# Kalkulus I.

Járai Antal

Ezek a programok csak szemléltetésre szolgálnak

# 1. Halmazok

# 2. Számok

# 3. Határérték

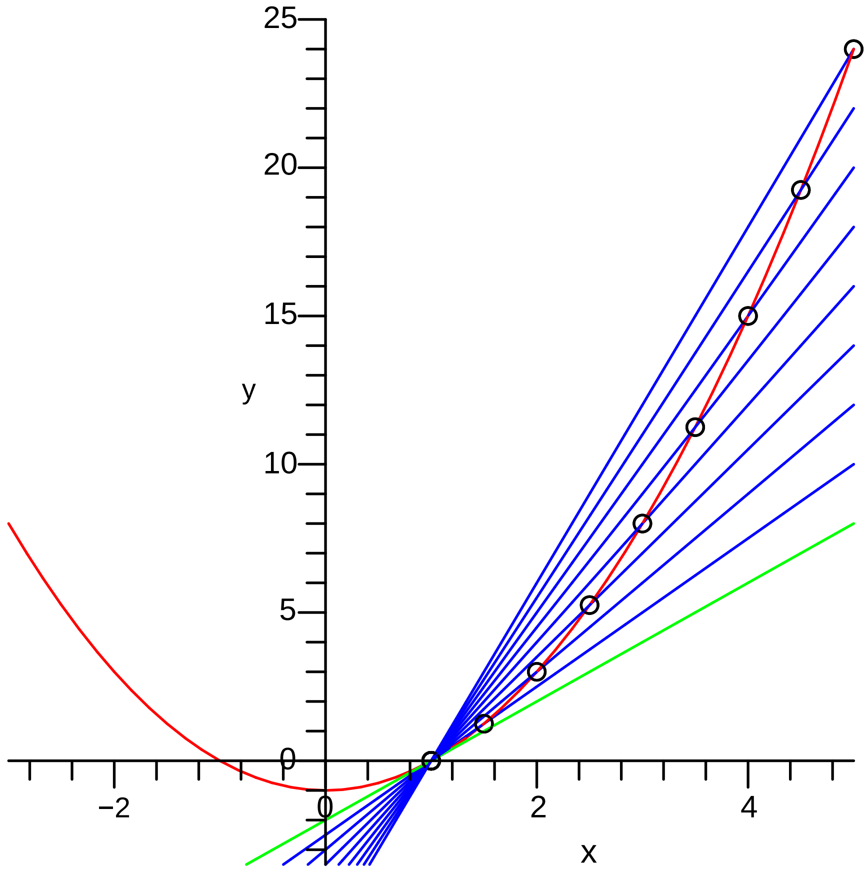
# 4. Differenciálszámítás

```
> restart;
```

