828318\ 10^{14}$$

$$54.98996575$$

$$2777143091314604471215621911501273214901533705874524377\backslash$$
$$4375474371978395728107173008782747458575903820497\backslash$$
$$3442611013331564691368332893280842294010575050052\backslash$$
$$1526107732841764980772053331059278317148795229698\backslash$$
$$3742789708502518237023426083874832018749447215424\backslash$$
$$7649280164135095538728368560952146724 30, 961.5108209$$

$$1016.500787 \qquad (1.19.168)$$

```
> P:=1445; f5:=qsAB(6,18,P);
  f5:=f5*(1-3/7)*(1-6/11)*(1-14/17)/(1-6/17)*(1-5/31)/(1-6/31);
  HCCI65:=H*f5; %/8; 2*%%;
  log[2.](H*f5); primeprod(P); op(2,[%])+%%;
```

$$P := 1445$$

$$f5 := 0.004790026085$$

$$f5 := 0.0003528897612$$

$$HCCI65 := 1.713827189\ 10^{14}$$

$$2.142283986\ 10^{13}$$

$$3.427654378\ 10^{14}$$

$$47.28421497$$

$$14035190102192877615491058304312201157559084136981124027484188803158848225968792931953530456906851740414\92985799236753957781188402079980573316101102666807053847184258236821659375569589235746486543818085\86090203878803160222717481113666508784009616753399464756155776230686191428602918437322026056816515\15823508028903858759438837139256280480693194051606928514707000049636241882348300313352955939810905\98844125637247807533443311519141169288269353400525081573843336158246625830959364476287275278180827\87794303258540573757278679748186024317619235344695160234778007078922368835190417752091176541624110\3624390, 1993.645906$$

$$2040.930121 \qquad (1.19.169)$$

```
> P:=2850; f6:=qsAB(6,18,P);
  f6:=f6*(1-3/7)*(1-6/11)*(1-14/17)/(1-6/17)*(1-5/31)/(1-6/31);
  HCCI66:=H*f6; %/8; 2*%%;
  log[2.](H*f); primeprod(P); op(2,[%])+%%;
```

$$P := 2850$$

$$f6 := 0.002788043984$$

$$f6 := 0.0002054001707$$

$$HCCI66 := 9.975364430\ 10^{13}$$

$$1.246920554\ 10^{13}$$

$$1.995072886\ 10^{14}$$

$$54.98996575$$

$$39525482918299574108511754760543232148255497423844694913568555279926724141580463905686635265060110286807\$$

5949182653575217522670010722609081764926566010133\
11889058655656388821719717922299841610317694907 01\
01760366135670947690705961076910993729909770129 18\
34406004956562052906900651845193728644026849444 25\
19484037847597598532699961628652263815281786505 27\
76866884275421277751764955049423242033939303911 87\
25577711338479292018276584965207736904621822379 73\
50936858258628318365547546831093064572408129828 18\
17711971039390051508205260998964095794780066647 59\
64706789569742428516669630238081984977035686616 05\
52510119680878021708585728986233235785455932664 50\
23089076043331576774676434057220273988241111359 31\
08109727514295733578278206251718921944559956029 55\
02116699272038912883203508359445115494104288480 99\
07669957849043493574102805423116974296981766789 18\
52365999593975113179424591740121410493561974613 13\
34552148079963269415372811103541350790847227574 91\
82672970666389954308730036279817873497240975648 15\
78893714964289491979776800816739808817032248152 26\
83619926465991284049481125046645952342789010599 82\
70069933574756581967720199322561497572994037348 87\
75068287402156977813427022311060997095285144993 88\
09394813815192274676800934432031 0, 4034.803490

4089.793456 (1.19.170)

```
> P:=5735; f7:=qsAB(6,18,P);
  f7:=f7*(1-3/7)*(1-6/11)*(1-14/17)/(1-6/17)*(1-5/31)/(1-6/31);
  HCCI67:=H*f7; %/8; 2*%%;
  log[2.](H*f); primeprod(P); op(2,[%])+%%;
```

$$P := 5735$$

$$f7 := 0.001697555556$$

$$f7 := 0.0001250619441$$

$$HCCI67 := 6.073697332\ 10^{13}$$

$$7.592121665\ 10^{12}$$

$$1.214739466\ 10^{14}$$

$$54.98996575$$

$$\begin{aligned}
&25877635365745943195479973710325254645005485323349982 15\backslash\\
&\quad 49508377125743308845165476593237396914252683644 47\backslash\\
&\quad 69563601263611453009159982294207199894940631796 80\backslash\\
&\quad 66406467607068713590532110560143541057461960394 30\backslash\\
&\quad 22798941452819001581628859284382154672411529941 70\backslash\\
&\quad 47762390213102303784377282110603753445701896818 04\backslash\\
&\quad 57010868555956812723170993108581254851573173540 36\backslash\\
&\quad 60174804807009336068472595324812534263909812797 55\backslash\\
&\quad 68477577825931774726280404626635833233811301497 14\backslash\\
&\quad 96090377094485635464728561208314857617681232399 70\backslash\\
&\quad 76792144664157308669810975606126422841962696332 32\backslash\\
&\quad 29152343876655311815592475921020363642913431602 74\backslash\\
&\quad 89665302076657998708741636138096578096806887209 54\backslash\\
&\quad 68260830253938281891316844947698729585282230310 00\backslash\\
&\quad 41285657095013866221153477079878602287862032968 31\backslash\\
&\quad 48862813133420376955646531788402444834594071535 12\backslash\\
&\quad 31011805540299064965751370701769259679528289249 73\backslash\\
&\quad 56989767177705146239397879515250096211907740867 38\backslash\\
&\quad 82769625510901787776785398514441263547531417907 69\backslash\\
&\quad 52451926620206090259933760897661746938752561186 87\backslash\\
&\quad 20701804891499376333966473701142076201395915500 77\backslash\\
&\quad 07109567031340021922475290308708917553709307273 29\backslash\\
&\quad 28471702151340411333601191981570485120797457197 30\backslash\\
&\quad 17109429128874918888174340777348670456994302940 50\backslash\\
&\quad 79380221766424604919266809326700313226622658778 41\backslash\\
&\quad 25464160034929014102080295235657910408457850796 46\backslash\\
&\quad 37204448018329098243301612196886595800323306966 08\backslash\\
&\quad 79885202240282558767628691438535935292875871838 16\backslash\\
&\quad 28779938215800823026327266085480275274295547215 23\backslash\\
&\quad 59873403439488213533950155733734686059827526082 56\backslash
\end{aligned}$$

