

0.4524463559
0.4634834176
0.4683756328
0.4705393823
0.4714954464
0.4719177069
0.4721041694
0.4721865010
0.4722228527
0.4722389028
0.4722459892
0.4722491180
0.4722504994
0.4722511093
0.4722513786
0.4722514975
0.4722515500
0.4722515732
0.4722515834
0.4722515879
0.4722515900
0.4722515908
0.4722515912
0.4722515914
0.4722515915
0.4722515914
0.4722515915
0.4722515914
0.4722515915

(3.3.2)

► **3.117. Feladat.**

► 3.118. Feladat.

▼ Feladat.

$$\begin{aligned}
 &> \text{solve}([x1^2+x2^2=1, 2*x1+x2=1], [x1, x2]); \\
 &\quad \left[[x1 = 0, x2 = 1], \left[x1 = \frac{4}{5}, x2 = -\frac{3}{5} \right] \right]
 \end{aligned} \tag{3.6.1}$$

$$\begin{aligned}
 &> T1:=x->[(1-x[2])/2, \text{sqrt}(1-x[1]^2)]; \\
 &\quad T1 := x \rightarrow \left[\frac{1}{2} - \frac{1}{2} x_2, \sqrt{1 - x_1^2} \right]
 \end{aligned} \tag{3.6.2}$$

$$\begin{aligned}
 &> x:=[-0.9, 0.9]; T1(%); T1(%); T1(%); T1(%); T1(%); T1(%); T1(%); T1(%) \\
 &\quad ; T1(%); T1(%); T1(%); \\
 &\quad \quad x := [-.9, 0.9] \\
 &\quad \quad [0.0500000000, 0.4358898944] \\
 &\quad \quad [0.2820550528, 0.9987492178] \\
 &\quad \quad [0.0006253911, 0.9593982214] \\
 &\quad \quad [0.0203008893, 0.9999998044] \\
 &\quad \quad [9.78 \cdot 10^{-8}, 0.9997939157] \\
 &\quad \quad [0.0001030422, 1.] \\
 &\quad \quad [0., 0.9999999947] \\
 &\quad \quad [2.6 \cdot 10^{-9}, 1.] \\
 &\quad \quad [0., 1.] \\
 &\quad \quad [0., 1.] \\
 &\quad \quad [0., 1.]
 \end{aligned} \tag{3.6.3}$$

$$\begin{aligned}
 &> T2:=x->[(1-x[2])/2, -\text{sqrt}(1-x[1]^2)]; \\
 &\quad T2 := x \rightarrow \left[\frac{1}{2} - \frac{1}{2} x_2, -\sqrt{1 - x_1^2} \right]
 \end{aligned} \tag{3.6.4}$$

$$\begin{aligned}
 &> x:=[0.9, 0.9]; T2(%); T2(%); T2(%); (%) ; T2(%); T2(%); T2(%); T2(%); T2 \\
 &\quad (%); T2(%); T2(%); T2(%); T2(%); T2(%); T2(%); T2(%); T2(%); T2 \\
 &\quad (%); T2(%); T2(%); T2(%); T2(%); T2(%); T2(%); T2(%); T2(%); T2 \\
 &\quad (%); T2(%); T2(%); T2(%); T2(%); T2(%); T2(%); T2(%); T2(%); T2 \\
 &\quad (%); T2(%); T2(%); T2(%); (%) ; T2(%); T2(%); T2(%); T2(%); T2(%); T2(%) \\
 &\quad ; \\
 &\quad \quad x := [0.9, 0.9] \\
 &\quad \quad [0.0500000000, -0.4358898944] \\
 &\quad \quad [0.7179444947, -0.9987492178] \\
 &\quad \quad [0.9993746089, -0.6960998871]
 \end{aligned}$$

[0.9993746089, -0.6960998871]
[0.8480499436, -0.03536086962]
[0.5176804348, -0.5299163077]
[0.7649581538, -0.8555740572]
[0.9277870286, -0.6440799818]
[0.8220399909, -0.3731102111]
[0.6865551056, -0.5694297616]
[0.7847148808, -0.7270777723]
[0.8635388862, -0.6198568834]
[0.8099284417, -0.5042822543]
[0.7521411272, -0.5865287029]
[0.7932643514, -0.6590020674]
[0.8295010337, -0.6088773840]
[0.8044386920, -0.5585051791]
[0.7792525896, -0.5940356814]
[0.7970178407, -0.6267099820]
[0.8133549910, -0.6039557613]
[0.8019778806, -0.5817677016]
[0.7908838508, -0.5973537302]
[0.7986768651, -0.6119662854]
[0.8059831427, -0.6017601393]
[0.8008800696, -0.5919384881]
[0.7959692440, -0.5988247775]
[0.7994123888, -0.6053370653]
[0.8026685326, -0.6007826833]
[0.8003913416, -0.5964253740]
[0.7982126870, -0.5994778564]
[0.7997389282, -0.6023757186]
[0.8011878593, -0.6003479380]
[0.8001739690, -0.5984129127]
[0.7992064564, -0.5997679712]
[0.7998839856, -0.6010566030]