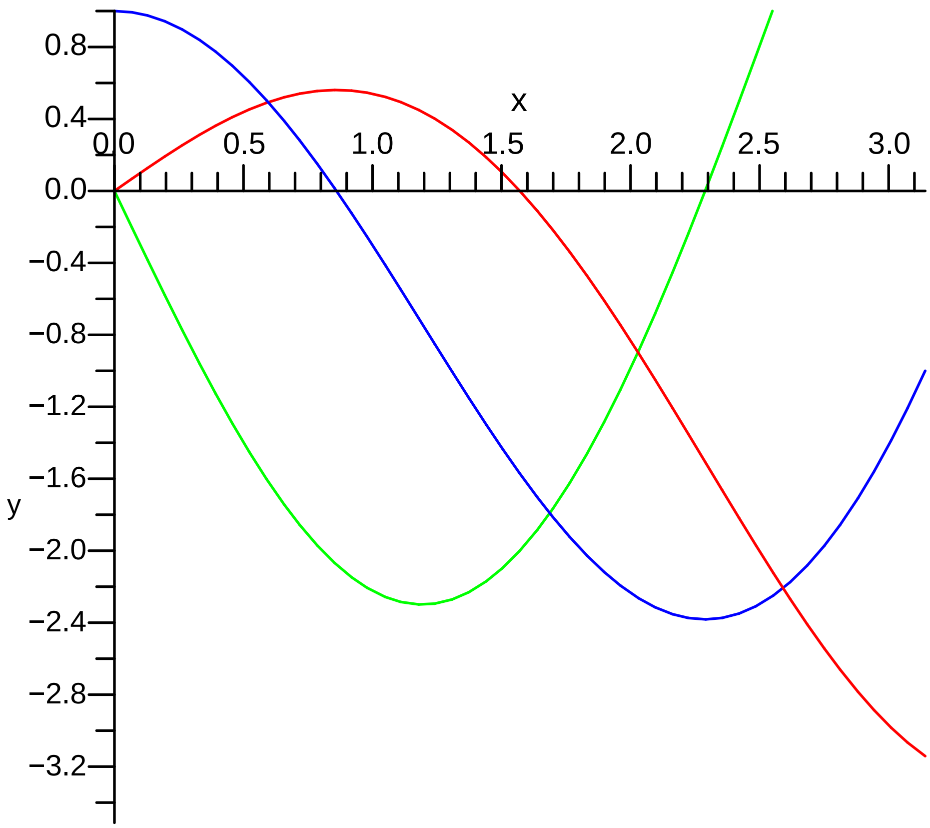
## 4.1. Derivált

### *4.1.1. Derivált.*

```
> Student[Calculus1][TangentTutor]();
```

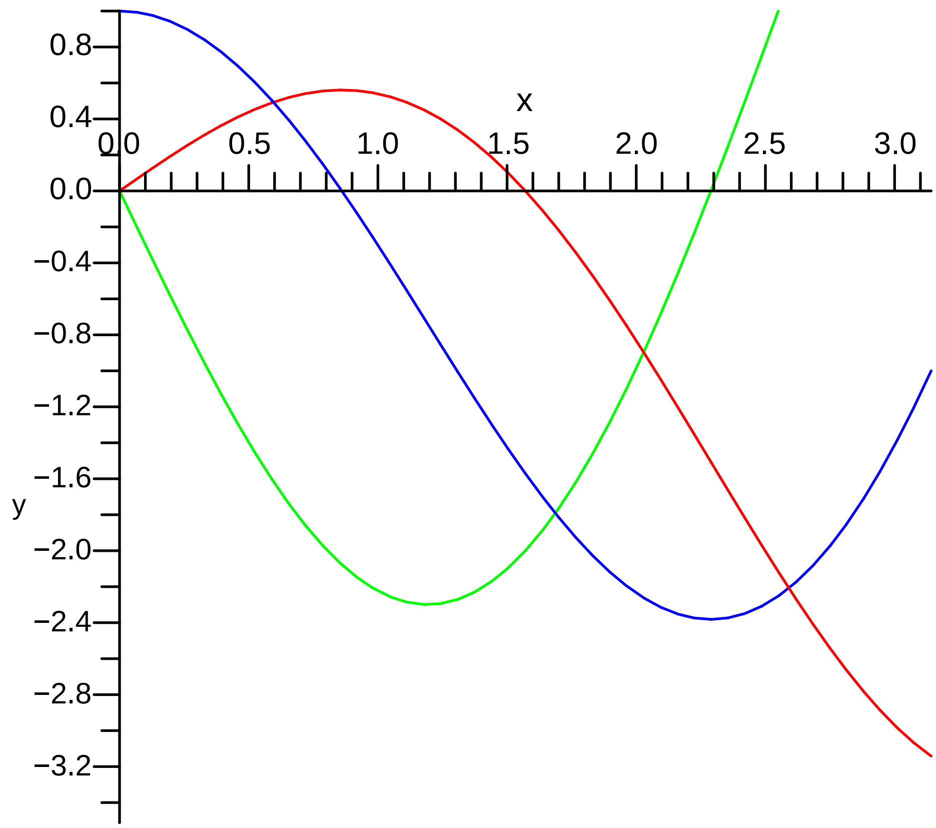



```
> Student[Calculus1][DerivativeTutor]();
```