491671882225191766907983529898079880913912698228\
2\
721338028266245629024684246505214676929367315972\
9\
385397717929512777438092268221287702316009937845\
4\
440024402603610086087510110614033213741390551701\
6\
449063402605406542745401812379166712873238083436\
1\
075891291422080863904561787182923945555984905646\
5\
734925930581082333035419324120853221195285636656\
6\
291710731209141101903893637700314151705035364887\
4\
320211768444746054628767407509596760209219541537\
9\
590977651004697427580290270877057640488418176076\
7\
268209442830575646807163786974873573576386711019\
7\
372232923540630015077171590326103053912583657878\
4\
135857442025316260545965832988158530834055664740\
9\
913167472570045685891990192197778082140572272189\
3\
522286260345596097585508465730834454462754104112\
8\
846621493682221035322611910909681788810090762545\
8\
732491940491486887978627265209898388773345386625\
2\
702609628861155718863595223017279958883408708885\
2\
691554012687173274486005615878499837575262023788\
2\
784381102943938930033106796769344570691 0,
8126.807826

8181.797792 (1.19.171)

```
> f10:=qsAB(6,18,2^16);
```

$$f10 := 0.0003856103068 \quad (1.19.172)$$

```
> f10:=f10*(1-3/7)*(1-6/11)*(1-14/17)/(1-6/17)*(1-5/31)/(1
  -6/31);
  H*f10; 2*%;
```

$$f10 := 0.00002840859875$$

$$1.379678140\ 10^{13}$$

$$2.759356280\ 10^{13} \quad (1.19.173)$$

```
> f13:=f10*(16/19)^6; H*f13; 2*%;
```

$$f13 := 0.00001013090174$$

$$4.920124288\ 10^{12}$$

(1.19.174)

$$9.840248576\ 10^{12} \tag{1.19.174}$$

```
> f15:=f13*(19/21)^6; H*f15; 2*%;
```

$$f15 := 0.000005557173300$$

$$2.698869659\ 10^{12}$$

$$5.397739318\ 10^{12} \tag{1.19.175}$$

```
> f18:=f15*(21/24)^6; H*f18; 2*%;
```

$$f18 := 0.000002494033362$$

$$1.211240069\ 10^{12}$$

$$2.422480138\ 10^{12} \tag{1.19.176}$$

```
> f26:=f18*(24/32)^6; H*f26; 2*%;
```

$$f26 := 4.438843557\ 10^{-7}$$

$$2.155747095\ 10^{11}$$

$$4.311494190\ 10^{11} \tag{1.19.177}$$

```
> f29:=f26*(32/35)^6; H*f29; 2*%;
```

$$f29 := 2.592754775\ 10^{-7}$$

$$1.259184628\ 10^{11}$$

$$2.518369256\ 10^{11} \tag{1.19.178}$$

```
> f34:=f29*(35/40)^6; H*f34; 2*%;
```

$$f34 := 1.163616205\ 10^{-7}$$

$$5.651161663\ 10^{10}$$

$$1.130232333\ 10^{11} \tag{1.19.179}$$

```
> f44:=f34*(40/50)^6; H*f44; 2*%;
```

$$f44 := 3.050350064\ 10^{-8}$$

$$1.481418123\ 10^{10}$$

$$2.962836246\ 10^{10} \tag{1.19.180}$$

```
> #128 bit multiplier
  H:=HCCI61; %/8; 2*%%; P:=58.;
```

$$H := 5.509771047\ 10^{15}$$

$$6.887213809\ 10^{14}$$

$$1.101954209\ 10^{16}$$

$$P := 58. \tag{1.19.181}$$

```
> 200*p50/3.2/10^9; # reciprocal modulo primes <=50 bit, sec
```

$$2.030412766\ 10^{6} \tag{1.19.182}$$

```
> # density and number after sieve with <=16 bit primes
  f10/f1; %*H;
```

$$0.002504057116$$

$$1.379678140\,10^{13} \tag{1.19.183}$$

```
> # sieve events for <=16 bit primes
  6*H*ln(16./ln(P));
```

$$4.533318311\,10^{16} \tag{1.19.184}$$

```
> # density and number after sieve with <=32 bit primes
  f26/f1; %*H;
```

$$0.00003912589246$$

$$2.155747095\,10^{11} \tag{1.19.185}$$

```
> # sieve events for <=32 bit primes
  6*H*ln(32./ln(P));
```

$$6.824767668\,10^{16} \tag{1.19.186}$$

```
> # sieve events for <=50 bit primes
  6*H*ln(50./ln(P));
```

$$8.300131524\,10^{16} \tag{1.19.187}$$

```
> %/0.8/10^9; # total sieve time in sec
```

$$1.037516440\,10^{8} \tag{1.19.188}$$

```
> #256 bit multiplier
  H:=HCCI62; %/8; 2*%%; P:=155.;
```

$$H := 1.468444550\,10^{15}$$

$$1.835555688\,10^{14}$$

$$2.936889100\,10^{15}$$

$$P := 155. \tag{1.19.189}$$

```
> 300*p50/3.2/10^9; # reciprocal modulo primes <=50 bit, sec
```

