Számítógépes számelmélet

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Ezek a programok csak szemléltetésre szolgálnak

▶ 1. A prímek eloszlása, szitálás

▶ 2. Egyszerű faktorizálási módszerek

▶ 3. Egyszerű prímtesztelési módszerek

▶ 4. Lucas–sorozatok

▶ 5. Alkalmazások

▶ 6. Számok és polinomok

▶ 7. Gyors Fourier–transzformáció

▶ 8. Elliptikus függvények

▼ 9. Számolás elliptikus görbéken

```
> restart;
```

▶ 9.1. Elliptikus görbék.

▼ 9.2. Hasse tétele.

```
> #
  # This routine randomly choose an elliptic "curve" modulo n,
  # where gcd(n,6)=1. The coordinates x,y are choosen
  # randomly, the parameter a too, and b is calculated.
  # The list [x,y,a,b] is given back or a divisor d of n.
  #

  ellrand:=proc(n) local x,y,a,b,r,d,f;
  r:=rand(n); d:=0;
```

```
   while d=0 do
     x:=r(n); y:=r(n); a:=r(n); b:=y^2-x^3-a*x mod n;
     d:=4*a^3+27*b^2 mod n; gcd(d,n);
   od;
   if %<n and %>1 then return % fi;
   [x,y,a,b]; end;
```

$$ellrand := \mathbf{proc}(n) \tag{9.2.1}$$

```
local x, y, a, b, r, d, f;
r := rand(n);
d := 0;
while d = 0 do
    x := r(n);
    y := r(n);
    a := r(n);
    b := mod(y^2 - x^3 - a*x, n);
    d := mod(4*a^3 + 27*b^2, n);
    gcd(d, n)
end do;
if % < n and 1 < % then
    return %
end if;
[x, y, a, b]
end proc
```

```
> #
  # This brute force procedure calculate the number of points
  # on an elliptic curve modulo p>3, a prime. The curve
  # parameters are a, b.
  #

  ellcount:=proc(a,b,p) local x,c;
  c:=1;
  for x from 0 to p-1 do c:=c+numtheory[jacobi](x^3+a*x+b,p)+1;
  od;
  c; end;
```

$$ellcount := \mathbf{proc}(a, b, p) \tag{9.2.2}$$

```
local x, c;
c := 1;
for x from 0 to p - 1 do
```

```
        c := c + numtheory[jacobi](x^3 + a*x + b, p) + 1
    end do;
    c
end proc
```

```
> ellrand(97); ellcount(%[3],%[4],97);
```