### *4.1.2. Magasabb deriváltak.*

```
> Student[Calculus1][DerivativeTutor]();
```




### 4.1.3. Állítás.

### 4.1.4. Tétel.

### 4.1.5. Példák.

```
> Student[Calculus1][DiffTutor](x^5-6*x+2);
```

$$\frac{d}{dx}\left(x^5 - 6\,x + 2\right) = 5\,x^4 - 6 \quad (4.1.5.1)$$

### 4.1.6. Tétel.

```
> sum(a[n]*(x-c)^n,n=0..infinity); diff(%,x); simplify(%);
```

$$\sum_{n=0}^{\infty} a_n \left(x - c\right)^n$$

$$\sum_{n=0}^{\infty} \frac{a_n (x-c)^n n}{x-c}$$

$$\sum_{n=0}^{\infty} a_n (x-c)^{n-1} n \qquad (4.1.6.1)$$

### 4.1.7. *Következmény: Taylor-tétel.*

### 4.1.8. *Következmény.*

```
> D(exp); D(sin); D(cos); D(sinh); D(cosh);
```

$$\exp$$
$$\cos$$
$$-\sin$$
$$\cosh$$
$$\sinh \qquad (4.1.8.1)$$

### 4.1.9. *Megjegyzés.*

### 4.1.10. *Segédtétel.*

### 4.1.11. *Láncszabály.*

### 4.1.12. *Az inverz függvény differenciálási szabálya.*

### 4.1.13. *Következmény.*

```
> D(ln); diff(x^r,x); simplify(%);
```

$$z \to \frac{1}{z}$$

$$\frac{x^r r}{x}$$

$$x^{r-1} r \qquad (4.1.13.1)$$

```
> Student[Calculus1][DiffTutor]();
```

$$\frac{d}{dx}(x \sin(x)) = \sin(x) + x \cos(x) \qquad (4.1.13.2)$$

## 4.2. Szélsőértékszámítás.

```
> restart;
```

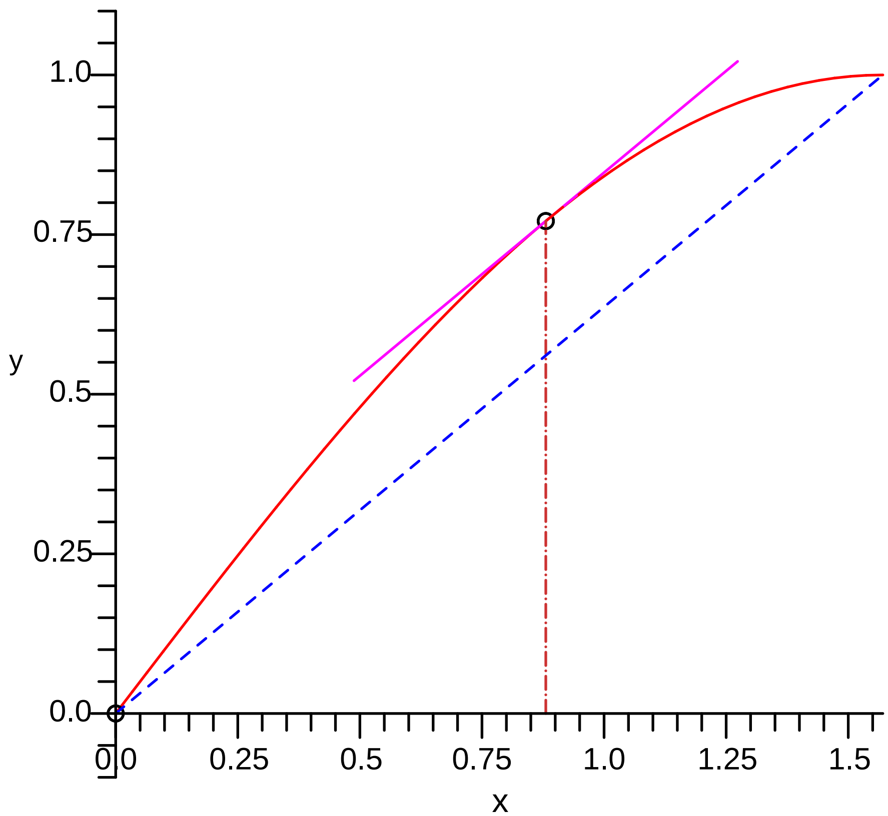
### 4.2.1. Tétel.

