Kalkulus I.

Járai Antal

Ezek a programok csak szemléltetésre szolgálnak

▶ 1. Halmazok

▶ 2. Számok

▶ 3. Határérték

▶ 4. Differenciálszámítás

▼ 5. Integrálszámítás

```
> restart;
```

▼ 5.1. Primitív függvények

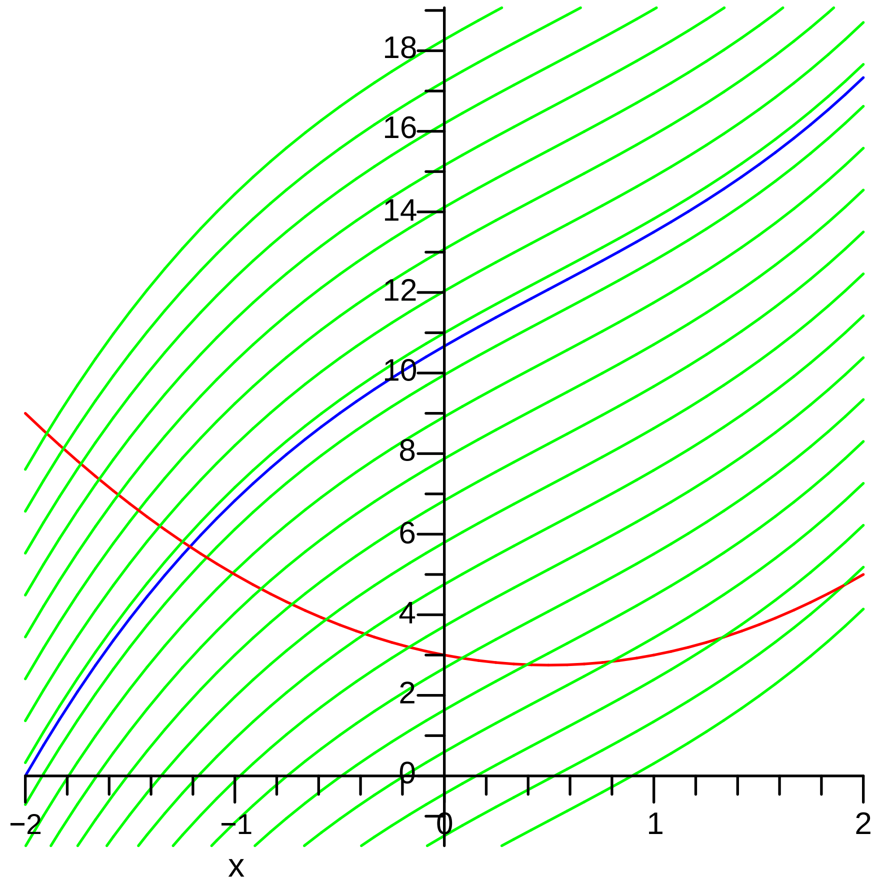
▼ *5.1.1. Primitív függvény.*

```
> int(cos(x),x); int(x/(x^3-1),x);
```

$$\sin(x)$$

$$-\frac{1}{6}\ln\left(x^2+x+1\right)+\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\left(2x+1\right)\sqrt{3}\right)+\frac{1}{3}\ln\left(x-1\right) \quad (5.1.1.1)$$

```
> Student[Calculus1][AntiderivativeTutor]();
```



▶ *5.1.2. Tagonkénti integrálás határozatlan integrálokra.*

▶ *5.1.3. Parciális integrálás határozatlan integrálokra.*

▶ *5.1.4. Helyettesítéses integrálás határozatlan integrálokra.*

▶ *5.1.5. Megjegyzés.*

▶ *5.1.6. Tétel.*

▼ *5.1.7. Alapintegrálok.*

```
> int(x^n,x);
```

$$\frac{x^{n+1}}{n+1} \tag{5.1.7.1}$$

```
> int(1/(1+x^2),x);
```

$$\arctan(x) \tag{5.1.7.2}$$

```
> int(exp(x),x);
```

$$e^{x} \tag{5.1.7.3}$$

```
> int(sin(x),x);
```
$$-\cos(x) \tag{5.1.7.4}$$

```
> int(cos(x),x);
```
$$\sin(x) \tag{5.1.7.5}$$

```
> int(sinh(x),x);
```
$$\cosh(x) \tag{5.1.7.6}$$

```
> int(cosh(x),x);
```
$$\sinh(x) \tag{5.1.7.7}$$

```
> int(1/cosh(x)^2,x);
```
$$\frac{\sinh(x)}{\cosh(x)} \tag{5.1.7.8}$$

```
> int(x^n,x);
```
$$\frac{x^{n+1}}{n+1} \tag{5.1.7.9}$$

```
> int(1/x,x);
```
$$\ln(x) \tag{5.1.7.10}$$

```
> int(1/sinh(x)^2,x);
```
$$-\frac{\cosh(x)}{\sinh(x)} \tag{5.1.7.11}$$

```
> int(x^n,x);
```
$$\frac{x^{n+1}}{n+1} \tag{5.1.7.12}$$

```
> int(1/sqrt(1-x^2),x);
```
$$\arcsin(x) \tag{5.1.7.13}$$

```
> int(1/sin(x)^2,x);
```
$$-\frac{\cos(x)}{\sin(x)} \tag{5.1.7.14}$$

```
> int(1/cos(x)^2,x);
```
$$\frac{\sin(x)}{\cos(x)} \tag{5.1.7.15}$$

