Modern alkalmazott analízis

Járai Antal

Ezek a programok csak szemléltetésre szolgálnak

Bevezetés

I. Mérték és integrál

▶ 1. Mértékelmélet

▶ 2. Integrálás

II. Funkcionálanalízis

▼ 3. Metrikus terek

```
> restart;
```

▼ 3.114. Banach–féle fixponttétel.

▶ 3.115. Megjegyzés.

▼ 3.116. Feladat.

```
> T:=x->(cos(x/2)-abs(x-0.5))/2;
```

$$T := x \to \frac{1}{2}\cos\left(\frac{1}{2}x\right) - \frac{1}{2}|x-0.5| \qquad (3.3.1)$$

```
> 0.6;T(%);T(%);T(%);T(%);T(%);T(%);T(%);T(%);T(%);T(%);T
  (%);T(%);T(%);T(%);T(%);T(%);T(%);T(%);T(%);T(%);T(%);T
  (%);T(%);T(%);T(%);T(%);T(%);T(%);
```

$$0.6$$

$$0.4276682446$$

0.4524463559
0.4634834176
0.4683756328
0.4705393823
0.4714954464
0.4719177069
0.4721041694
0.4721865010
0.4722228527
0.4722389028
0.4722459892
0.4722491180
0.4722504994
0.4722511093
0.4722513786
0.4722514975
0.4722515500
0.4722515732
0.4722515834
0.4722515879
0.4722515900
0.4722515908
0.4722515912
0.4722515914
0.4722515915
0.4722515914
0.4722515915
0.4722515914
0.4722515915 (3.3.2)

▶ 3.117. Feladat.

▶ 3.118. Feladat.

▼ Feladat.

```
> solve([x1^2+x2^2=1,2*x1+x2=1],[x1,x2]);
```

$$\left[[x1 = 0,\ x2 = 1],\ \left[x1 = \frac{4}{5},\ x2 = -\frac{3}{5}\right]\right] \tag{3.6.1}$$

```
> T1:=x->[(1-x[2])/2,sqrt(1-x[1]^2)];
```

$$T1 := x \to \left[\frac{1}{2} - \frac{1}{2}\,x_2,\ \sqrt{1 - x_1^2}\right] \tag{3.6.2}$$

```
> x:=[-0.9,0.9];T1(%);T1(%);T1(%);T1(%);T1(%);T1(%);T1(%);T1(%)
  ;T1(%);T1(%);T1(%);
```

$$x := [-.9,\ 0.9]$$
$$[0.0500000000,\ 0.4358898944]$$
$$[0.2820550528,\ 0.9987492178]$$
$$[0.0006253911,\ 0.9593982214]$$
$$[0.0203008893,\ 0.9999998044]$$
$$[9.78\,10^{-8},\ 0.9997939157]$$
$$[0.0001030422,\ 1.]$$
$$[0.,\ 0.9999999947]$$
$$[2.6\,10^{-9},\ 1.]$$
$$[0.,\ 1.]$$
$$[0.,\ 1.]$$
$$[0.,\ 1.] \tag{3.6.3}$$

```
> T2:=x->[(1-x[2])/2,-sqrt(1-x[1]^2)];
```

$$T2 := x \to \left[\frac{1}{2} - \frac{1}{2}\,x_2,\ -\sqrt{1 - x_1^2}\right] \tag{3.6.4}$$

```
> x:=[0.9,0.9];T2(%);T2(%);T2(%);(%);T2(%);T2(%);T2(%);T2(%);T2
  (%);T2(%);T2(%);T2(%);T2(%);T2(%);T2(%);T2(%);T2(%);T2(%);T2
  (%);T2(%);T2(%);T2(%);T2(%);T2(%);T2(%);T2(%);T2(%);T2(%);T2
  (%);T2(%);T2(%);T2(%);T2(%);T2(%);T2(%);T2(%);T2(%);T2(%);T2
  (%);T2(%);T2(%);T2(%);(%);T2(%);T2(%);T2(%);T2(%);T2(%);T2(%)
  ;
```