$$3.045619148\,10^{6} \tag{1.19.190}$$

```
> # density and number after sieve with <=16 bit primes
  f10/f2; %*H;
```

$$0.009395507240$$

$$1.379678140\,10^{13} \tag{1.19.191}$$

```
> # sieve events for <=16 bit primes
  6*H*ln(16./ln(P));
```

$$1.017194433\,10^{16} \tag{1.19.192}$$

```
> # density and number after sieve with <=32 bit primes
```

```
f26/f2; %*H;
```

$$0.0001468048007$$

$$2.155747095\ 10^{11} \tag{1.19.193}$$

```
> # sieve events for <=32 bit primes
  6*H*ln(32./ln(P));
```

$$1.627903354\ 10^{16} \tag{1.19.194}$$

```
> # sieve events for <=50 bit primes
  6*H*ln(50./ln(P));
```

$$2.021112072\ 10^{16} \tag{1.19.195}$$

```
> %/0.8/10^9; # total sieve time in sec
```

$$2.526390090\ 10^{7} \tag{1.19.196}$$

```
> #512 bit multiplier
  H:=HCCI63; %/8; 2*%%; P:=345.;
```

$$H := 6.321849954\ 10^{14}$$

$$7.902312442\ 10^{13}$$

$$1.264369991\ 10^{15}$$

$$P := 345. \tag{1.19.197}$$

```
> 500*p50/3.2/10^9; # reciprocal modulo primes <=50 bit, sec
```

$$5.076031914\ 10^{6} \tag{1.19.198}$$

```
> # density and number after sieve with <=16 bit primes
  f10/f3; %*H;
```

$$0.02182396213$$

$$1.379678140\ 10^{13} \tag{1.19.199}$$

```
> # sieve events for <=16 bit primes
  6*H*ln(16./ln(P));
```

$$3.820614530\ 10^{15} \tag{1.19.200}$$

```
> # density and number after sieve with <=32 bit primes
  f26/f3; %*H;
```

$$0.0003409994085$$

$$2.155747095\ 10^{11} \tag{1.19.201}$$

```
> # sieve events for <=32 bit primes
  6*H*ln(32./ln(P));
```

$$6.449798016\ 10^{15} \tag{1.19.202}$$

```
> # sieve events for <=50 bit primes
  6*H*ln(50./ln(P));
```

$$8.142614070\ 10^{15} \tag{1.19.203}$$

```
> %/0.8/10^9; # total sieve time in sec
```

$$1.017826759\ 10^{7} \tag{1.19.204}$$

```
> #1024 bit multiplier
  H:=HCCI64; %/8; 2*%%; P:=700.;
```

$$H := 3.152414159\ 10^{14}$$

$$3.940517699\ 10^{13}$$

$$6.304828318\ 10^{14}$$

$$P := 700. \tag{1.19.205}$$

```
> 900*p50/3.2/10^9; # reciprocal modulo primes <=50 bit, sec
```

$$9.136857445\ 10^{6} \tag{1.19.206}$$

```
> # density and number after sieve with <=16 bit primes
  f10/f4; %*H;
```

$$0.04376576397$$

$$1.379678140\ 10^{13} \tag{1.19.207}$$

```
> # sieve events for <=16 bit primes
  6*H*ln(16./ln(P));
```

$$1.688985483\ 10^{15} \tag{1.19.208}$$

```
> # density and number after sieve with <=32 bit primes
  f26/f4; %*H;
```

$$0.0006838400623$$

$$2.155747095\ 10^{11} \tag{1.19.209}$$

```
> # sieve events for <=32 bit primes
  6*H*ln(32./ln(P));
```

$$3.000037675\ 10^{15} \tag{1.19.210}$$

```
> # sieve events for <=50 bit primes
  6*H*ln(50./ln(P));
```

$$3.844166744\ 10^{15} \tag{1.19.211}$$

```
> %/0.8/10^9; # total sieve time in sec
```

$$4.805208430\ 10^{6} \tag{1.19.212}$$

```
> #2048 bit multiplier
  H:=HCCI65; %/8; 2*%%; P:=1445.;
```

$$H := 1.713827189\ 10^{14}$$

$$2.142283986\ 10^{13}$$

$$3.427654378\ 10^{14}$$

$$P := 1445. \tag{1.19.213}$$

```
> 1700*p50/3.2/10^9; # reciprocal modulo primes <=50 bit, sec
```

$$1.725850851\ 10^{7} \tag{1.19.214}$$

```
> # density and number after sieve with <=16 bit primes
  f10/f5; %*H;
```

$$0.08050275716$$

$$1.379678140\ 10^{13} \tag{1.19.215}$$

```
> # sieve events for <=16 bit primes
  6*H*ln(16./ln(P));
```

$$8.103243036\ 10^{14} \tag{1.19.216}$$

```
> # density and number after sieve with <=32 bit primes
  f26/f5; %*H;
```

$$0.001257855581$$

$$2.155747095\ 10^{11} \tag{1.19.217}$$

```
> # sieve events for <=32 bit primes
  6*H*ln(32./ln(P));
```

$$1.523084995\ 10^{15} \tag{1.19.218}$$

```
> # sieve events for <=50 bit primes
  6*H*ln(50./ln(P));
```

$$1.982000376\ 10^{15} \tag{1.19.219}$$

```
> %/0.8/10^9; # total sieve time in sec
```

$$2.477500470\ 10^{6} \tag{1.19.220}$$

```
> #4096 bit multiplier
  H:=HCCI66; %/8; 2*%%; P:=2850.;
```

$$H := 9.975364430\ 10^{13}$$

$$1.246920554\ 10^{13}$$

$$1.995072886\ 10^{14}$$

$$P := 2850. \tag{1.19.221}$$

```
> 3300*p50/3.2/10^9; # reciprocal modulo primes <=50 bit, sec
```

$$3.350181063\ 10^{7} \tag{1.19.222}$$

```
> # density and number after sieve with <=16 bit primes
  f10/f6; %*H;
```

$$0.1383085450$$

$$1.379678140\ 10^{13} \tag{1.19.223}$$

```
> # sieve events for <=16 bit primes
  6*H*ln(16./ln(P));
```