▶ ***5.1.8. Megjegyzés.***

▼ ***5.1.9. Példák.***

```
> Student[Calculus1][IntTutor]();
```
$$\int \sin(x)^2 \, dx \tag{5.1.9.1}$$

▶ ***5.1.10. Tétel.***

▼ ***5.1.11. Elemien integrálható függvények.***

```
> int(exp(x^2),x); int(cos(x^2),x); int(sin(x^2),x); int(sin
  (x)/x,x);
  int(cos(x)/x,x); int(exp(x)/x,x); int(sqrt(1+x^3),x); int
  (1/ln(x),x);
```

$$-\frac{1}{2}\,\mathrm{I}\sqrt{\pi}\,\mathrm{erf}(\mathrm{I}\,x)$$

$$\frac{1}{2}\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)$$

$$\frac{1}{2}\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\,x}{\sqrt{\pi}}\right)$$

$$\mathrm{Si}(x)$$

$$\mathrm{Ci}(x)$$

$$-\mathrm{Ei}(1,-x)$$

$$\frac{2}{5}\,x\sqrt{1+x^3}+\frac{6}{5}\,\frac{1}{\sqrt{1+x^3}}\left(\left(\frac{3}{2}-\frac{1}{2}\,\mathrm{I}\sqrt{3}\right)\sqrt{\frac{x+1}{\frac{3}{2}-\frac{1}{2}\,\mathrm{I}\sqrt{3}}}\,\sqrt{\frac{x-\frac{1}{2}-\frac{1}{2}\,\mathrm{I}\sqrt{3}}{-\frac{3}{2}-\frac{1}{2}\,\mathrm{I}\sqrt{3}}}\,\sqrt{\frac{x-\frac{1}{2}+\frac{1}{2}\,\mathrm{I}\sqrt{3}}{-\frac{3}{2}+\frac{1}{2}\,\mathrm{I}\sqrt{3}}}\,\mathrm{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2}-\frac{1}{2}\,\mathrm{I}\sqrt{3}}},\sqrt{\frac{-\frac{3}{2}+\frac{1}{2}\,\mathrm{I}\sqrt{3}}{-\frac{3}{2}-\frac{1}{2}\,\mathrm{I}\sqrt{3}}}\right)\right)$$

$$-\mathrm{Ei}(1,-\ln(x)) \tag{5.1.11.1}$$

5.1.12. Segédtétel.

```
> f:=x^3+1; g:=x^2-3; gcdex(f,g,x,'u','v'); f*u+g*v; expand
  (%);
```

$$f:=1+x^3$$

$$g:=x^2-3$$

$$1$$

$$(1+x^3)\left(-\frac{1}{26}+\frac{3}{26}\,x\right)+(x^2-3)\left(-\frac{9}{26}-\frac{3}{26}\,x^2+\frac{1}{26}\,x\right)$$

$$1 \tag{5.1.12.1}$$

5.1.13. Parciális törtekre bontás tétele.

```
> convert(x^2/(x^3+1),parfrac,x);
```

$$\frac{1}{3}\,\frac{2\,x-1}{x^2-x+1}+\frac{1}{3\,(x+1)} \qquad (5.1.13.1)$$

5.1.14. Racionális törtfüggvények integrálása.

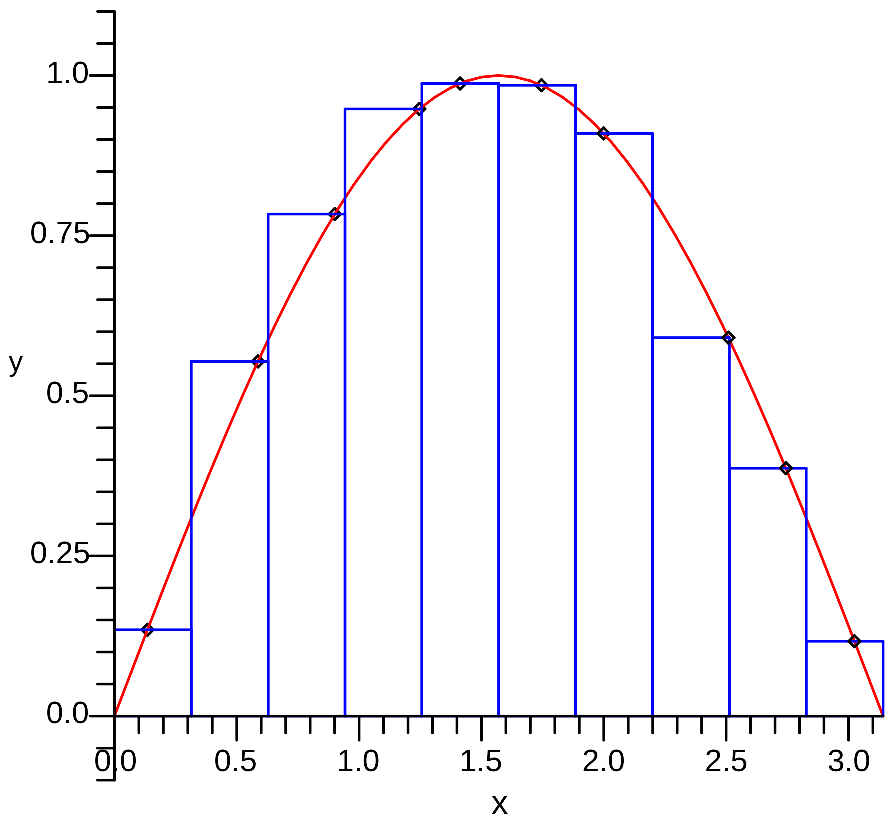
5.1.15. Racionális törtfüggvények integrálására visszavezethető integrálok.