$$x := [0.9,\ 0.9]$$
$$[0.0500000000,\ -.4358898944]$$
$$[0.7179449472,\ -.9987492178]$$
$$[0.9993746089,\ -.6960998871]$$

$$[0.9993746089, -.6960998871]$$
$$[0.8480499436, -0.03536086962]$$
$$[0.5176804348, -.5299163077]$$
$$[0.7649581538, -.8555740572]$$
$$[0.9277870286, -.6440799818]$$
$$[0.8220399909, -.3731102111]$$
$$[0.6865551056, -.5694297616]$$
$$[0.7847148808, -.7270777723]$$
$$[0.8635388862, -.6198568834]$$
$$[0.8099284417, -.5042822543]$$
$$[0.7521411272, -.5865287029]$$
$$[0.7932643514, -.6590020674]$$
$$[0.8295010337, -.6088773840]$$
$$[0.8044386920, -.5585051791]$$
$$[0.7792525896, -.5940356814]$$
$$[0.7970178407, -.6267099820]$$
$$[0.8133549910, -.6039557613]$$
$$[0.8019778806, -.5817677016]$$
$$[0.7908838508, -.5973537302]$$
$$[0.7986768651, -.6119662854]$$
$$[0.8059831427, -.6017601393]$$
$$[0.8008800696, -.5919384881]$$
$$[0.7959692440, -.5988247775]$$
$$[0.7994123888, -.6053370653]$$
$$[0.8026685326, -.6007826833]$$
$$[0.8003913416, -.5964253740]$$
$$[0.7982126870, -.5994778564]$$
$$[0.7997389282, -.6023757186]$$
$$[0.8011878593, -.6003479380]$$
$$[0.8001739690, -.5984129127]$$
$$[0.7992064564, -.5997679712]$$
$$[0.7998839856, -.6010566030]$$

$$
\begin{bmatrix}
[0.8005283015, -.6001546547] \\
[0.8000773274, -.5992949512] \\
[0.7996474756, -.5998968830] \\
[0.7999484415, -.6004697451] \\
[0.8002348726, -.6000687385] \\
[0.8000343692, -.5996867088] \\
[0.7998433544, -.5999541717] \\
[0.7998433544, -.5999541717] \\
[0.7999770858, -.6002088040] \\
[0.8001044020, -.6000305511] \\
[0.8000152756, -.5998607721] \\
[0.7999303860, -.5999796320] \\
[0.7999898160, -.6000928075] \\
[0.8000464038, -.6000135784]
\end{bmatrix} \tag{3.6.5}
$$

```
> with(LinearAlgebra); with(VectorCalculus);
```

```
[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix,
  BidiagonalForm, BilinearForm, CharacteristicMatrix,
  CharacteristicPolynomial, Column, ColumnDimension,
  ColumnOperation, ColumnSpace, CompanionMatrix,
  ConditionNumber, ConstantMatrix, ConstantVector, Copy,
  CreatePermutation, CrossProduct, DeleteColumn, DeleteRow,
  Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions,
  DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal,
  ForwardSubstitute, FrobeniusForm, GaussianElimination,
  GenerateEquations, GenerateMatrix, GetResultDataType,
  GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix,
  HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix,
  HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite,
  IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm,
  LA_Main, LUDecomposition, LeastSquares, LinearSolve, Map, Map2,
  MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse,
  MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply,
  MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply,
```

NoUserValue, *Norm*, *Normalize*, *NullSpace*, *OuterProductMatrix*, *Permanent*, *Pivot*, *PopovForm*, *QRDecomposition*, *RandomMatrix*, *RandomVector*, *Rank*, *RationalCanonicalForm*, *ReducedRowEchelonForm*, *Row*, *RowDimension*, *RowOperation*, *RowSpace*, *ScalarMatrix*, *ScalarMultiply*, *ScalarVector*, *SchurForm*, *SingularValues*, *SmithForm*, *SubMatrix*, *SubVector*, *SumBasis*, *SylvesterMatrix*, *ToeplitzMatrix*, *Trace*, *Transpose*, *TridiagonalForm*, *UnitVector*, *VandermondeMatrix*, *VectorAdd*, *VectorAngle*, *VectorMatrixMultiply*, *VectorNorm*, *VectorScalarMultiply*, *ZeroMatrix*, *ZeroVector*, *Zip*]