(1.19.224)

$$4.182343457\,10^{14} \tag{1.19.224}$$

```
> # density and number after sieve with <=32 bit primes
  f26/f6; %*H;
```

$$0.002161071017$$

$$2.155747095\,10^{11} \tag{1.19.225}$$

```
> # sieve events for <=32 bit primes
  6*H*ln(32./ln(P));
```

$$8.330980896\,10^{14} \tag{1.19.226}$$

```
> # sieve events for <=50 bit primes
  6*H*ln(50./ln(P));
```

$$1.100210678\,10^{15} \tag{1.19.227}$$

```
> %/0.8/10^9; # total sieve time in sec
```

$$1.375263348\,10^{6} \tag{1.19.228}$$

```
> #8192 bit multiplier
  H:=HCCI67; %/8; 2*%%; P:=5735.;
```

$$H := 6.073697332\,10^{13}$$

$$7.592121665\,10^{12}$$

$$1.214739466\,10^{14}$$

$$P := 5735. \tag{1.19.229}$$

```
> 6500*p50/3.2/10^9; # reciprocal modulo primes <=50 bit, sec
```

$$6.598841488\,10^{7} \tag{1.19.230}$$

```
> # density and number after sieve with <=16 bit primes
  f10/f7; %*H;
```

$$0.2271562221$$

$$1.379678140\,10^{13} \tag{1.19.231}$$

```
> # sieve events for <=16 bit primes
  6*H*ln(16./ln(P));
```

$$2.239472215\,10^{14} \tag{1.19.232}$$

```
> # density and number after sieve with <=32 bit primes
  f26/f7; %*H;
```

$$0.003549315972$$

$$2.155747095\,10^{11} \tag{1.19.233}$$

```
> # sieve events for <=32 bit primes
  6*H*ln(32./ln(P));
```

$$4.765451921\,10^{14} \tag{1.19.234}$$

```
> # sieve events for <=50 bit primes
  6*H*ln(50./ln(P));
```

$$6.391819596\,10^{14} \tag{1.19.235}$$

```
> %/0.8/10^9; # total sieve time in sec
```

$$7.989774495\,10^{5} \tag{1.19.236}$$

```
> H*f44/f7*300/128^2/2.373951103; # SPU time in sec
```

$$1.142634149\,10^{8} \tag{1.19.237}$$

```
> %/24/3600/420; # time for all SPU in days
```

$$3.148793400 \tag{1.19.238}$$

```
> # Cunningham chain II, length 2 + twinprime
> qsAB(2,16,1000000);
```

$$0.04409389230 \tag{1.19.239}$$

```
> q2:=%*(ln(1000000.)/(50*ln(2.)))^2;
```

$$q2 := 0.007006826832 \tag{1.19.240}$$

```
> qsAB(3,16,1000000);
```

$$0.009000759447 \tag{1.19.241}$$

```
> q3:=%*(ln(1000000.)/(50*ln(2.)))^3;
```

$$q3 := 0.0005701558391 \tag{1.19.242}$$

```
> expval:=29.62739122*q3/q2; # Expected number of twins/Cullens
```

$$expval := 2.410824544 \tag{1.19.243}$$

```
> H:=2.^35; q3*H*260/expval; # SPU time in sec
```

$$H := 3.435973837\,10^{10}$$

$$2.112764876\,10^{9} \tag{1.19.244}$$

```
> %/24/3600/420; # time for all SPU in days
```

$$58.22213613 \tag{1.19.245}$$

```
> # Cunningham chain II, length 3
> f:=qsAB(3,4,64); H:=2.^43; H/8; H*f; 2*%;
```

$$f := 0.03943182080$$

$$H := 8.796093022\,10^{12}$$

$$1.099511628\,10^{12}$$

$$3.468459638\,10^{11}$$

$$6.936919276\,10^{11} \tag{1.19.246}$$

```
> P:=20; f:=qsAB(3,4,P); H*f; %/8; 2*%%;
  log[2.](H*f); primeprod(P); op(2,[%])+%%;
```

$$P := 20$$

$$f := 0.08867912278$$

$$7.800298131\ 10^{11}$$

$$9.750372664\ 10^{10}$$

$$1.560059626\ 10^{12}$$

$$39.50473831$$

$$9699690,\ 23.20950721$$

$$62.71424552 \qquad (1.19.247)$$

```
> P:=72; f:=qsAB(3,4,P); H*f; %/8; 2*%%;
  log[2.](H*f); primeprod(P); op(2,[%])+%%;
```

$$P := 72$$

$$f := 0.03607468659$$

$$3.173162990\ 10^{11}$$

$$3.966453738\ 10^{10}$$

$$6.346325980\ 10^{11}$$

$$38.20713067$$

$$5579408301266989609674153 90,\ 88.85024260$$

$$127.0573733 \qquad (1.19.248)$$

```
> P:=165; f:=qsAB(3,4,P); H*f; %/8; 2*%%;
  log[2.](H*f); primeprod(P); op(2,[%])+%%;
```

$$P := 165$$

$$f := 0.02197966828$$

$$1.933352068\ 10^{11}$$

$$2.416690085\ 10^{10}$$

$$3.866704136\ 10^{11}$$

$$37.49231342$$

$$576615221997595165902363003533613430656538401560606631 9\backslash$$

$$856068810,\ 211.8090789$$

$$249.3013923 \qquad (1.19.249)$$

```
> P:=358; f:=qsAB(3,4,P); H*f; %/8; 2*%%;
  log[2.](H*f); primeprod(P); op(2,[%])+%%;
```