[0.8005283015, -0.6001546547]
 [0.8000773274, -0.5992949512]
 [0.7996474756, -0.5998968830]
 [0.7999484415, -0.6004697451]
 [0.8002348726, -0.6000687385]
 [0.8000343692, -0.5996867088]
 [0.7998433544, -0.5999541717]
 [0.7998433544, -0.5999541717]
 [0.7999770858, -0.6002088040]
 [0.8001044020, -0.6000305511]
 [0.8000152756, -0.5998607721]
 [0.7999303860, -0.5999796320]
 [0.7999898160, -0.6000928075]
 [0.8000464038, -0.6000135784]

(3.6.5)

> with(LinearAlgebra); with(VectorCalculus);
 [&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix,
 BidiagonalForm, BilinearForm, CharacteristicMatrix,
 CharacteristicPolynomial, Column, ColumnDimension,
 ColumnOperation, ColumnSpace, CompanionMatrix,
 ConditionNumber, ConstantMatrix, ConstantVector, Copy,
 CreatePermutation, CrossProduct, DeleteColumn, DeleteRow,
 Determinant, Diagonal, DiagonalMatrix, Dimension, Dimensions,
 DotProduct, EigenConditionNumbers, Eigenvalues, Eigenvectors, Equal,
 ForwardSubstitute, FrobeniusForm, GaussianElimination,
 GenerateEquations, GenerateMatrix, GetResultDataType,
 GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix,
 HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix,
 HouseholderMatrix, IdentityMatrix, IntersectionBasis, IsDefinite,
 IsOrthogonal, IsSimilar, IsUnitary, JordanBlockMatrix, JordanForm,
 LA_Main, LUdecomposition, LeastSquares, LinearSolve, Map, Map2,
 MatrixAdd, MatrixExponential, MatrixFunction, MatrixInverse,
 MatrixMatrixMultiply, MatrixNorm, MatrixPower, MatrixScalarMultiply,
 MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply,

NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot, PopovForm, QRDecomposition, RandomMatrix, RandomVector, Rank, RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation, RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues, SmithForm, SubMatrix, SubVector, SumBasis, SylvesterMatrix, ToeplitzMatrix, Trace, Transpose, TridiagonalForm, UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm, VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip]

[&x, *, +, -, ., <, >, </>, AddCoordinates, ArcLength, BasisFormat, Binormal, CrossProd, CrossProduct, Curl, Curvature, D, Del, DirectionalDiff, Divergence, DotProd, DotProduct, Flux, GetCoordinateParameters, GetCoordinates, Gradient, Hessian, Jacobian, Laplacian, LineInt, MapToBasis, Nabla, Norm, Normalize, PathInt, PrincipalNormal, RadiusOfCurvature, ScalarPotential, SetCoordinateParameters, SetCoordinates, SurfaceInt, TNBFrame, Tangent, TangentLine, TangentPlane, TangentVector, Torsion, Vector, VectorField, VectorPotential, Wronskian, diff, evalVF, int, limit, series]

> **x:='x'; T1:=<(1-y)/2, sqrt(1-x^2)>;**
 $x := x$

$$T1 := \left(\frac{1}{2} - \frac{1}{2} y \right) e_x + \left(\sqrt{1-x^2} \right) e_y \quad (3.6.7)$$

> **Jacobian(T1, [x, y]);**

$$\begin{bmatrix} 0 & -\frac{1}{2} \\ -\frac{x}{\sqrt{1-x^2}} & 0 \end{bmatrix} \quad (3.6.8)$$

> **a:=subs(x=0, y=1, %);**

$$a := \begin{bmatrix} 0 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} \quad (3.6.9)$$

> **MatrixNorm(a);**

(3.6.10)

$$\frac{1}{2} \quad (3.6.10)$$

> T2:=<(1-y)/2,-sqrt(1-x^2)>;

$$T2 := \left(\frac{1}{2} - \frac{1}{2} y \right) e_x - \sqrt{1-x^2} e_y \quad (3.6.11)$$

> Jacobian(T2,[x,y]);

$$\begin{bmatrix} 0 & -\frac{1}{2} \\ \frac{x}{\sqrt{1-x^2}} & 0 \end{bmatrix} \quad (3.6.12)$$