### 4.2.2. Rolle tétele.

### 4.2.3. Cauchy középérték tétele.

### 4.2.4. Következmény: Lagrange középérték tétele.

```
> Student[Calculus1][MeanValueTheoremTutor]();
```




### 4.2.5. Megjegyzés.

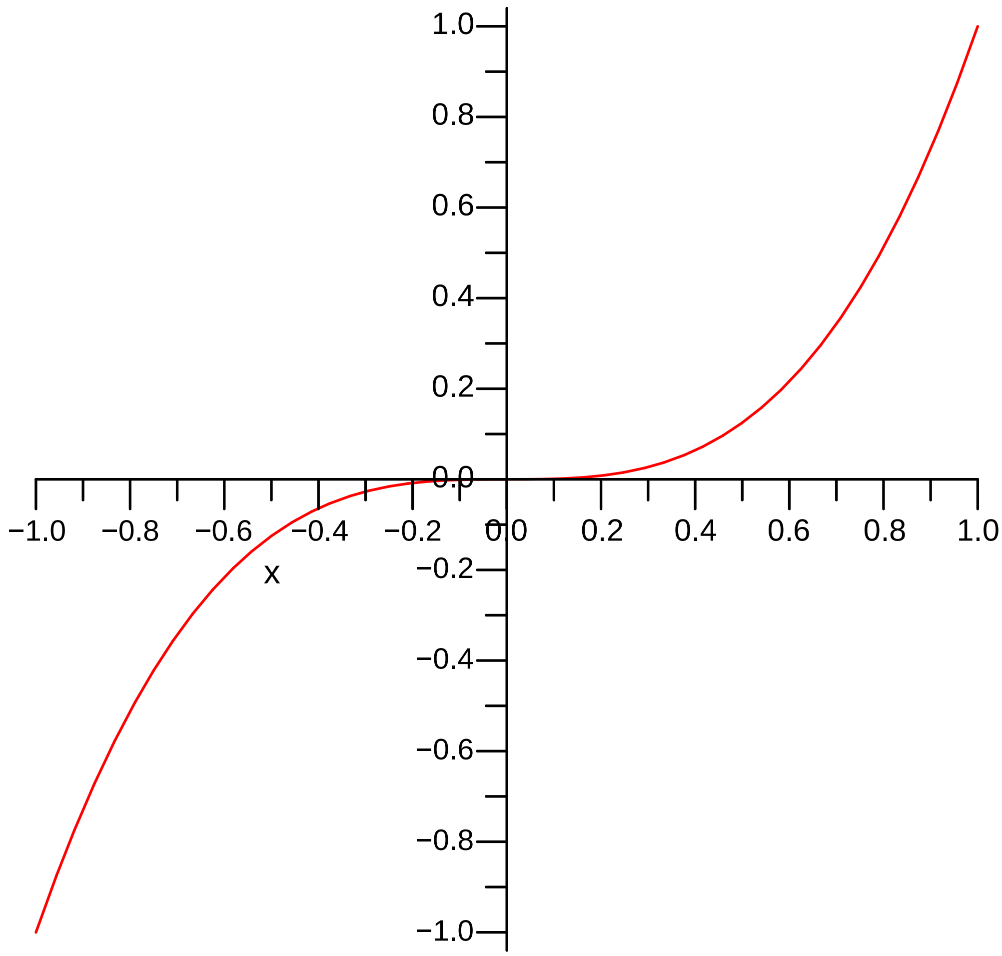