$$P := 358$$

$$f := 0.01463869425$$

$$1.287633163\ 10^{11}$$

$$1.609541454\ 10^{10}$$

$$2.575266326\ 10^{11}$$

$$36.90593068$$

$$923259870916926031672467589164424478488970695878801562 4\backslash$$

$$8625475693453150629759757766750033011164044499544\backslash$$

$$0632646862354019996149474298560056206 10,\ 474.9205263$$

$$511.8264570 \tag{1.19.250}$$

```
> f:=qsAB(3,4,2^19); H*f; 2*%;
```

$$f := 0.001328332788$$

$$1.168413877\ 10^{10}$$

$$2.336827754\ 10^{10} \tag{1.19.251}$$

```
> f:=f*(19/21)^3; H*f; 2*%;
```

$$f := 0.0009838067804$$

$$8.653655956\ 10^{9}$$

$$1.730731191\ 10^{10} \tag{1.19.252}$$

```
> f:=f*(21/24)^3; H*f; 2*%;
```

$$f := 0.0006590736830$$

$$5.797273424\ 10^{9}$$

$$1.159454685\ 10^{10} \tag{1.19.253}$$

```
> f:=f*(24/35)^3; H*f; 2*%;
```

$$f := 0.0002125022646$$

$$1.869189687\ 10^{9}$$

$$3.738379374\ 10^{9} \tag{1.19.254}$$

```
> f:=f*(35/40)^3; H*f; 2*%;
```

$$f := 0.0001423599155$$

$$1.252211059\ 10^{9}$$

$$2.504422118\ 10^{9} \tag{1.19.255}$$

```
> f:=f*(40/50)^3; H*f; 2*%;
```

$$f := 0.00007288827674$$

$$6.411320624\ 10^{8}$$

$$1.282264125\ 10^{9} \tag{1.19.256}$$

```
> H*f*300/16^2/2.380149958; # SPU time in sec
```

$$3.156635712\ 10^{8} \tag{1.19.257}$$

```
> %/24/3600/420; # time for all SPU in days
```

$$8.698841799 \tag{1.19.258}$$

```
> # Cunningham chain II, length 4
> f:=qsAB(4,6,64); f:=f*(1-3/7)/(1-4/7); H:=2.^49; H/8; H*f; 2*
  %;
```

$$f := 0.03802790523$$

$$f := 0.05070387364$$

$$H := 5.629499534\ 10^{14}$$

$$7.036874418\ 10^{13}$$

$$2.854374330\ 10^{13}$$

$$5.708748660\ 10^{13} \tag{1.19.259}$$

```
> P:=16; f:=qsAB(4,6,P); f:=f*(1-3/7)/(1-4/7); H*f; %/8; 2*%%;
  log[2.](H*f); primeprod(P); op(2,[%])+%%;
```

$$P := 16$$

$$f := 0.1888111888$$

$$f := 0.2517482517$$

$$1.417216666\ 10^{14}$$

$$1.771520832\ 10^{13}$$

$$2.834433332\ 10^{14}$$

$$47.01005366$$

$$30030,\ 14.87411685$$

$$61.88417051 \tag{1.19.260}$$

```
> P:=70; f:=qsAB(4,6,P); f:=f*(1-3/7)/(1-4/7); H*f; %/8; 2*%%;
  log[2.](H*f); primeprod(P); op(2,[%])+%%;
```

$$P := 70$$

$$f := 0.03575758253$$

$$f := 0.04767677671$$

$$2.683963923\ 10^{13}$$

$$3.354954904\ 10^{12}$$

$$5.367927846\ 10^{13}$$

$$44.60943051$$

$$7858321551080267055879090,\ 82.70049548$$

$$127.3099260 \tag{1.19.261}$$

```
> P:=165; f:=qsAB(4,6,P); f:=f*(1-3/7)/(1-4/7); H*f; %/8; 2*%%;
  log[2.](H*f); primeprod(P); op(2,[%])+%%;
```

$$P := 165$$

$$f := 0.01737216914$$

$$f := 0.02316289219$$

$$1.303954908\ 10^{13}$$

$$1.629943635\ 10^{12}$$

$$2.607909816\ 10^{13}$$

$$43.56795921$$

$$5766152219975951659023630035336134306565384015606066319\backslash$$
$$856068810,\ 211.8090789$$

$$255.3770381 \qquad (1.19.262)$$

```
> P:=352; f:=qsAB(4,6,P); f:=f*(1-3/7)/(1-4/7); H*f; %/8; 2*%%;
  log[2.](H*f); primeprod(P); op(2,[%])+%%;
```

$$P := 352$$

$$f := 0.01020782566$$

$$f := 0.01361043421$$

$$7.661993304\ 10^{12}$$

$$9.577491630\ 10^{11}$$

$$1.532398661\ 10^{13}$$

$$42.80085690$$

$$2615467056421886775276112150607434783254874492574508675\backslash$$
$$5984554020808257968770469622308791221292930453128\backslash$$
$$6298200244292923511657074872113330370,\ 466.4570019$$

$$509.2578588 \qquad (1.19.263)$$

```
> P:=718; f:=qsAB(4,6,P); f:=f*(1-3/7)/(1-4/7); H*f; %/8; 2*%%;
  log[2.](H*f); primeprod(P); op(2,[%])+%%;
```