$$[55, 42, 82, 47]$$

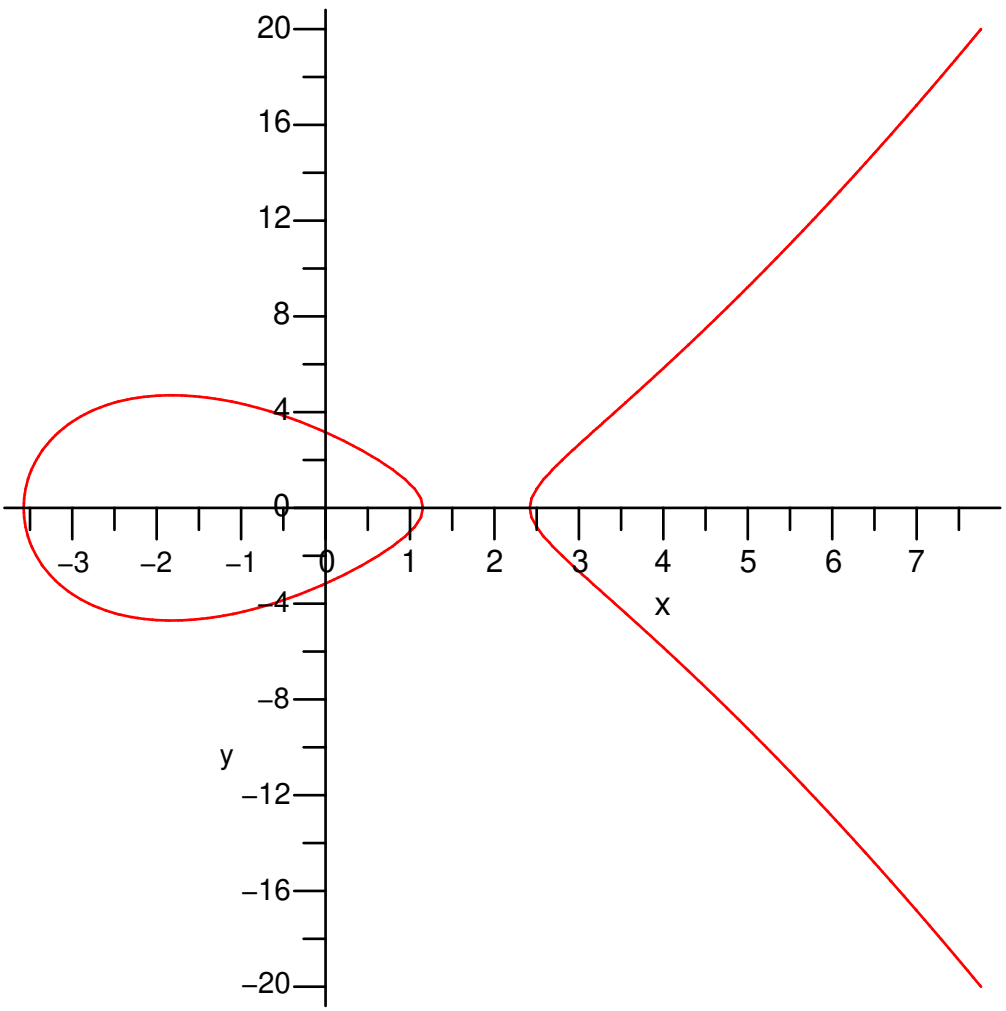
$$106 \tag{9.2.3}$$

9.3. Gyakorlat.

```
> with(plots);
```

$$[Interactive, animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, cylinderplot, densityplot, display, display3d, fieldplot, fieldplot3d, gradplot, gradplot3d, graphplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, replot, rootlocus, semilogplot, setoptions, setoptions3d, spacecurve, sparsematrixplot, sphereplot, surfdata, textplot, textplot3d, tubeplot] \tag{9.3.1}$$

```
> implicitplot(y^2=x^3-10*x+10,x=-5..10,y=-20..20,numpoints=
  10000);
```



9.4. Gyakorlat.

```
> #
  # Addition on an elliptic "curve" modulo n, where gcd(n,6)=1.
  # P and Q are the points to add and a,b are the parameters.
  # The return value is the sum of the points or a divisor d of
  n.
  #

  elladd:=proc(P,Q,a,b,n) local l,d;
  if P[3]=0 then return Q fi;
  if Q[3]=0 then return P fi;
  if P=Q then return elldou(P,a,b,n) fi;
  if P[1]=Q[1] then return [0,1,0] fi;
  d:=igcdex(P[1]-Q[1],n,'l');
  if 1<d and d<n then return d fi;
  l:=(P[2]-Q[2])*l mod n;
  l^2-P[1]-Q[1] mod n;
```

```
   [%,l*(P[1]-%)-P[2] mod n,1];
   end;
```

$$elladd := \mathbf{proc}(P, Q, a, b, n) \tag{9.4.1}$$

```
    local l, d;
    if P[3] = 0 then
        return Q
    end if;
    if Q[3] = 0 then
        return P
    end if;
    if P = Q then
        return elldou(P, a, b, n)
    end if;
    if P[1] = Q[1] then
        return [0, 1, 0]
    end if;
    d := igcdex(P[1] - Q[1], n, 'l');
    if 1 < d and d < n then
        return d
    end if;
    l := mod((P[2] - Q[2]) * l, n);
    mod(l^2 - P[1] - Q[1], n);
    [%, mod(l * (P[1] - %) - P[2], n), 1]
end proc
```

```
> #
  # Doubling on an elliptic "curve" modulo n, where gcd(n,6)=1.
  # P is the point to double and a, b are the parameters.
  # The return value is the double of the point P or
  # a divisor d of n.
  #

  elldou:=proc(P,a,b,n) local l,d;
  if P[3]=0 then return P fi;
  if P[2]=0 then return [0,1,0] fi;
  d:=igcdex(2*P[2],n,'l');
  if 1<d and d<n then return d fi;
  l:=(3*P[1]^2+a)*l mod n;
```

```
   l^2-2*P[1] mod n;
   [%,l*(P[1]-%)-P[2] mod n,1];
   end;
```

$$elldou := \mathbf{proc}(P, a, b, n) \tag{9.4.2}$$

```
    local l, d;
    if P[3] = 0 then
        return P
    end if;
    if P[2] = 0 then
        return [0, 1, 0]
    end if;
    d := igcdex(2 * P[2], n, 'l');
    if 1 < d and d < n then
        return d
    end if;
    l := mod((3 * P[1]^2 + a) * l, n);
    mod(l^2 - 2 * P[1], n);
    [%, mod(l * (P[1] - %) - P[2], n), 1]
end proc
```

```
> #
  # This program compute k*P, k>=0 or a divisor of n
  # on an elliptic "curve" modulo n, where gcd(n,6)=1.
  # It use the left-to-right binary method.
  #

  ellmul:=proc(P,k,a,b,n) local L,i,Q;
  if k=0 then return [0,1,0] fi;
  if P[3]=0 then return P fi;
  L:=convert(k,base,2);
  Q:=P;
  for i from nops(L)-1 to 1 by -1 do
    Q:=elldou(Q,a,b,n);
    if type(Q,integer) then return Q fi;
    if L[i]=1 then
      Q:=elladd(P,Q,a,b,n);
      if type(Q,integer) then return Q fi;
    fi;
  od; Q; end;
```

$$ellmul := \mathbf{proc}(P, k, a, b, n) \tag{9.4.3}$$

```
    local L, i, Q;
```

```
    if k = 0 then
        return [0, 1, 0]
    end if;
    if P[3] = 0 then
        return P
    end if;
    L := convert(k, base, 2);
    Q := P;
    for i from nops(L) - 1 by -1 to 1 do
        Q := elldou(Q, a, b, n);
        if type(Q, integer) then
            return Q
        end if;
        if L[i] = 1 then
            Q := elladd(P, Q, a, b, n);
            if type(Q, integer) then
                return Q
            end if
        end if
    end do;
    Q
end proc
```

▶ 10. Faktorizálás elliptikus görbékkel

▶ 11. Prímteszt elliptikus görbékkel

▶ 12. Polinomfaktorizálás

▶ 13. Az AKS–teszt

▶ 14. A szita módszerek alapjai

► 15. Számtest szita

► 16. Vegyes problémák