5.2. Határozott integrál

```
> restart;
```

5.2.1. Definíció.

```
> Student[Calculus1][ApproximateIntTutor]();
```



▶ ***5.2.2. Tétel.***

▶ ***5.2.3. Tétel: az integrál egyértelműsége.***

▶ ***5.2.4. Tétel: komplex értékű függvények integrálja.***

▶ ***5.2.5. Tétel: az integrál linearitása.***

▶ ***5.2.6. Tétel: az integrál nemnegativitása.***

▶ ****5.2.7. Tétel: Cauchy-kritérium.***

▶ ****5.2.8. Segédtétel.***

▼ ***5.2.9. Tétel: az integrál intervallum-additivitása.***

```
> Student[Calculus1][IntTutor](x^3,x=0..3);
```

$$\int_0^3 x^3\,dx = \int_0^2 x^3\,dx + \int_2^3 x^3\,dx \qquad (5.2.9.1)$$

▶ ***5.2.10. Definíció.***

▶ *5.2.11. Nullahalmazok.*

▶ *5.2.12. Lebesgue-feltétel.*

▶ *5.2.13. Következmény.*

▶ *5.2.14. Tétel.*

▶ *5.2.15. Megjegyzés.*

▶ *5.2.16. Az integrál, mint a felső határ függvénye.*

▶ *5.2.17. Következmény.*

▼ *5.2.18. Newton-Leibniz-formula.*

```
> Student[Calculus1][IntTutor](sin(x)^2,x=0..Pi);
```

$$\int_0^{\pi} \sin(x)^2 \, dx = \frac{1}{2}\pi \qquad (5.2.18.1)$$

▼ *5.2.19. Parciális integrálás határozott integrálokra.*

```
> Student[Calculus1][IntTutor](x*exp(x),x=-1..1);
```

$$\int_{-1}^{1} x\, e^x \, dx = 2\, e^{-1} \qquad (5.2.19.1)$$

▼ *5.2.20. Helyettesítéses integrálás határozott integrálokra.*

```
> Student[Calculus1][IntTutor](1/(1+cos(x)),x=-Pi/2..Pi/2);
```

$$\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \frac{1}{1+\cos(x)} \, dx = 2 \qquad (5.2.20.1)$$

▼ *5.2.21. Tétel.*

```
> int(1/sqrt(x),x=0..1);
```

$$2 \qquad (5.2.21.1)$$

▼ *5.2.22. Példák.*

```
> int(x^a,x=0..1) assuming a<0 and -1<a; int(x^a,x=0..1)
  assuming a<=-1;
```

$$\frac{1}{a+1}$$

(5.2.22.1)

$$\infty \quad (5.2.22.1)$$

```
> int(1/x,x=-1..1);
```

$$undefined \quad (5.2.22.2)$$

▶ 5.2.23. Megjegyzés.
▶ 5.2.24. Abszolút integrálható függvények.
▶ 5.2.25. Tétel.
▶ 5.2.26. Improprius integrál.
▼ 5.2.27. Példák.

```
> int(x^a,x=1..infinity) assuming a<-1;
```

$$-\frac{1}{a+1} \quad (5.2.27.1)$$

```
> int(x^a,x=1..infinity) assuming a>=-1;
```

$$\infty \quad (5.2.27.2)$$

```
> int(exp(a*x),x=0..infinity) assuming a<0;
```

$$-\frac{1}{a} \quad (5.2.27.3)$$

```
> int(exp(a*x),x=0..infinity) assuming a>=0;
```

$$\infty \quad (5.2.27.4)$$

▶ 5.2.28. Tétel.
▶ 5.2.29. További improprius integrálok.

▼ 5.3. Alkalmazások

```
> restart;
```

▶ 5.3.1. Végtelen kicsinyek.
▼ 5.3.2. Görbe hossza, heurisztikusan.

