Komputeralgebrai algoritmusok

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Ezek a programok csak szemléltetésre szolgálnak.

1. Történet

```
> restart;
```

E 1.1. Példa.

```
> 33!/2^31+41^41;
```

$$1330877630632711998713399240963346255989932815022128910520\backslash 902250516 \quad (1.1.1)$$

```
> 43!/(2^43-1);
```

$$\frac{60415263063373835637355132068513997507264512000000000}{8796093022207} \quad (1.1.2)$$

```
> 483952545774574373476/122354323571234 mod 1000003;
```

$$887782 \quad (1.1.3)$$

```
> 10*(8+6*I)^(-1/2);
```

$$\frac{10}{\sqrt{8+6\,\mathrm{I}}} \quad (1.1.4)$$

```
> evalc(%);
```

$$3-\mathrm{I} \quad (1.1.5)$$

```
> sqrt(15523/3-98/2);
```

$$\frac{124}{3}\sqrt{3} \quad (1.1.6)$$

```
> a:=sin(Pi/3)*exp(2+ln(33));
```

$$a := \frac{1}{2}\sqrt{3}\,\mathrm{e}^{2+\ln(33)} \quad (1.1.7)$$

```
> simplify(a);
```

$$\frac{33}{2}\sqrt{3}\,\mathrm{e}^{2} \quad (1.1.8)$$

```
> evalf(a);
```

$$211.1706396 \quad (1.1.9)$$

```
> evalf(a,60);
```

$$211.170639624855418173457016949952935319763238458535271 73\backslash 1859 \quad (1.1.10)$$

```
> n:=193802871990921965256085980559909428418 20;
```

$$n := 193802871990921965256085980559909428 41820 \quad (1.1.11)$$

```
> isprime(n);
```

$$false \quad (1.1.12)$$

```
> ifactor(n);
```

$$(2)^2\,(3)^2\,(5)\,(19)^3\,(101)^4\,(12282045523619)^2 \quad (1.1.13)$$

```
> nextprime(n);
```

$$193802871990921965256085980559909428 42043 \quad (1.1.14)$$

```
> igcd(15990335972848346968323925788771404985,
15163659044370489780);
```

$$12636382536975 40815 \quad (1.1.15)$$

```
> a:=(x+y)^12-(x-y)^12;
```

$$a := (x+y)^{12} - (x-y)^{12} \quad (1.1.16)$$

```
> expand(a);
```

$$24\,y\,x^{11} + 440\,y^3\,x^9 + 1584\,y^5\,x^7 + 1584\,y^7\,x^5 + 440\,y^9\,x^3 + 24\,y^{11}\,x \quad (1.1.17)$$

```
> quo(x^3*y-x^3*z+2*x^2*y^2-2*x^2*z^2+x*y^3+x*y^2*z-x*z^3,x+y+
z,x);
```

$$(y-z)\,x^2 + (y^2 - z^2)\,x + y\,z^2 \quad (1.1.18)$$

```
> gcd(x^3*y-x^3*z+2*x^2*y^2-2*x^2*z^2+x*y^3+x*y^2*z-x*z^3,x+y+
z);
```

$$1 \quad (1.1.19)$$

```
> b:=(x^4-y^4)/(x^3+y^3)-(x^5+y^5)/(x^4-y^4);
```

$$b := \frac{x^4 - y^4}{x^3 + y^3} - \frac{x^5 + y^5}{x^4 - y^4} \quad (1.1.20)$$

```
> normal(b);
```

$$-\frac{x^3\,y^3}{(x^3 - x^2\,y + x\,y^2 - y^3)\,(x^2 - x\,y + y^2)} \quad (1.1.21)$$

```
> f:=(x+y)*(x-y)^6; g:=(x^2-y^2)*(x-y)^3; f/g;
```

$$f := (x+y)\,(x-y)^6$$

$$g := (x^2 - y^2)\,(x-y)^3$$

$$\frac{(x+y)\,(x-y)^3}{x^2 - y^2} \quad (1.1.22)$$

```
> normal(f/g);
```

$$(x-y)^2 \quad (1.1.23)$$

```
> factor(x^6-x^5+x^2+1);
```

$$x^6 - x^5 + x^2 + 1 \quad (1.1.24)$$

```
> factor(5*x^4-4*x^3-48*x^2+44*x+3);
```

$$(x-1)\,(x-3)\,(5\,x^2 + 16\,x + 1) \quad (1.1.25)$$

```
> Factor(x^6-x^5+x^2+1) mod 13;
```

$$\left(x^3+10\,x^2+8\,x+11\right)\left(x^3+2\,x^2+11\,x+6\right) \tag{1.1.26}$$

```
> factor(x^12-y^12);
```

$$(x-y)\left(y^2+x^2+x\,y\right)(x+y)\left(x^2-x\,y+y^2\right)\left(x^2+y^2\right)\left(x^4-x^2\,y^2+y^4\right) \tag{1.1.27}$$

```
> restart;
> alias(a=RootOf(x^4-2));
```