```
> diff(exp(I*t),t);
```

$$I\,e^{I\,t} \tag{4.2.5.1}$$

### 4.2.6. Tétel.

### 4.2.7. Megjegyzés.

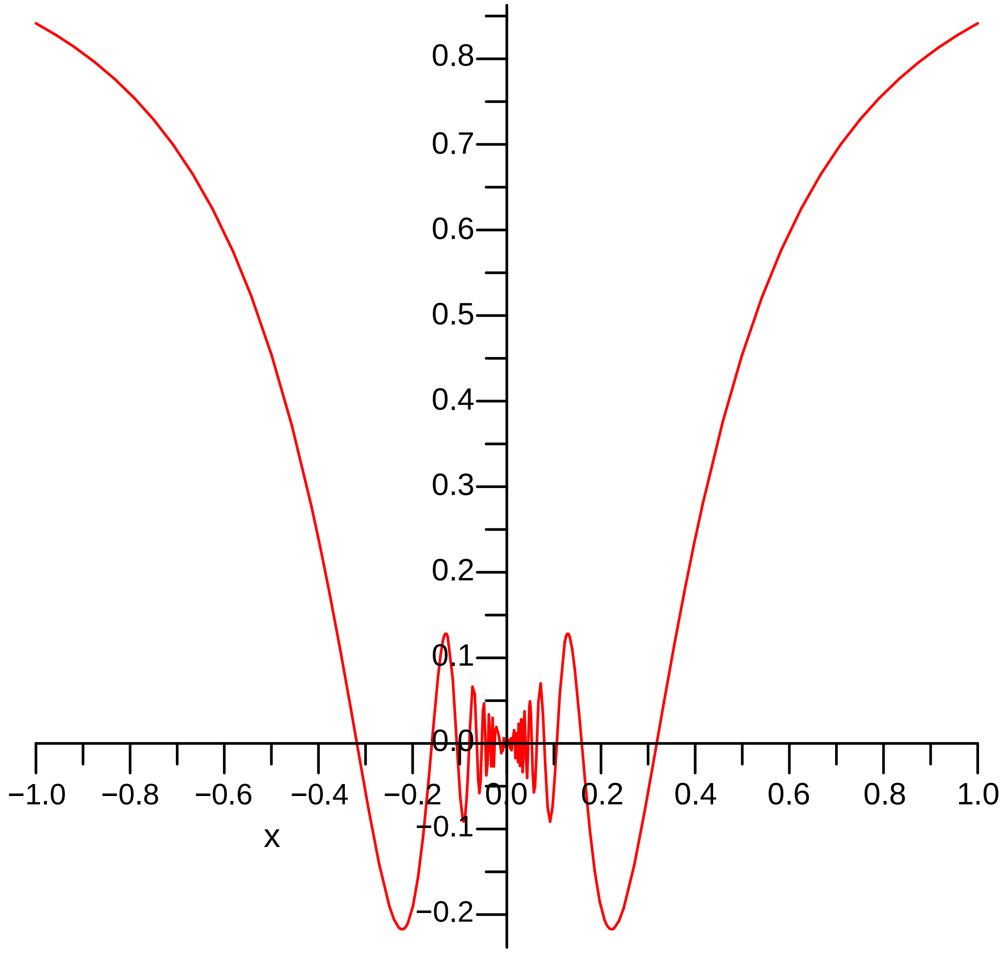
```
> plot(x^3,x=-1..1);
```




### 4.2.8. Példák.

```
> f:=x*sin(1/x); plot(f,x=-1..1);