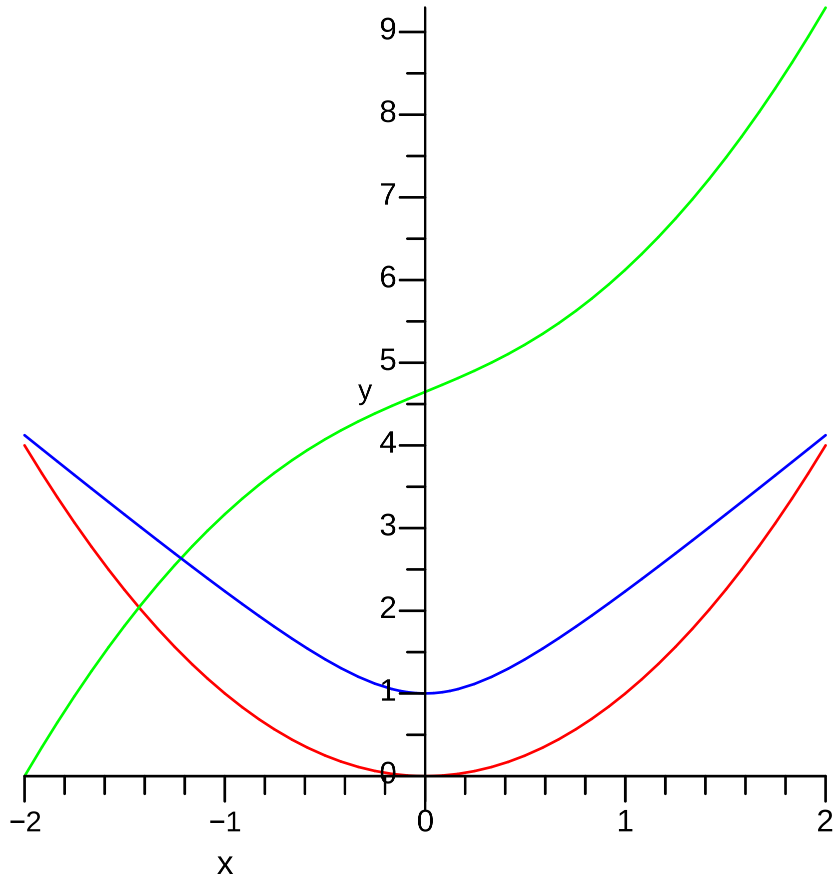
```
> z:=exp(I*t); zp:=diff(z,t); int(evalc(abs(zp)),t=0..2*Pi);
```

$$z := e^{I t}$$

$$zp := I e^{I t}$$

$$2\pi \quad (5.3.2.1)$$

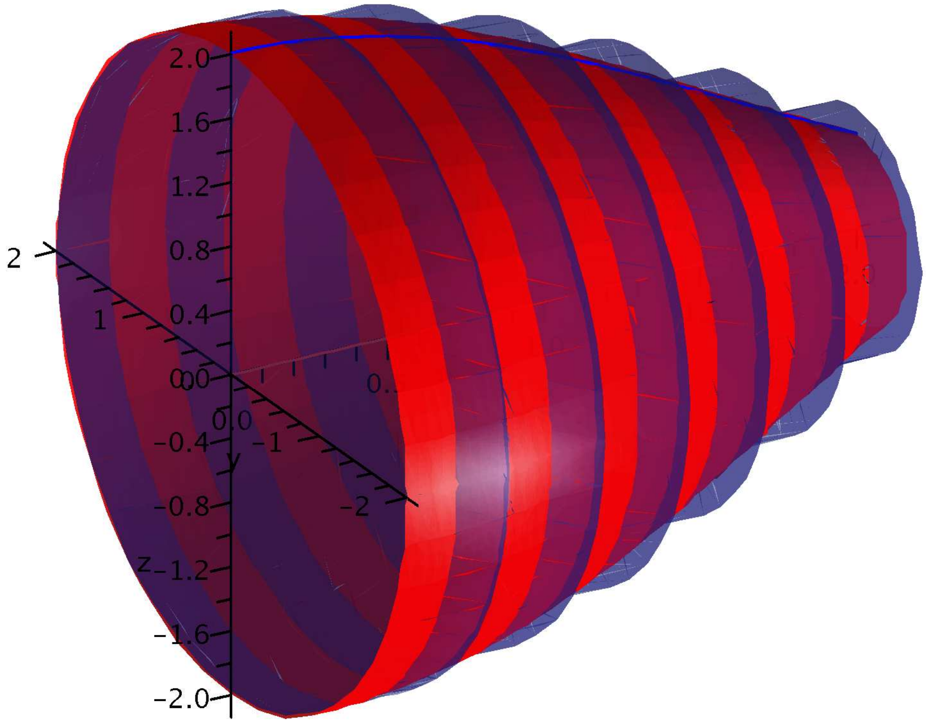
```
> Student[Calculus1][ArcLengthTutor]();
```