$$P := 718$$

$$f := 0.006501210698$$

$$f := 0.008668280930$$

$$4.879808346\ 10^{12}$$

$$6.099760432\ 10^{11}$$

$$9.759616692\ 10^{12}$$

$$42.14996163$$

$$1380265110671180253634405030613336299264996365622991486\backslash$$

```
305858014214261048243081794992210453163935138192\
1\
564573865712490763569228376295661814770390189505\
1\
370316927819195277132853745401644085712780556831\
7\
159302017023312808646477597452054638680664401109\
1\
046992108509661969860784773011026549129761870,
980.4337334
```

$$1022.583695 \tag{1.19.264}$$

```
> f:=qsAB(4,6,2^19); f:=f*(1-3/7)/(1-4/7); H*f; 2*%;
```

$$f := 0.0004111818238$$

$$f := 0.0005482424317$$

$$3.086330514\ 10^{11}$$

$$6.172661028\ 10^{11} \tag{1.19.265}$$

```
> f:=f*(19/21)^4; H*f; 2*%;
```

$$f := 0.0003673752292$$

$$2.068138682\ 10^{11}$$

$$4.136277364\ 10^{11} \tag{1.19.266}$$

```
> f:=f*(21/24)^4; H*f; 2*%;
```

$$f := 0.0002153486146$$

$$1.212304926\ 10^{11}$$

$$2.424609852\ 10^{11} \tag{1.19.267}$$

```
> f:=f*(24/35)^4; H*f; 2*%;
```

$$f := 0.00004761182971$$

$$2.680307732\ 10^{10}$$

$$5.360615464\ 10^{10} \tag{1.19.268}$$

```
> f:=f*(35/40)^4; H*f; 2*%;
```

$$f := 0.00002790918045$$

$$1.571147183\ 10^{10}$$

$$3.142294366\ 10^{10} \tag{1.19.269}$$

```
> f:=f*(40/50)^4; H*f; 2*%;
```

$$f := 0.00001143160031$$

$$6.435418862\ 10^{9}$$

$$1.287083772\ 10^{10} \tag{1.19.270}$$

```
> H*f*300/64^2/1.731884327; # SPU time in sec
```

$$2.721568338\,10^{8} \tag{1.19.271}$$

```
> %/24/3600/420; # time for all SPU in days
```

$$7.499912748 \tag{1.19.272}$$

```
> # Cunningham chain II, length 5
> f:=qsAB(5,6,64); f:=f*(1-2/7)/(1-4/7); H:=2.^54; H/8; H*f; 2*
  %;
```

$$f := 0.01236551651$$

$$f := 0.02060919418$$

$$H := 1.801439851\,10^{16}$$

$$2.251799814\,10^{15}$$

$$3.712622369\,10^{14}$$

$$7.425244738\,10^{14} \tag{1.19.273}$$

```
> P:=12; f:=qsAB(5,6,P); f:=f*(1-2/7)/(1-4/7); H*f; %/8; 2*%%;
  log[2.](H*f); primeprod(P); op(2,[%])+%%;
```

$$P := 12$$

$$f := 0.1558441558$$

$$f := 0.2597402597$$

$$4.679064547\,10^{15}$$

$$5.848830684\,10^{14}$$

$$9.358129094\,10^{15}$$

$$52.05514155$$

$$2310,\ 11.17367714$$

$$63.22881869 \tag{1.19.274}$$

```
> P:=65; f:=qsAB(5,6,P); f:=f*(1-2/7)/(1-4/7); H*f; %/8; 2*%%;
  log[2.](H*f); primeprod(P); op(2,[%])+%%;
```

$$P := 65$$

$$f := 0.01236551651$$

$$f := 0.02060919418$$

$$3.712622369\,10^{14}$$

$$4.640777961\,10^{13}$$

$$7.425244738\,10^{14}$$

$$48.39943191$$

$$117288381359406970983270,\ 76.63440629$$

$$125.0338382 \tag{1.19.275}$$

```
> P:=162; f:=qsAB(5,6,P); f:=f*(1-2/7)/(1-4/7); H*f; %/8; 2*%%;
  log[2.](H*f); primeprod(P); op(2,[%])+%%;
```

$$P := 162$$

$$f := 0.004765729034$$

$$f := 0.007942881723$$

$$1.430862367\ 10^{14}$$

$$1.788577959\ 10^{13}$$

$$2.861724734\ 10^{14}$$

$$47.02387824$$

$$3537516699371749484063576708795174421205757064788997742\backslash$$
$$2429870, 204.4603507$$

$$251.4842289 \qquad (1.19.276)$$

```
> P:=348; f:=qsAB(5,6,P); f:=f*(1-2/7)/(1-4/7); H*f; %/8; 2*%%;
  log[2.](H*f); primeprod(P); op(2,[%])+%%;
```

$$P := 348$$

$$f := 0.002407576746$$

$$f := 0.004012627910$$

$$7.228507824\ 10^{13}$$

$$9.035634780\ 10^{12}$$

$$1.445701565\ 10^{14}$$

$$46.03876310$$

$$749417494676758388331264226535081599786496989276363517 3\backslash$$
$$6345426993719936873267821267360433298833611613549\backslash$$
$$0825788665595769374375572699465130, 458.0099187$$

$$504.0486818 \qquad (1.19.277)$$

```
> P:=708; f:=qsAB(5,6,P); f:=f*(1-2/7)/(1-4/7); H*f; %/8; 2*%%;
  log[2.](H*f); primeprod(P); op(2,[%])+%%;
```

$$P := 708$$

$$f := 0.001358967666$$

$$f := 0.002264946110$$

$$4.080164183\ 10^{13}$$

$$5.100205229\ 10^{12}$$

$$8.160328366\ 10^{13}$$

45.21369244

19467773070115377343221509599623925236459751278180415881\
5837207534756855405403128279156705968461708578168\
6383270320345426848649201358189870448101413110086\
5589801520722077251521209385072554100321305456018\
5603695585660265284153421684796257245143362498012\
7602145395058701972648586361227454853734301,
970.9640916