```

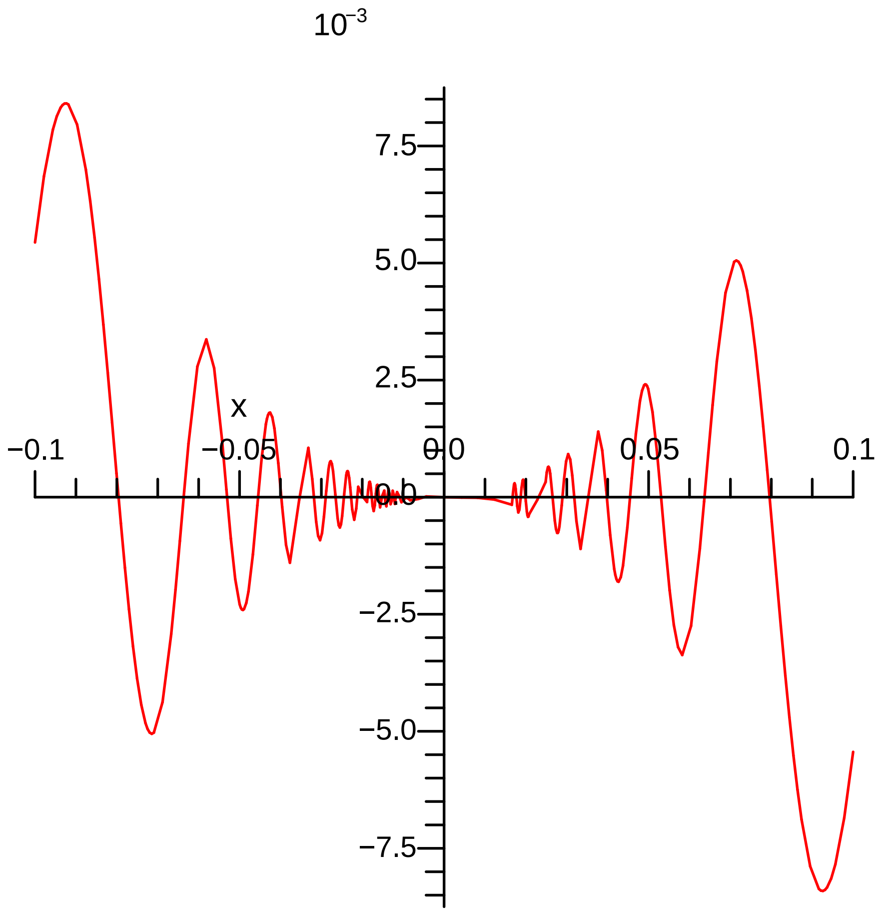
$$f := x \sin\left(\frac{1}{x}\right)$$



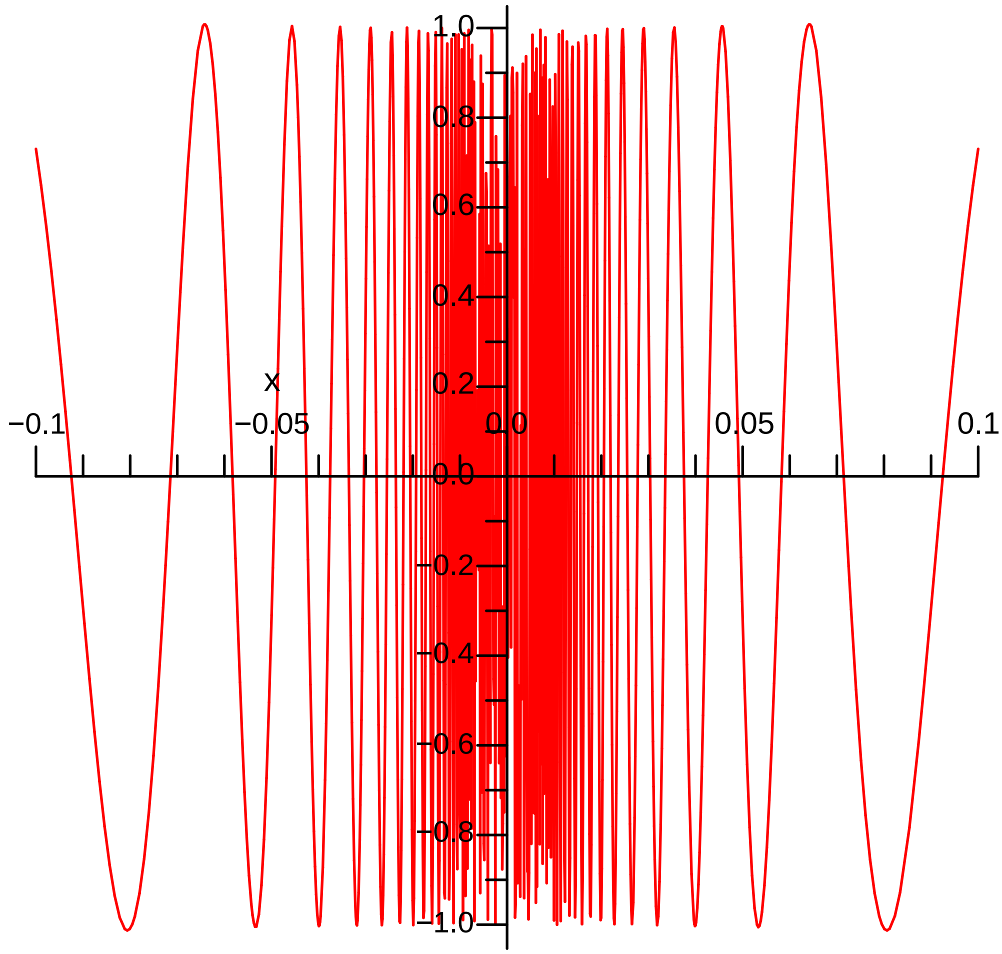

```
> f:=x^2*sin(1/x); g:=diff(f,x); plot(f,x=-0.1..0.1); plot(g,
  x=-0.1..0.1);
```