[*&x*, *, +, -, ., <,>, </>, *AddCoordinates*, *ArcLength*, *BasisFormat*, *Binormal*, *CrossProd*, *CrossProduct*, *Curl*, *Curvature*, D, *Del*, *DirectionalDiff*, *Divergence*, *DotProd*, *DotProduct*, *Flux*, *GetCoordinateParameters*, *GetCoordinates*, *Gradient*, *Hessian*, *Jacobian*, *Laplacian*, *LineInt*, *MapToBasis*, *Nabla*, *Norm*, *Normalize*, *PathInt*, *PrincipalNormal*, *RadiusOfCurvature*, *ScalarPotential*, *SetCoordinateParameters*, *SetCoordinates*, *SurfaceInt*, *TNBFrame*, *Tangent*, *TangentLine*, *TangentPlane*, *TangentVector*, *Torsion*, *Vector*, *VectorField*, *VectorPotential*, *Wronskian*, *diff*, *evalVF*, *int*, *limit*, *series*] (3.6.6)

```
> x:='x';T1:=<(1-y)/2,sqrt(1-x^2)>;
```

$$x := x$$

$$T1 := \left(\frac{1}{2} - \frac{1}{2}\,y\right) e_x + \left(\sqrt{1-x^2}\right) e_y \tag{3.6.7}$$

```
> Jacobian(T1,[x,y]);
```

$$\begin{bmatrix} 0 & -\frac{1}{2} \\ -\frac{x}{\sqrt{1-x^2}} & 0 \end{bmatrix} \tag{3.6.8}$$

```
> a:=subs(x=0,y=1,%);
```

$$a := \begin{bmatrix} 0 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} \tag{3.6.9}$$

```
> MatrixNorm(a);
```

(3.6.10)

$$\frac{1}{2} \tag{3.6.10}$$

```
> T2:=<(1-y)/2,-sqrt(1-x^2)>;
```

$$T2 := \left(\frac{1}{2} - \frac{1}{2}\,y\right) e_x - \sqrt{1-x^2}\,e_y \tag{3.6.11}$$

```
> Jacobian(T2,[x,y]);
```

$$\begin{bmatrix} 0 & -\frac{1}{2} \\ \frac{x}{\sqrt{1-x^2}} & 0 \end{bmatrix} \tag{3.6.12}$$

```
> a:=subs(x=4/5,y=-3/5,%);
```

$$a := \begin{bmatrix} 0 & -\frac{1}{2} \\ \frac{4}{45}\sqrt{9}\sqrt{25} & 0 \end{bmatrix} \tag{3.6.13}$$

```
> MatrixNorm(a); evalf(%);
```

$$\frac{4}{45}\sqrt{9}\sqrt{25}$$

$$1.333333333 \tag{3.6.14}$$

```
> Eigenvalues(a); evalf(%);
```

$$\begin{bmatrix} \frac{1}{3}\,\mathrm{I}\sqrt{6} \\ -\frac{1}{3}\,\mathrm{I}\sqrt{6} \end{bmatrix}$$

$$\begin{bmatrix} 0.8164965809\,\mathrm{I} \\ -0.8164965809\,\mathrm{I} \end{bmatrix} \tag{3.6.15}$$

▶ 3.119. Feladat.

▶ 3.120. Feladat: a fixpont folytonos függése a paraméterektől.

▶ ->3.121. Feladat.

▶ Fixpont tulajdonság.

▶ Retraktum.