> a:=subs(x=4/5,y=-3/5,%);

$$a := \begin{bmatrix} 0 & -\frac{1}{2} \\ \frac{4}{45} \sqrt{9} \sqrt{25} & 0 \end{bmatrix} \quad (3.6.13)$$

> MatrixNorm(a); evalf(%);

$$\frac{4}{45} \sqrt{9} \sqrt{25} \\ 1.333333333 \quad (3.6.14)$$

> Eigenvalues(a); evalf(%);

$$\begin{bmatrix} \frac{1}{3} I\sqrt{6} \\ -\frac{1}{3} I\sqrt{6} \end{bmatrix} \\ \begin{bmatrix} 0.8164965809 I \\ -0.8164965809 I \end{bmatrix} \quad (3.6.15)$$

► 3.119. Feladat.

► 3.120. Feladat: a fixpont folytonos függése a paramétereiktől.

► -> 3.121. Feladat.

► Fixpont tulajdonság.

► Retraktum.

► Tétel.

► Összehúzóhatóság.

► Tétel.

► Tétel.

► Tétel.

► Megjegyzés.

▼ *3.122. Egyenletek megoldása és fixponttételek.

```
> v:='v'; f:=Vector(2,[v[1]^2+v[2]^2-1,2*v[1]+v[2]-1]);  
v:=v
```

$$f := (v_1^2 + v_2^2 - 1)e_x + (2v_1 + v_2 - 1)e_y \quad (3.18.1)$$

```
> J:=Jacobian(f,[v[1],v[2]]);
```

$$J := \begin{bmatrix} 2v_1 & 2v_2 \\ 2 & 1 \end{bmatrix} \quad (3.18.2)$$

```
> Jinv:=MatrixInverse(J);
```

$$Jinv := \begin{bmatrix} \frac{1}{2(v_1 - 2v_2)} & -\frac{v_2}{v_1 - 2v_2} \\ -\frac{1}{v_1 - 2v_2} & \frac{v_1}{v_1 - 2v_2} \end{bmatrix} \quad (3.18.3)$$

```
> Jinv0:=subs(v[1]=-2.,v[2]=2.,Jinv); v:=Vector(2,[-2.,2.]);
```

$$Jinv0 := \begin{bmatrix} -0.083333333335 & 0.33333333333 \\ 0.16666666667 & 0.33333333334 \end{bmatrix} \\ v := (-2.)e_x + (2.)e_y \quad (3.18.4)$$

```
> v:=v-Jinv0.f;v:=v-Jinv0.f;v:=v-Jinv0.f;v:=v-Jinv0.f;v:=v-  
Jinv0.f;v:=v-Jinv0.f;v:=v-Jinv0.f;v:=v-Jinv0.f;v:=v-Jinv0.f;  
v:=v-Jinv0.f;v:=v-Jinv0.f;
```

$$v := (-.416666667000000102)e_x + (1.833333333300000012)e_y$$

$$v := (-.2054398149000000101)e_x + (1.410879629700000014)e_y$$

$$v := (-.1193742442200000104)e_x + (1.238748488300000018)e_y$$

$$v := (-0.0736452418300000966)e_x + (1.14729048391000022)e_y$$

$$\begin{aligned}
v &:= (-0.0468369854100001010)e_x + (1.09367397107000032)e_y \\
v &:= (-0.0303106139100000997)e_x + (1.06062122806000025)e_y \\
v &:= (-0.0198242704100001014)e_x + (1.03964854106000026)e_y \\
v &:= (-0.0130524294920001012)e_x + (1.02610485922000017)e_y \\
v &:= (-0.00863063382400010119)e_x + (1.01726126788500015)e_y \\
v &:= (-0.00572271924000010077)e_x + (1.01144543871700021)e_y \\
v &:= (-0.00380150073900010089)e_x + (1.00760300071700026)e_y \quad (3.18.5)
\end{aligned}$$

```

> v:=Vector(2, [-0.9, 0.9]);
v:=v-Jinv.f;v:=v-Jinv.f;v:=v-Jinv.f;v:=v-Jinv.f;v:=v-Jinv.f;
v:=v-Jinv.f;v:=v-Jinv.f;
      v:= (-.9)e_x + (0.9)e_y
      v:= (-.151851851899999946)e_x + (1.30370370370000010)e_y
      v:= (-0.0208923687999999497)e_x + (1.04178473739999999)e_y
      v:= (-0.000518530579999950531)e_x + (1.00103706156999994)e_y
      v:= (-3.35580999950586128 10^-7)e_x + (1.00000067057199993)e_y
      v:= (1.87184494138486878 10^-11)e_x + (0.99999999957256292)e_y
      v:= (1.87184493138486877 10^-11)e_x + (0.9999999997256296)e_y
      v:= (1.87184493138486877 10^-11)e_x + (0.9999999997256296)e_y \quad (3.18.6)