$$f := x^2 \sin\left(\frac{1}{x}\right)$$

$$g := 2\,x \sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)$$



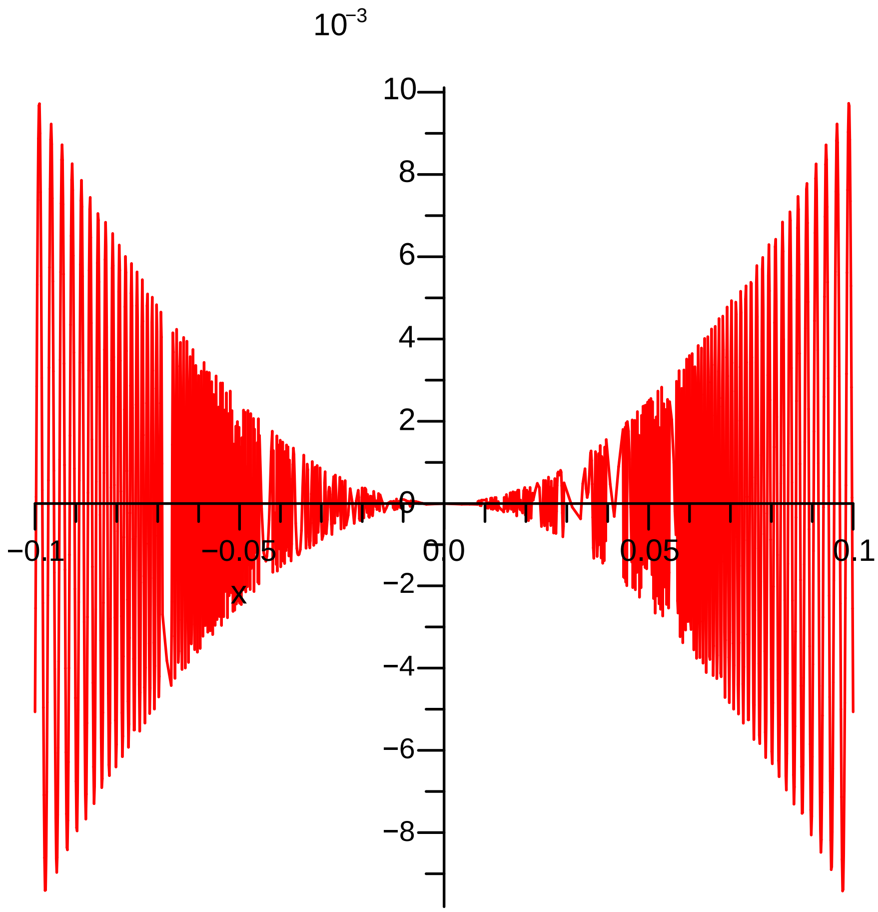
10−3
7.5
5.0
2.5
x
−0.1
−0.05
0.0
0.05
0.1
0.0
−2.5
−5.0
−7.5



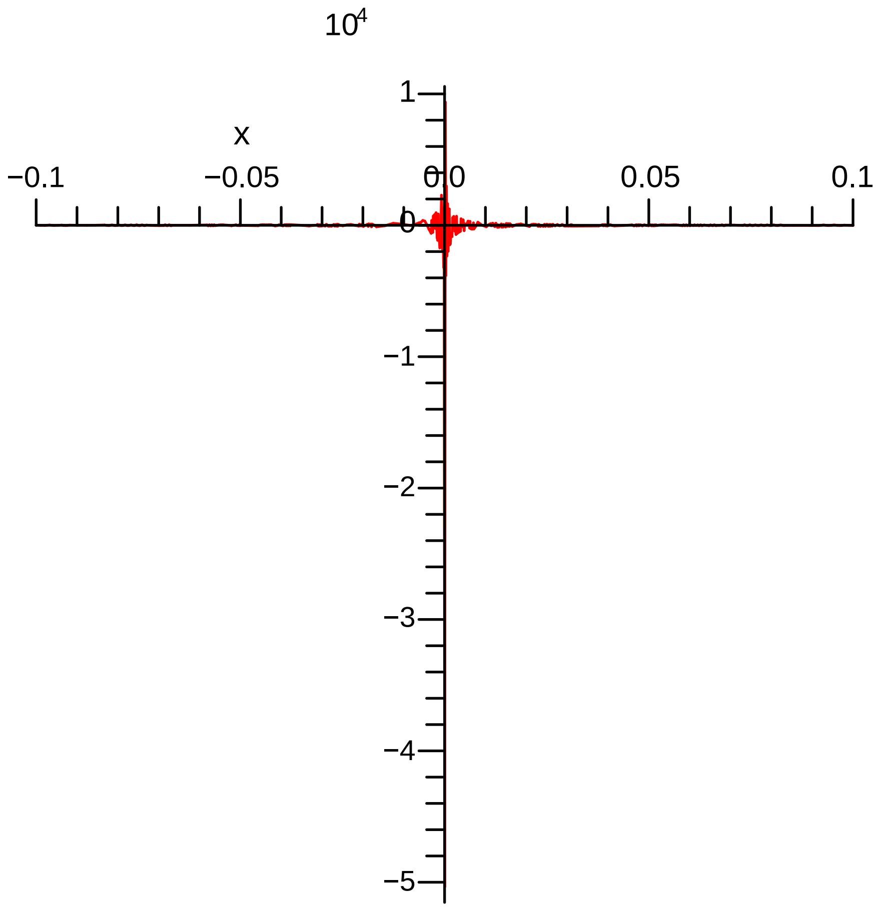

```
> f:=x^2*sin(1/x^2); g:=diff(f,x); plot(f,x=-0.1..0.1); plot
  (g,x=-0.1..0.1);
```