5.3.3. *Polárkoordinátákkal adott szektor területe, heurisztikusan.*

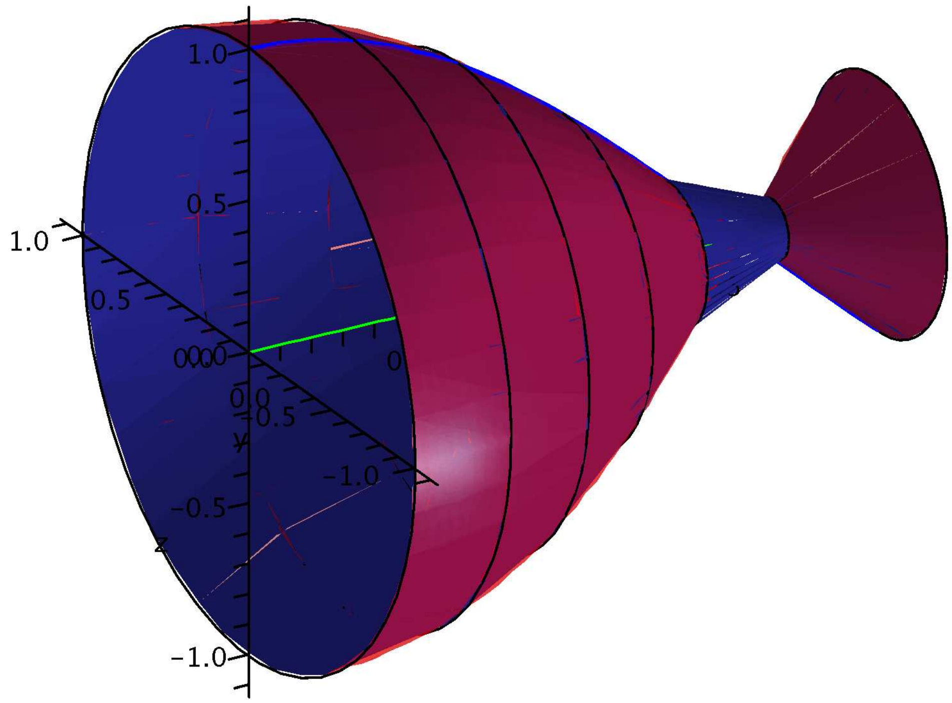
5.3.4. *Forgástest térfogata, heurisztikusan.*

```
> Student[Calculus1][VolumeOfRevolutionTutor]();
```



5.3.5. Forgástest felszíne, heurisztikusan.

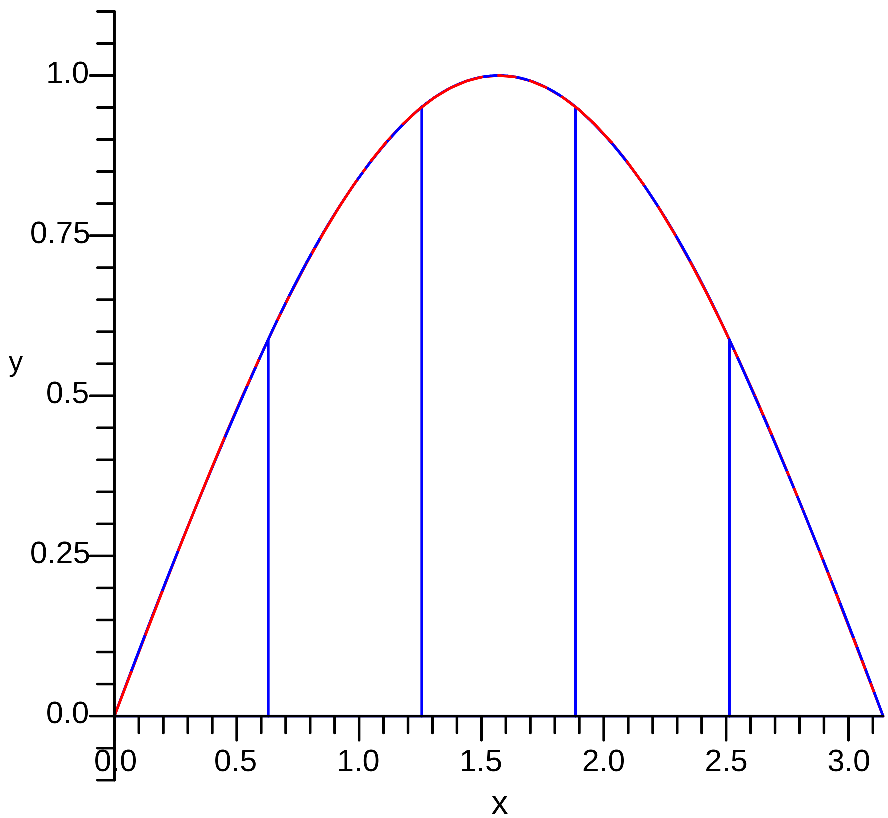
```
> Student[Calculus1][SurfaceOfRevolutionTutor]();
```