```

▼ 3.123. Aitken Δ^2 -módszere.

```

> Aitken:=proc(x1,x2,x3) x1-(x2-x1)^2/((x3-x2)-(x2-x1)) end;
Aitken:=proc(x1, x2, x3) \quad (3.19.1)
  VectorCalculus:-+(x1, VectorCalculus:-
-(VectorCalculus:-*(VectorCalculus:-+(x2, VectorCalculus:-
-(x1))^2, 1 / VectorCalculus:-+(VectorCalculus:-
+(x3, VectorCalculus:--(x2)), VectorCalculus:--(VectorCalculus:-
+(x2, VectorCalculus:--(x1))))))
end proc

```

▼ *3.124. Feladat.

> **fsolve(x=cos(x));**

0.7390851332

(3.20.1)

> **x:=1.5;cos(%);cos(%);cos(%);cos(%);cos(%);cos(%);cos(%);cos
(%);cos(%);cos(%);cos(%);cos(%);cos(%);cos(%);cos(%);cos
cos(%);cos(%);cos(%);cos(%);cos(%);cos(%);cos(%);cos(%);cos
(%);cos(%);cos(%);cos(%);cos(%);cos(%);cos(%);**

x:= 1.5

0.07073720167

0.9974991672

0.5424049923

0.8564697090

0.6551088017

0.7929816458

0.7017241683

0.7637303113

0.7222610821

0.7503128857

0.7314755580

0.7441895867

0.7356370983

0.7414033729

0.7375215545

0.7401374748

0.7383758542

0.7395627261

0.7387633366

0.7393018610

0.7389391254

0.7391834780

0.7390188834

0.7391297583

0.7390550725

0.7391053821

0.7390714932

0.7390943212

0.7390789440

0.7390893023

0.7390823249

(3.20.2)

```
> x:=0.;cos(%);cos(%);Aitken(%%,%,%);cos(%);cos(%);Aitken
(%%,%,%);cos(%);cos(%);Aitken(%%,%,%);cos(%);cos(%);
Aitken(%%,%,%);
```

x:= 0.

1.

0.5403023059

0.6850733574

0.7743726338

0.7148598716

0.7386601562

0.7393713361

0.7388923132

0.7390851063

0.7390851513

0.7390851210

0.7390851332

(3.20.3)

▼ *3.125. Feladat.

```
> x:=1.;2/%;2/%;2/%;2/%;2/%;2/%;2/%;2/%;2/%;2/%;2/%;2/%;
2/%;2/%;2/%;2/%;
```

x:= 1.

2.

1.000000000

2.000000000

1.000000000

2.000000000

1.000000000

2.000000000

1.000000000

2.000000000

1.000000000
2.000000000
1.000000000
2.000000000
1.000000000
2.000000000
1.000000000
2.000000000

(3.21.1)

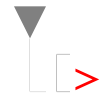
> x:=1.;2/%;2/%;Aitken(%%,%,%);2/%;2/%;Aitken(%%,%,%);2/%;
2/%;Aitken(%%,%,%);2/%;2/%;Aitken(%%,%,%);2/%;2/%;Aitken
(%%,%,%);2/%;2/%;Aitken(%%,%,%);2/%;2/%;Aitken(%%,%,%);
x:= 1.

2.

1.000000000
1.500000000
1.333333333
1.500000000
1.416666666
1.411764707
1.416666666
1.414215686
1.414211439
1.414215686
1.414213562
1.414213563
1.414213562
1.414213562
1.414213563
1.414213562
1.414213562
1.414213563
1.414213562
1.414213562
1.414213563
1.414213562
1.414213562

(3.21.2)

▶ ***3.126. Feladat.**



- ▶ **4. Normált terek**
- ▶ **5. Lineáris operátorok**
- ▶ **6. Hilbert-terek**
- ▶ **7. Spektrálelmélet**
- ▶ **8. Kompakt operátorok**
- ▶ **9. Differenciálszámítás**

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- ▶ **10. A differenciálgeometria alapjai**

- ▶ 11. Stieltjes–integrál és görbe menti integrál
- ▶ 12. Differenciálformák integrálása

IV. Komplex függvénytan

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- ▶ 14. Holomorf függvények
- ▶ 15. Meromorf függvények

V. Fourier–elmélet

- ▶ 16. Klasszikus Fourier–sorok
- ▶ 17. Ortogonális polinomok
- ▶ 18. Fourier–transzformáció

VI. Variációszámítás

- ▶ 19. Az Euler–Lagrange–egyenletek

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- ▶ 20. Alapfogalmak
- ▶ 21. Elemi megoldási módszerek
- ▶ 22. Általános eredmények

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