$$a \tag{1.1.28}$$

```
> factor(x^12-2*x^8+4*x^4-8,a);
```

$$\left(x^4-2\,x^2+2\right)\left(x^4+2\,x^2+2\right)\left(x^2+a^2\right)(x+a)(x-a) \tag{1.1.29}$$

```
> Factor(x^6-2*x^4+4*x^2-8,a) mod 5;
```

$$(x+3)(x+2)(x+1)(x+4)\left(x+\mathrm{RootOf}\left(\_Z^4+3\right)^2\right)\left(x+4\,\mathrm{RootOf}\left(\_Z^4+3\right)^2\right) \tag{1.1.30}$$

```
> V:=vandermonde([x,y,z]);
```

$$V := vandermonde([x, y, z]) \tag{1.1.31}$$

```
> with(linalg);
```

[*BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian*] (1.1.32)

```
> V:=vandermonde([x,y,z]);
```

$$V := \begin{bmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{bmatrix} \tag{1.1.33}$$

```
> inverse(V);
```

$$\begin{bmatrix} \frac{zy}{zy-xz+x^2-xy} & \frac{xz}{xz-xy+y^2-zy} & -\frac{xy}{xz-xy-z^2+zy} \\ -\frac{z+y}{zy-xz+x^2-xy} & -\frac{x+z}{xz-xy+y^2-zy} & \frac{x+y}{xz-xy-z^2+zy} \\ \frac{1}{zy-xz+x^2-xy} & \frac{1}{xz-xy+y^2-zy} & -\frac{1}{xz-xy-z^2+zy} \end{bmatrix} \quad (1.1.34)$$

```
> det(V);
```

$$y z^2 - y^2 z - x z^2 + x^2 z + x y^2 - x^2 y \quad (1.1.35)$$

```
> factor(%);
```

$$(z-y)(-y+x)(-z+x) \quad (1.1.36)$$

```
> e1:=(1-eps)*x+2*y-4*z-1=0;
```

$$e1 := (1-eps)\,x + 2\,y - 4\,z - 1 = 0 \quad (1.1.37)$$

```
> e2:=(3/2-eps)*x+3*y-5*z-2=0;
```

$$e2 := \left(\frac{3}{2} - eps\right) x + 3\,y - 5\,z - 2 = 0 \quad (1.1.38)$$

```
> e3:=(5/2+eps)*x+5*y-7*z-3=0;
```

$$e3 := \left(\frac{5}{2} + eps\right) x + 5\,y - 7\,z - 3 = 0 \quad (1.1.39)$$

```
> sols:=solve([e1,e2,e3],[x,y,z]);
```

$$sols := \left[\left[x = -\frac{1}{2\,eps},\ y = \frac{1}{4}\,\frac{1+7\,eps}{eps},\ z = \frac{3}{4}\right]\right] \quad (1.1.40)$$

```
> subs(eps=10^(-20),sols);
```

$$\left[\left[x = -50000000000000000000,\ y = \frac{100000000000000000007}{4},\ z = \frac{3}{4}\right]\right] \quad (1.1.41)$$

```
> f:=x^2*y*(1-x-y)^3;
```

$$f := x^2\,y\,(1-x-y)^3 \quad (1.1.42)$$

```
> e1:=diff(f,x); e2:=diff(f,y);
```

$$e1 := 2\,x\,y\,(1-x-y)^3 - 3\,x^2\,y\,(1-x-y)^2$$

$$e2 := x^2\,(1-x-y)^3 - 3\,x^2\,y\,(1-x-y)^2 \quad (1.1.43)$$

```
> solve([e1,e2],[x,y]);
```

$$\left[[x=0,\ y=y],\ \left[x=\frac{1}{3},\ y=\frac{1}{6}\right],\ [x=1-y,\ y=y],\ [x=1-y,\ y=y]\right] \quad (1.1.44)$$

```
> limit(tan(x)/x,x=0);
```

$$1 \quad (1.1.45)$$

```
> diff(ln(sec(x)),x);
```

$$\tan(x) \quad (1.1.46)$$

```
> series(tan(sinh(x))-sinh(tan(x)),x=0,15);
```

$$\frac{1}{90}\, x^{7} + \frac{13}{756}\, x^{9} + \frac{1451}{75600}\, x^{11} + \frac{6043}{332640}\, x^{13} + \mathrm{O}\left(x^{15}\right) \qquad (1.1.47)$$

```
> series(BesselJ(0,x)/BesselJ(1,x),x,12);
```

$$2\, x^{-1} - \frac{1}{4}\, x - \frac{1}{96}\, x^{3} - \frac{1}{1536}\, x^{5} - \frac{1}{23040}\, x^{7} - \frac{13}{4423680}\, x^{9} + \mathrm{O}\left(x^{10}\right) \qquad (1.1.48)$$

```
> int(((3*x^2-7*x+15)*exp(x)+3*x^2-14)/(x-exp(x))^2,x);
```

$$\frac{14 + 3\, x^{2} - \mathrm{e}^{x}}{x - \mathrm{e}^{x}} \qquad (1.1.49)$$

```
> int((3*x^3-x+14)/(x^2+4*x-4),x);
```