5.3.6. Tömeg, tömegközéppont, tehetetlenségi nyomaték.

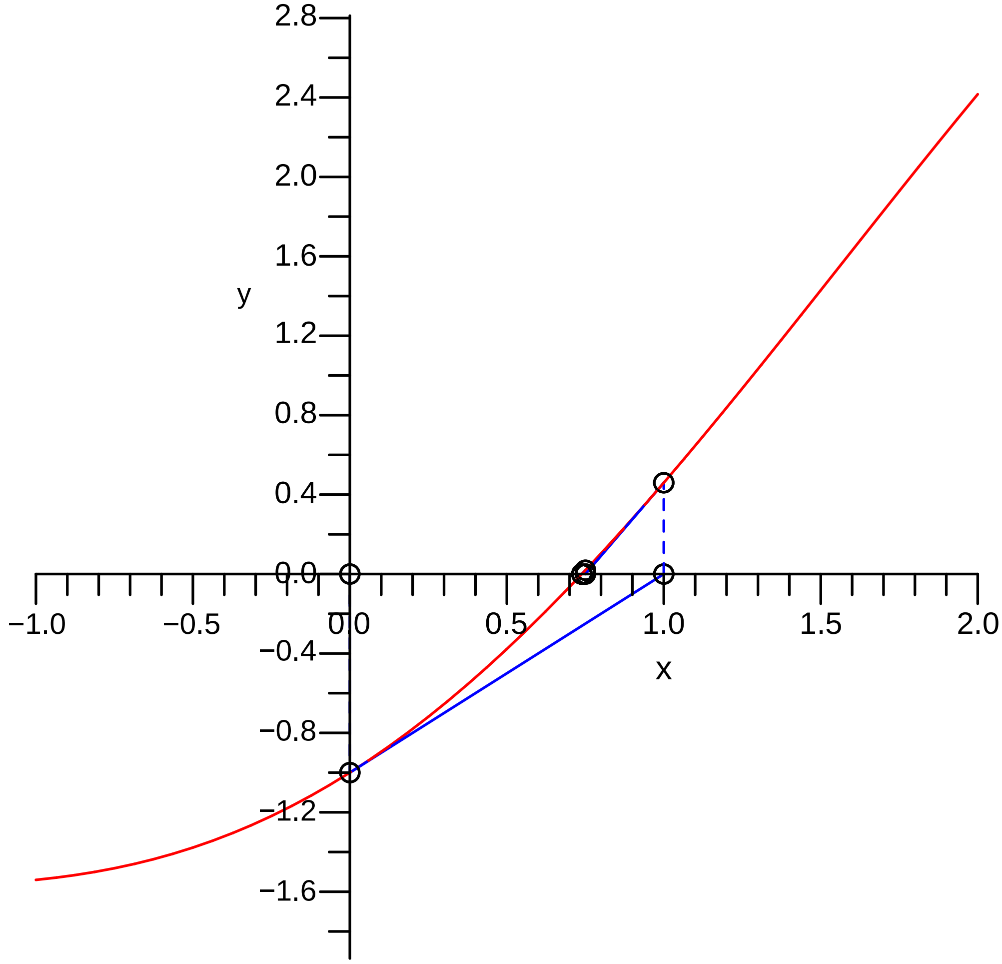
5.3.7. Közelítő számítások.

```
> Student[Calculus1][ApproximateIntTutor]();
```



5.3.8. Newton-módszer.

```
> Student[Calculus1][NewtonsMethodTutor](x-cos(x),0);
```




```
> relaxnewton:=proc(f::procedure,fp::procedure,x0,epsilon)
  local x,xx,t; x:=x0; t:=1.; print(x,t,f(x),abs(f(x)));
  do if abs(f(x))<epsilon then return(x) fi;
    t:=1.;
    do xx:=x-t*f(x)/fp(x); print(xx,t,f(xx),abs(f(xx)));
      if abs(f(xx))<abs(f(x)) then break else t:=t/2 fi;
    od; x:=xx; od; end;

  relaxnewton(z->z^5-z+3,z->5*z^4-1,3.+10.*I,0.0000001);

  relaxnewton(z->z^5-z+3,z->5*z^4-1,(1/5.)^(1/4)+0.01,
  0.0000001);
```

relaxnewton := **proc**(*f*::*procedure*, *fp*::*procedure*, *x0*, ε)
local *x*, *xx*, *t*;
x := *x0*;
t := 1.;
print(*x*, *t*, *f*(*x*), abs(*f*(*x*)));

```
    do
        if abs(f(x)) < ε then
            return x
        end if;
        t := 1.;
        do
            xx := x - t*f(x) / fp(x);
            print(xx, t, f(xx), abs(f(xx)));
            if abs(f(xx)) < abs(f(x)) then
                break
            else
                t := 1 / 2*t
            end if
        end do;
        x := xx
    end do
end proc
```