▶ **Tétel.**

▶ **Összehúzhatóság.**

▶ **Tétel.**

▶ **Tétel.**

▶ **Tétel.**

▶ **Megjegyzés.**

▼ ***3.122. Egyenletek megoldása és fixponttételek.**

```
> v:='v'; f:=Vector(2,[v[1]^2+v[2]^2-1,2*v[1]+v[2]-1]);
```

$$v := v$$

$$f := \left(v_1^2 + v_2^2 - 1\right) e_x + \left(2\,v_1 + v_2 - 1\right) e_y \tag{3.18.1}$$

```
> J:=Jacobian(f,[v[1],v[2]]);
```

$$J := \begin{bmatrix} 2\,v_1 & 2\,v_2 \\ 2 & 1 \end{bmatrix} \tag{3.18.2}$$

```
> Jinv:=MatrixInverse(J);
```

$$Jinv := \begin{bmatrix} \frac{1}{2\left(v_1 - 2\,v_2\right)} & -\frac{v_2}{v_1 - 2\,v_2} \\ -\frac{1}{v_1 - 2\,v_2} & \frac{v_1}{v_1 - 2\,v_2} \end{bmatrix} \tag{3.18.3}$$

```
> Jinv0:=subs(v[1]=-2.,v[2]=2.,Jinv); v:=Vector(2,[-2.,2.]);
```

$$Jinv0 := \begin{bmatrix} -0.08333333335 & 0.3333333333 \\ 0.1666666667 & 0.3333333334 \end{bmatrix}$$

$$v := (-2.)\,e_x + (2.)\,e_y \tag{3.18.4}$$

```
> v:=v-Jinv0.f;v:=v-Jinv0.f;v:=v-Jinv0.f;v:=v-Jinv0.f;v:=v-
  Jinv0.f;v:=v-Jinv0.f;v:=v-Jinv0.f;v:=v-Jinv0.f;v:=v-Jinv0.f;
  v:=v-Jinv0.f;v:=v-Jinv0.f;
```

$$v := (-.41666666700000102)\,e_x + (1.83333333300000012)\,e_y$$

$$v := (-.20543981490000101)\,e_x + (1.41087962970000014)\,e_y$$

$$v := (-.11937424422000104)\,e_x + (1.23874848830000018)\,e_y$$

$$v := (-0.07364524183000009 66)\,e_x + (1.14729048391000022)\,e_y$$

$$v := (-0.0468369854100001010)e_x + (1.09367397107000032)e_y$$
$$v := (-0.0303106139100000997)e_x + (1.06062122806000025)e_y$$
$$v := (-0.0198242704100001014)e_x + (1.03964854106000026)e_y$$
$$v := (-0.0130524294920001012)e_x + (1.02610485922000017)e_y$$
$$v := (-0.00863063382400010119)e_x + (1.01726126788500015)e_y$$
$$v := (-0.00572271924000010077)e_x + (1.01144543871700021)e_y$$
$$v := (-0.00380150073900010089)e_x + (1.00760300071700026)e_y \quad (3.18.5)$$

```
> v:=Vector(2,[-0.9,0.9]);
  v:=v-Jinv.f;v:=v-Jinv.f;v:=v-Jinv.f;v:=v-Jinv.f;v:=v-Jinv.f;
  v:=v-Jinv.f;v:=v-Jinv.f;
```

$$v := (-.9)e_x + (0.9)e_y$$
$$v := (-.151851851899999946)e_x + (1.30370370370000010)e_y$$
$$v := (-0.0208923687999999497)e_x + (1.04178473739999999)e_y$$
$$v := (-0.000518530579999950531)e_x + (1.00103706156999994)e_y$$
$$v := (-3.35580999950586128\,10^{-7})e_x + (1.00000067057199993)e_y$$
$$v := (1.87184494138486878\,10^{-11})e_x + (0.99999999957256292)e_y$$
$$v := (1.87184493138486877\,10^{-11})e_x + (0.99999999997256296)e_y$$
$$v := (1.87184493138486877\,10^{-11})e_x + (0.99999999997256296)e_y \quad (3.18.6)$$