$$\frac{3}{2}\, x^{2} - 12\, x + \frac{59}{2}\, \ln\left(x^{2} + 4\, x - 4\right) + 38\, \sqrt{2}\, \operatorname{arctanh}\left(\frac{1}{8}\, (2\, x + 4)\, \sqrt{2}\right) \qquad (1.1.50)$$

```
> int(x*exp(x^3),x);
```

$$-\frac{1}{3}\, (-1)^{1/3} \left(\frac{x^{2}\, (-1)^{2/3}\, \Gamma\left(\frac{2}{3}\right)}{\left(-x^{3}\right)^{2/3}} - \frac{x^{2}\, (-1)^{2/3}\, \Gamma\left(\frac{2}{3}, -x^{3}\right)}{\left(-x^{3}\right)^{2/3}} \right) \qquad (1.1.51)$$

```
> diff_eqn:=diff(y(x),x$2)+t*diff(y(x),x)-2*t^2*y(x)=0;
```

$$\mathit{diff\_eqn} := \frac{\mathrm{d}^{2}}{\mathrm{d}x^{2}}\, y(x) + t \left(\frac{\mathrm{d}}{\mathrm{d}x}\, y(x)\right) - 2\, t^{2}\, y(x) = 0 \qquad (1.1.52)$$

```
> init_conds:=y(0)=t,D(y)(0)=2*t^2;
```

$$\mathit{init\_conds} := y(0) = t,\ \mathrm{D}(y)(0) = 2\, t^{2} \qquad (1.1.53)$$

```
> dsolve({diff_eqn,init_conds},y(x));
```

$$y(x) = \frac{4}{3}\, t\, \mathrm{e}^{t x} - \frac{1}{3}\, t\, \mathrm{e}^{-2 t x} \qquad (1.1.54)$$

```
> Cheby:=proc(n,x) local T,k;
    T[0]:=1; T[1]:=x;
    for k from 2 to n do
      T[k]:=expand(2*x*T[k-1]-T[k-2]);
     od; T[n];
  end;
```

Cheby := **proc**(*n*, *x*) (1.1.55)

```
    local T, k;
    T[0] := 1;
    T[1] := x;
    for k from 2 to n do
        T[k] := expand(2*x*T[k - 1] - T[k - 2])
    end do;
    T[n]
end proc
```

```
> Cheby(7,x);
```

(1.1.56)

$$64 x^7 - 112 x^5 + 56 x^3 - 7 x \quad (1.1.56)$$

Ex1.3. Feladat.

```
> int(x/(1+exp(x)),x);
  int(exp(x^2),x);
  int(sqrt((x^2-1)*(x^2-4)),x);
  int(sqrt((x-1)*(x-4)),x);
  int(sqrt((1+x)/(1-x)),x);
  int(log(x^2-5*x+4),x);
  int(log(x)/(1+x),x);
  int(1/log(x),x);
```

$$\frac{1}{2} x^2 - x \ln\left(1 + e^x\right) - \operatorname{polylog}\left(2, -e^x\right)$$

$$-\frac{1}{2} I \sqrt{\pi} \operatorname{erf}(I x)$$

$$\frac{1}{3} x \sqrt{4 + x^4 - 5 x^2} + \frac{4}{3} \frac{\sqrt{1 - x^2} \sqrt{4 - x^2} \operatorname{EllipticF}\left(x, \frac{1}{2}\right)}{\sqrt{4 + x^4 - 5 x^2}} - \frac{10}{3} \frac{\sqrt{1 - x^2} \sqrt{4 - x^2} \left(\operatorname{EllipticF}\left(x, \frac{1}{2}\right) - \operatorname{EllipticE}\left(x, \frac{1}{2}\right)\right)}{\sqrt{4 + x^4 - 5 x^2}}$$

$$\frac{1}{4} (2 x - 5) \sqrt{4 + x^2 - 5 x} - \frac{9}{8} \ln\left(-\frac{5}{2} + x + \sqrt{4 + x^2 - 5 x}\right)$$

$$\frac{\sqrt{-\frac{x+1}{x-1}} (x - 1) \left(\sqrt{1 - x^2} - \arcsin(x)\right)}{\sqrt{-(x-1)(x+1)}}$$

$$x \ln\left(4 + x^2 - 5 x\right) - 2 x - \ln(x - 1) - 4 \ln(x - 4)$$

$$\operatorname{dilog}(x + 1) + \ln(x) \ln(x + 1)$$

$$-\operatorname{Ei}(1, -\ln(x)) \quad (1.2.1)$$

▶ 2. Algebrai alapok

▶ 3. Normál formák, reprezentáció

▶ 4. Aritmetika

- **5. Kínai maradékolás**
- **6. Newton–iteráció, Hensel–felemelés**
- **7. Legnagyobb közös osztó**
- **8. Faktorizálás**
- **9. Egyenletrendszerek**
- **10. Gröbner–bázisok**
- **11. Racionális törtfüggvények integrálása**
- **12. A Risch–algoritmus.**