$3. + 10.\,I,\ 1.,\ 1.23243\ 10^5 + 14040.\,I,\ 1.240401493\ 10^5$

$2.399872243 + 8.000043773\,I,\ 1.,\ 40384.62044 + 4599.180768\,I,\ 40645.66437$

$1.919638552 + 6.400097857\,I,\ 1.,\ 13233.67341 + 1505.902344\,I,\ 13319.07856$

$1.535180765 + 5.120145726\,I,\ 1.,\ 4336.920528 + 492.5281458\,I,\ 4364.798236$

$1.227051191 + 4.096113408\,I,\ 1.,\ 1421.688163 + 160.6504428\,I,\ 1430.736103$

$0.9793624175 + 3.276555586\,I,\ 1.,\ 466.4696379 + 52.04789824\,I,\ 469.3643646$

$0.7786832399 + 2.619787728\,I,\ 1.,\ 153.5006812 + 16.57703671\,I,\ 154.3931905$

$0.6126365287 + 2.091022469\,I,\ 1.,\ 50.98085391 + 5.042396308\,I,\ 51.22961278$

$0.4673540873 + 1.658543942\,I,\ 1.,\ 17.42867285 + 1.321921898\,I,\ 17.47873322$

$0.3202538500 + 1.285961515\,I,\ 1.,\ 6.518954023 + 0.117325617\,I,\ 6.520009728$

$0.1022977568 + 0.9060193817\,I,\ 1.,\ 3.233581963 - 0.3728503660\,I,\ 3.255006867$

$-0.9977750502 + 0.3000912412\,I,\ 1.,$
$3.862941208 + 0.9204442098\,I,\ 3.971087045$
$-0.4477386469 + 0.6030553114\,I,\ 0.5000000000,$
$3.460083711 - 0.8417804237\,I,\ 3.561007381$
$-0.1727204450 + 0.7545373466\,I,\ 0.2500000000,$
$2.921980382 - 0.6347627568\,I,\ 2.990132624$
$0.1060046936 + 2.855401862\,I,\ 1.,\ 38.03105906 + 184.3477905\,I,$
188.2298311
$-0.0333578757 + 1.804969604\,I,\ 0.5000000000,$
$1.264262686 + 17.28757601\,I,\ 17.33374294$
$-0.1030391604 + 1.279753475\,I,\ 0.2500000000,$
$1.739041227 + 1.931106675\,I,\ 2.598737651$
$-0.2843003719 + 1.182761857\,I,\ 1.,\ 0.8220340289 - 0.166825940\,I,$
0.8387912964
$-0.3118996189 + 1.258603280\,I,\ 1.,$
$-0.1236837561 + 0.019656712\,I,\ 0.1252360086$
$-0.3084767808 + 1.250051687\,I,\ 1.,$
$-0.0018307092 + 0.000149634\,I,\ 0.001836814228$
$-0.3084151213 + 1.249926939\,I,\ 1.,\ -4.087\ 10^{-7} - 3.2\ 10^{-8}\,I,$
$4.099508385\ 10^{-7}$
$-0.3084151033 + 1.249926914\,I,\ 1.,\ -2.7\ 10^{-9} + 2.\ 10^{-9}\,I,$
$3.360059523\ 10^{-9}$

$$-0.3084151033 + 1.249926914\,I$$

$$0.6787403050,\ 1.,\ 2.465311331,\ 2.465311331$$
$$-39.62452844,\ 1.,\ -9.768330493\ 10^{7},\ 9.768330493\ 10^{7}$$
$$-19.47289408,\ 0.5000000000,\ -2.799941916\ 10^{6},\ 2.799941916\ 10^{6}$$
$$-9.397076885,\ 0.2500000000,\ -73263.96516,\ 73263.96516$$
$$-4.359168288,\ 0.1250000000,\ -1566.689577,\ 1566.689577$$
$$-1.840213992,\ 0.06250000000,\ -16.26266179,\ 16.26266179$$
$$-.5807368430,\ 0.03125000000,\ 3.514683182,\ 3.514683182$$
$$0.0490017308,\ 0.01562500000,\ 2.950998552,\ 2.950998552$$
$$0.3638710180,\ 0.007812500000,\ 2.642507758,\ 2.642507758$$
$$0.5213056615,\ 0.003906250000,\ 2.517194470,\ 2.517194470$$
$$0.6000229832,\ 0.001953125000,\ 2.477751911,\ 2.477751911$$
$$0.6393816441,\ 0.0009765625000,\ 2.467474825,\ 2.467474825$$
$$0.6590609746,\ 0.0004882812500,\ 2.465283926,\ 2.465283926$$

$44.17609570, 1., 1.682428220\ 10^8, 1.682428220\ 10^8$

$22.41757833, 0.5000000000, 5.661636599\ 10^6, 5.661636599\ 10^6$

$11.53831965, 0.2500000000, 2.045006528\ 10^5, 2.045006528\ 10^5$

$6.098690317, 0.1250000000, 8433.801381, 8433.801381$

$3.378875646, 0.06250000000, 440.0350502, 440.0350502$

$2.018968310, 0.03125000000, 34.52755452, 34.52755452$

$1.339014642, 0.01562500000, 5.965524190, 5.965524190$

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$-108.1303429, 0.2500000000, -1.478215997\ 10^{10}, 1.478215997\ 10^{10}$

$-53.73032884, 0.1250000000, -4.478137638\ 10^8, 4.478137638\ 10^8$

$-26.53032179, 0.06250000000, -1.314351764\ 10^7, 1.314351764\ 10^7$

$-12.93031827, 0.03125000000, -3.614322763\ 10^5, 3.614322763\ 10^5$

$-6.130316509, 0.01562500000, -8648.808030, 8648.808030$

$-2.730315628, 0.007812500000, -145.9971749, 145.9971749$

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$0.4571851957, 0.0004882812500, 2.562788609, 2.562788609$

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$144.9521918, 0.2500000000, 6.399174192\ 10^{10}, 6.399174192\ 10^{10}$

$72.81010846, 0.1250000000, 2.046248505\ 10^{9}, 2.046248505\ 10^{9}$

$36.73906676, 0.06250000000, 6.693301407\ 10^{7}, 6.693301407\ 10^{7}$

$18.70354593, 0.03125000000, 2.288847052\ 10^{6}, 2.288847052\ 10^{6}$

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0.8089276006, 0.0002441406250, 2.537448779, 2.537448779