1016.177784 (1.19.278)

```
> P:=1445; f:=qsAB(5,6,P); f:=f*(1-2/7)/(1-4/7); H*f; %/8; 2*
  %%;
  log[2.](H*f); primeprod(P); op(2,[%])+%%;
```

$P := 1445$

$f := 0.0008238876656$

$f := 0.001373146109$

$2.473640122\ 10^{13}$

$3.092050152\ 10^{12}$

$4.947280244\ 10^{13}$

44.49170086

14035190102192877615491058304312201157559084136981124021\
7484188803158848225968792931953530456906851740414\
9298579923675395778118840207998057331610110266680\
7053847184258236821659375569589235746486543818085\
8609020387880316022271748111366650878400961675339\
9464756155776230686191428602918437322026056816515\
1582350802890385875943883713925628048069319405160\
6928514707000049636241882348300313352955939810905\
9884412563724780753344331151914116928826935340052\
5081573843336158246625830959364476287275278180827\
8779430325854057375727867974818602431761923534469\
5160234778007078922368835190417752091176541624110\
3624390, 1993.645906

2038.137607 (1.19.279)

```
> f:=qsAB(5,6,2^19); f:=f*(1-2/7)/(1-4/7); H*f; 2*%;
```

$$f := 0.00004277933606$$

$$f := 0.00007129889343$$

$$1.284406680\ 10^{12}$$

$$2.568813360\ 10^{12} \tag{1.19.280}$$

```
> f:=f*(19/21)^5; H*f; 2*%;
```

$$f := 0.00004322692282$$

$$7.787070140\ 10^{11}$$

$$1.557414028\ 10^{12} \tag{1.19.281}$$

```
> f:=f*(21/24)^5; H*f; 2*%;
```

$$f := 0.00002217147497$$

$$3.994057857\ 10^{11}$$

$$7.988115714\ 10^{11} \tag{1.19.282}$$

```
> f:=f*(24/35)^5; H*f; 2*%;
```

$$f := 0.000003361325518$$

$$6.055225740\ 10^{10}$$

$$1.211045148\ 10^{11} \tag{1.19.283}$$

```
> f:=f*(35/40)^5; H*f; 2*%;
```

$$f := 0.000001724053893$$

$$3.105779388\ 10^{10}$$

$$6.211558776\ 10^{10} \tag{1.19.284}$$

```
> f:=f*(40/50)^5; H*f; 2*%;
```

$$f := 5.649379797\ 10^{-7}$$

$$1.017701790\ 10^{10}$$

$$2.035403580\ 10^{10} \tag{1.19.285}$$

```
> H*f*300/128^2/2.132358865; # SPU time in sec
```

$$8.738996349\ 10^{7} \tag{1.19.286}$$

```
> %/24/3600/420; # time for all SPU in days
```

$$2.408233121 \tag{1.19.287}$$

```
> # Cunningham chain II, length 6
> f:=qsAB(6,6,64); f:=f*(1-1/7)/(1-4/7)*(1-25/31)/(1-26/31);
  H:=2.^58; H/8; H*f; 2*%;
```

$$f := 0.002821881978$$

$$f := 0.006772516747$$

$$H := 2.882303762\ 10^{17}$$

$$3.602879702\ 10^{16}$$

$$1.952045050\ 10^{15}$$

$$3.904090100\ 10^{15} \tag{1.19.288}$$

```
> f:=qsAB(6,6,68); f:=f*(1-1/7)/(1-4/7)*(1-25/31)/(1-26/31); H*
  f; 2*%;
```

$$f := 0.002569176129$$

$$f := 0.006166022710$$

$$1.777235045\ 10^{15}$$

$$3.554470090\ 10^{15} \tag{1.19.289}$$

```
> f:=qsAB(6,6,2^19); f:=f*(1-1/7)/(1-4/7)*(1-25/31)/(1-26/31);
  H*f; 2*%;
```

$$f := 0.000003113467638$$

$$f := 0.000007472322331$$

$$2.153750277\ 10^{12}$$

$$4.307500554\ 10^{12} \tag{1.19.290}$$

```
> f:=f*(19/21)^6; H*f; 2*%;
```

$$f := 0.000004098844428$$

$$1.181411471\ 10^{12}$$

$$2.362822942\ 10^{12} \tag{1.19.291}$$

```
> f:=f*(21/24)^6; H*f; 2*%;
```

$$f := 0.000001839542191$$

$$5.302119377\ 10^{11}$$

$$1.060423875\ 10^{12} \tag{1.19.292}$$

```
> f:=f*(24/35)^6; H*f; 2*%;
```

$$f := 1.912356856\ 10^{-7}$$

$$5.511993360\ 10^{10}$$

$$1.102398672\ 10^{11} \tag{1.19.293}$$

```
> f:=f*(35/40)^6; H*f; 2*%;
```

$$f := 8.582568045\ 10^{-8}$$

$$2.473756816\ 10^{10}$$

$$4.947513632\ 10^{10} \tag{1.19.294}$$

```
> f:=f*(40/50)^6; H*f; 2*%;
```

$$f := 2.249868718\ 10^{-8}$$

$$6.484805070\ 10^{9}$$

$$1.296961014\ 10^{10} \tag{1.19.295}$$

```
> H*f*300/256^2/1.927056039; # SPU time in sec
```

$$1.540436874\ 10^{7} \tag{1.19.296}$$

```
> %/24/3600/420; # time for all SPU in days
```

$$0.4245031068 \tag{1.19.297}$$

1.20. Kérdés.

```
> #
  # This triviality makes a list of the differences
  # between odd primes up to a given limit. The
  # half of the differences is stored. The first
  # item is 1=(5-3)/2.
  #

  primes:=proc(N) local p,pp,L;
  L:=[]; pp:=3;
  for p from 5 while p<=N by 2 do
    if isprime(p) then L:=[op(L),(p-pp)/2]; pp:=p; fi;
  od; L end;
```