3.123. Aitken Δ^2-módszere.

```
> Aitken:=proc(x1,x2,x3) x1-(x2-x1)^2/((x3-x2)-(x2-x1)) end;
```

$$Aitken := \mathbf{proc}(x1, x2, x3) \quad (3.19.1)$$
$$VectorCalculus:-+(x1, VectorCalculus:-+(VectorCalculus:-*(VectorCalculus:-+(x2, VectorCalculus:--(x1))^2, 1 / VectorCalculus:-+(VectorCalculus:-+(x3, VectorCalculus:--(x2)), VectorCalculus:--(VectorCalculus:-+(x2, VectorCalculus:--(x1))))))))$$
$$\mathbf{end\ proc}$$

*3.124. Feladat.

```
> fsolve(x=cos(x));
```

$$0.7390851332 \tag{3.20.1}$$

```
> x:=1.5;cos(%);cos(%);cos(%);cos(%);cos(%);cos(%);cos(%);cos
  (%);cos(%);cos(%);cos(%);cos(%);cos(%);cos(%);cos(%);cos(%);
  cos(%);cos(%);cos(%);cos(%);cos(%);cos(%);cos(%);cos(%);cos
  (%);cos(%);cos(%);cos(%);cos(%);cos(%);cos(%);
```

$$x := 1.5$$

$$0.07073720167$$

$$0.9974991672$$

$$0.5424049923$$

$$0.8564697090$$

$$0.6551088017$$

$$0.7929816458$$

$$0.7017241683$$

$$0.7637303113$$

$$0.7222610821$$

$$0.7503128857$$

$$0.7314755580$$

$$0.7441895867$$

$$0.7356370983$$

$$0.7414033729$$

$$0.7375215545$$

$$0.7401374748$$

$$0.7383758542$$

$$0.7395627261$$

$$0.7387633366$$

$$0.7393018610$$

$$0.7389391254$$

$$0.7391834780$$

$$0.7390188834$$

$$0.7391297583$$

$$0.7390550725$$

$$0.7391053821$$

$$0.7390714932$$

0.7390943212

0.7390789440

0.7390893023

0.7390823249 (3.20.2)

```
> x:=0.;cos(%);cos(%);Aitken(%%%,%%,%);cos(%);cos(%);Aitken
(%%%,%%,%);cos(%);cos(%);Aitken(%%%,%%,%);cos(%);cos(%);
Aitken(%%%,%%,%);
```

$x := 0.$

1.

0.5403023059

0.6850733574

0.7743726338

0.7148598716

0.7386601562

0.7393713361

0.7388923132

0.7390851063

0.7390851513

0.7390851210

0.7390851332 (3.20.3)

*3.125. Feladat.

```
> x:=1.;2/%;2/%;2/%;2/%;2/%;2/%;2/%;2/%;2/%;2/%;2/%;2/%;2/%;
2/%;2/%;2/%;2/%;
```

$x := 1.$

2.

1.000000000

2.000000000

1.000000000

2.000000000

1.000000000

2.000000000

1.000000000

2.000000000

1.000000000

2.000000000

1.000000000

2.000000000

1.000000000

2.000000000

1.000000000

2.000000000 (3.21.1)

```
> x:=1.;2/%;2/%;Aitken(%%%,%%,%);2/%;2/%;Aitken(%%%,%%,%);2/%;
  2/%;Aitken(%%%,%%,%);2/%;2/%;Aitken(%%%,%%,%);2/%;2/%;Aitken
  (%%%,%%,%);2/%;2/%;Aitken(%%%,%%,%);2/%;2/%;Aitken(%%%,%%,%);
```

$x := 1.$

2.

1.000000000

1.500000000

1.333333333

1.500000000

1.416666666

1.411764707

1.416666666

1.414215686

1.414211439

1.414215686

1.414213562

1.414213563

1.414213562

1.414213562

1.414213563

1.414213562

1.414213562

1.414213563

1.414213562

1.414213562 (3.21.2)

▶ *3.126. Feladat.

>

>

>

>

>

>

>

4. Normált terek

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6. Hilbert–terek

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