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0.6768315007, 0.00001525878906, 2.465205933, 2.465205933

0.6724282973, 0.000007629394530, 2.465048658, 2.465048658

0.6702266957, 0.000003814697265, 2.465014378, 2.465014378

0.6691258948, 0.000001907348632, 2.465008201, 2.465008201

$-1067.192127, 1., -1.384245318\ 10^{15}, 1.384245318\ 10^{15}$

$-533.2615002, 0.5000000000, -4.312222394\ 10^{13}, 4.312222394\ 10^{13}$

$-266.2961872, 0.2500000000, -1.339136173\ 10^{12}, 1.339136173\ 10^{12}$

$-132.8135307, 0.1250000000, -4.132488076\ 10^{10}, 4.132488076\ 10^{10}$

$-66.07220241, 0.06250000000, -1.259197619\ 10^{9}, 1.259197619\ 10^{9}$

$-32.70153825, 0.03125000000, -3.739732126\ 10^{7}, 3.739732126\ 10^{7}$

$-16.01620618, 0.01562500000, -1.053878194\ 10^{6}, 1.053878194\ 10^{6}$

$-7.673540141, 0.007812500000, -26595.28111, 26595.28111$

$-3.502207123, 0.003906250000, -520.3746648, 520.3746648$

$-1.416540614, 0.001953125000, -1.287008084, 1.287008084$

$$-1.349270566,\ 1.,\ -.122661857,\ 0.122661857$$
$$-1.341393320,\ 1.,\ -0.001515350,\ 0.001515350$$
$$-1.341293547,\ 1.,\ -2.32\ 10^{-7},\ 2.32\ 10^{-7}$$
$$-1.341293532,\ 1.,\ -5.\ 10^{-9},\ 5.\ 10^{-9}$$
$$-1.341293532 \tag{5.3.8.1}$$

5.3.9. Differenciálegyenletek.

```
> de:=diff(y(x),x,x)=2*y(x)+1; dsolve(de);
```

$$de := \frac{d^2}{dx^2}\, y(x) = 2\, y(x) + 1$$

$$y(x) = e^{\sqrt{2}\, x}\, \_C2 + e^{-\sqrt{2}\, x}\, \_C1 - \frac{1}{2} \tag{5.3.9.1}$$

5.3.10. Szeparábilsi differenciálegyenletek megoldása.

5.3.11. Elsőrendű lineáris differenciálegyenletek megoldása.

*5.3.12. Variációszámítás.

```
> with(VariationalCalculus); g:='g';
```

$$[ConjugateEquation,\ Convex,\ EulerLagrange,\ Jacobi,\ Weierstrass]$$

$$g := g \tag{5.3.12.1}$$

```
> L:=g*rho*y(x)*sqrt(1+diff(y(x),x)^2);
```

$$L := g\, \rho\, y(x)\, \sqrt{1 + \left(\frac{d}{dx}\, y(x)\right)^2} \tag{5.3.12.2}$$

```
> EulerLagrange(L,x,y(x)); simplify(%); %[2];
```

$$\left\{ g\, \rho\, y(x)\, \sqrt{1 + \left(\frac{d}{dx}\, y(x)\right)^2} - \frac{\left(\frac{d}{dx}\, y(x)\right)^2 g\, \rho\, y(x)}{\sqrt{1 + \left(\frac{d}{dx}\, y(x)\right)^2}} = K_1, \right.$$

$$g\, \rho\, \sqrt{1 + \left(\frac{d}{dx}\, y(x)\right)^2} - \frac{g\, \rho \left(\frac{d}{dx}\, y(x)\right)^2}{\sqrt{1 + \left(\frac{d}{dx}\, y(x)\right)^2}}$$

$$+\frac{g\,\rho\,y(x)\left(\frac{d}{dx}\,y(x)\right)^2\left(\frac{d^2}{dx^2}\,y(x)\right)}{\left(1+\left(\frac{d}{dx}\,y(x)\right)^2\right)^{3/2}}-\frac{g\,\rho\,y(x)\left(\frac{d^2}{dx^2}\,y(x)\right)}{\sqrt{1+\left(\frac{d}{dx}\,y(x)\right)^2}}\Bigg\}$$

$$\left\{-\frac{g\,\rho\left(-1-\left(\frac{d}{dx}\,y(x)\right)^2+y(x)\left(\frac{d^2}{dx^2}\,y(x)\right)\right)}{\left(1+\left(\frac{d}{dx}\,y(x)\right)^2\right)^{3/2}},\right.$$

$$\left.\frac{g\,\rho\,y(x)}{\sqrt{1+\left(\frac{d}{dx}\,y(x)\right)^2}}=K_1\right\}$$

$$\frac{g\,\rho\,y(x)}{\sqrt{1+\left(\frac{d}{dx}\,y(x)\right)^2}}=K_1 \qquad (5.3.12.3)$$

*5.3.13. Integrálkritérium.

```
> sum(n^a,n=1..infinity) assuming a<-1;
```

$$\zeta(-a) \qquad (5.3.13.1)$$