$$primes := \mathbf{proc}(N) \tag{1.20.1}$$

$$\mathbf{local}\ p,\ pp,\ L;$$

$$L := [\,];$$

$$pp := 3;$$

$$\mathbf{for}\ p\ \mathbf{from}\ 5\ \mathbf{by}\ 2\ \mathbf{while}\ p <= N\ \mathbf{do}$$

$$\mathbf{if}\ isprime(p)\ \mathbf{then}$$

$$L := [op(L),\ 1 / 2 * p - 1 / 2 * pp];$$

$$pp := p$$

$$\mathbf{end\ if}$$

$$\mathbf{end\ do};$$

$$L$$

$$\mathbf{end\ proc}$$

```
> L:=primes(1000);
```

$$L := [1,\ 1,\ 2,\ 1,\ 2,\ 1,\ 2,\ 3,\ 1,\ 3,\ 2,\ 1,\ 2,\ 3,\ 3,\ 1,\ 3,\ 2,\ 1,\ 3,\ 2,\ 3,\ 4,\ 2,\ 1,\ 2,\ 1, \tag{1.20.2}$$

$$2,\ 7,\ 2,\ 3,\ 1,\ 5,\ 1,\ 3,\ 3,\ 2,\ 3,\ 3,\ 1,\ 5,\ 1,\ 2,\ 1,\ 6,\ 6,\ 2,\ 1,\ 2,\ 3,\ 1,\ 5,\ 3,\ 3,\ 3,$$

1, 3, 2, 1, 5, 7, 2, 1, 2, 7, 3, 5, 1, 2, 3, 4, 3, 3, 2, 3, 4, 2, 4, 5, 1, 5, 1, 3,
2, 3, 4, 2, 1, 2, 6, 4, 2, 4, 2, 3, 6, 1, 9, 3, 5, 3, 3, 1, 3, 5, 3, 3, 1, 3, 3, 2,
1, 6, 5, 1, 2, 3, 3, 1, 6, 2, 3, 4, 5, 4, 5, 4, 3, 3, 2, 4, 3, 2, 4, 2, 7, 5, 6, 1,
5, 1, 2, 1, 5, 7, 2, 1, 2, 7, 2, 1, 2, 10, 2, 4, 5, 4, 2, 3, 3, 7, 2, 3, 3, 4, 3]

1.21. Kérdés.

```
> save(L,"primediffs"); read("primediffs");
> L := [1, 1, 2, 1, 2, 1, 2, 3, 1, 3, 2, 1, 2, 3, 3, 1, 3,
2, 1, 3, 2, 3, 4, 2,
> 1, 2, 1, 2, 7, 2, 3, 1, 5, 1, 3, 3, 2, 3, 3, 1, 5, 1, 2,
1, 6, 6, 2, 1, 2, 3,
> 1, 5, 3, 3, 3, 1, 3, 2, 1, 5, 7, 2, 1, 2, 7, 3, 5, 1, 2,
3, 4, 3, 3, 2, 3, 4,
> 2, 4, 5, 1, 5, 1, 3, 2, 3, 4, 2, 1, 2, 6, 4, 2, 4, 2, 3,
6, 1, 9, 3, 5, 3, 3,
> 1, 3, 5, 3, 3, 1, 3, 3, 2, 1, 6, 5, 1, 2, 3, 3, 1, 6, 2,
3, 4, 5, 4, 5, 4, 3,
> 3, 2, 4, 3, 2, 4, 2, 7, 5, 6, 1, 5, 1, 2, 1, 5, 7, 2, 1,
2, 7, 2, 1, 2, 10, 2,
> 4, 5, 4, 2, 3, 3, 7, 2, 3, 3, 4, 3];
```

$L := [1, 1, 2, 1, 2, 1, 2, 3, 1, 3, 2, 1, 2, 3, 3, 1, 3, 2, 1, 3, 2, 3, 4, 2, 1, 2, 1,$ (1.21.1)
2, 7, 2, 3, 1, 5, 1, 3, 3, 2, 3, 3, 1, 5, 1, 2, 1, 6, 6, 2, 1, 2, 3, 1, 5, 3, 3, 3,
1, 3, 2, 1, 5, 7, 2, 1, 2, 7, 3, 5, 1, 2, 3, 4, 3, 3, 2, 3, 4, 2, 4, 5, 1, 5, 1, 3,
2, 3, 4, 2, 1, 2, 6, 4, 2, 4, 2, 3, 6, 1, 9, 3, 5, 3, 3, 1, 3, 5, 3, 3, 1, 3, 3, 2,
1, 6, 5, 1, 2, 3, 3, 1, 6, 2, 3, 4, 5, 4, 5, 4, 3, 3, 2, 4, 3, 2, 4, 2, 7, 5, 6, 1,
5, 1, 2, 1, 5, 7, 2, 1, 2, 7, 2, 1, 2, 10, 2, 4, 5, 4, 2, 3, 3, 7, 2, 3, 3, 4, 3]

1.22. Kérdés.

1.23. Kérdés.

1.24. Kérdés.

1.25. Kérdés.

2. Egyszerű faktorizálási módszerek

3. Egyszerű prímtesztelési módszerek

▶ 4. Lucas–sorozatok
▶ 5. Alkalmazások
▶ 6. Számok és polinomok
▶ 7. Gyors Fourier–transzformáció
▶ 8. Elliptikus függvények
▶ 9. Számolás elliptikus görbéken
▶ 10. Faktorizálás elliptikus görbékkel
▶ 11. Prímteszt elliptikus görbékkel
▶ 12. Polinomfaktorizálás
▶ 13. Az AKS–teszt
▶ 14. A szita módszerek alapjai
▶ 15. Számtest szita
▶ 16. Vegyes problémák