$$f := x^2 \sin\left(\frac{1}{x^2}\right)$$

$$g := 2\,x \sin\left(\frac{1}{x^2}\right) - \frac{2\cos\left(\frac{1}{x^2}\right)}{x}$$

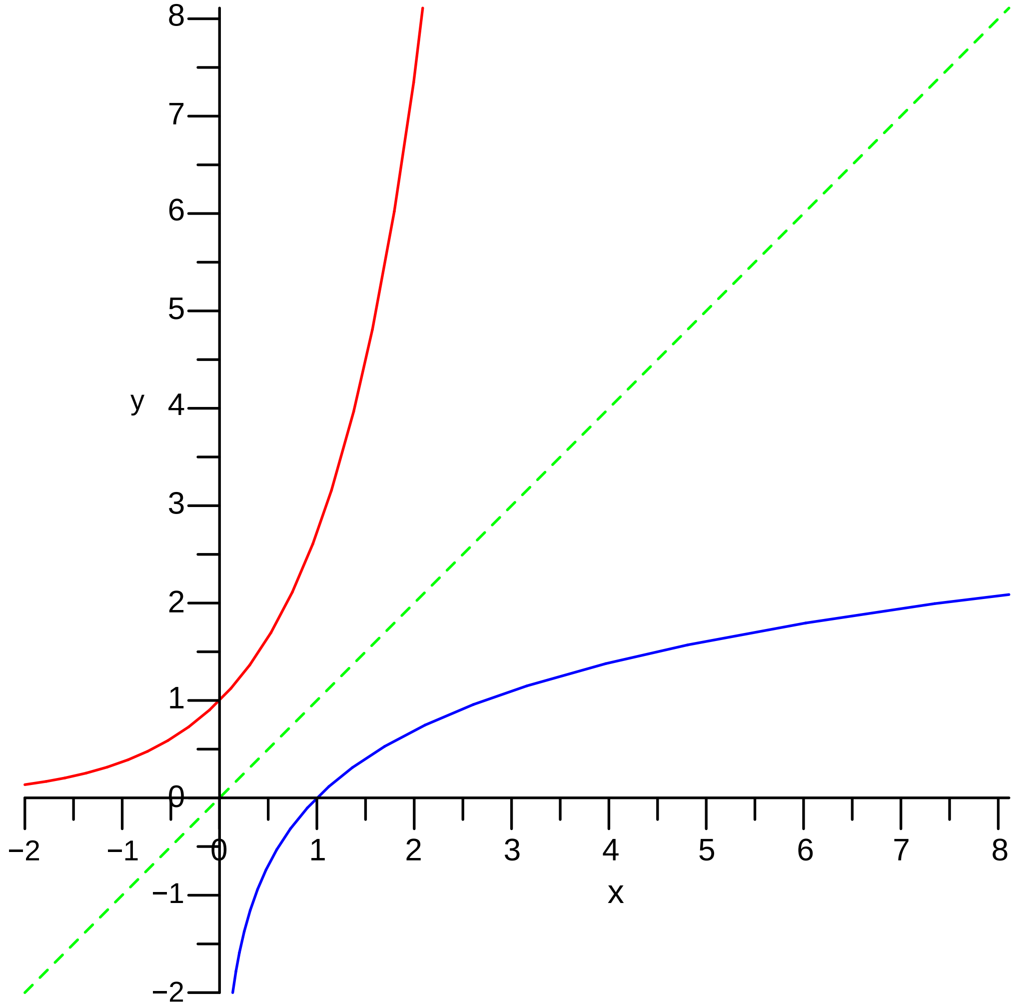


10⁻³
10
8
6
4
2
0
−2
−4
−6
−8
−0.1
−0.05
0.0
0.05
0.1
x

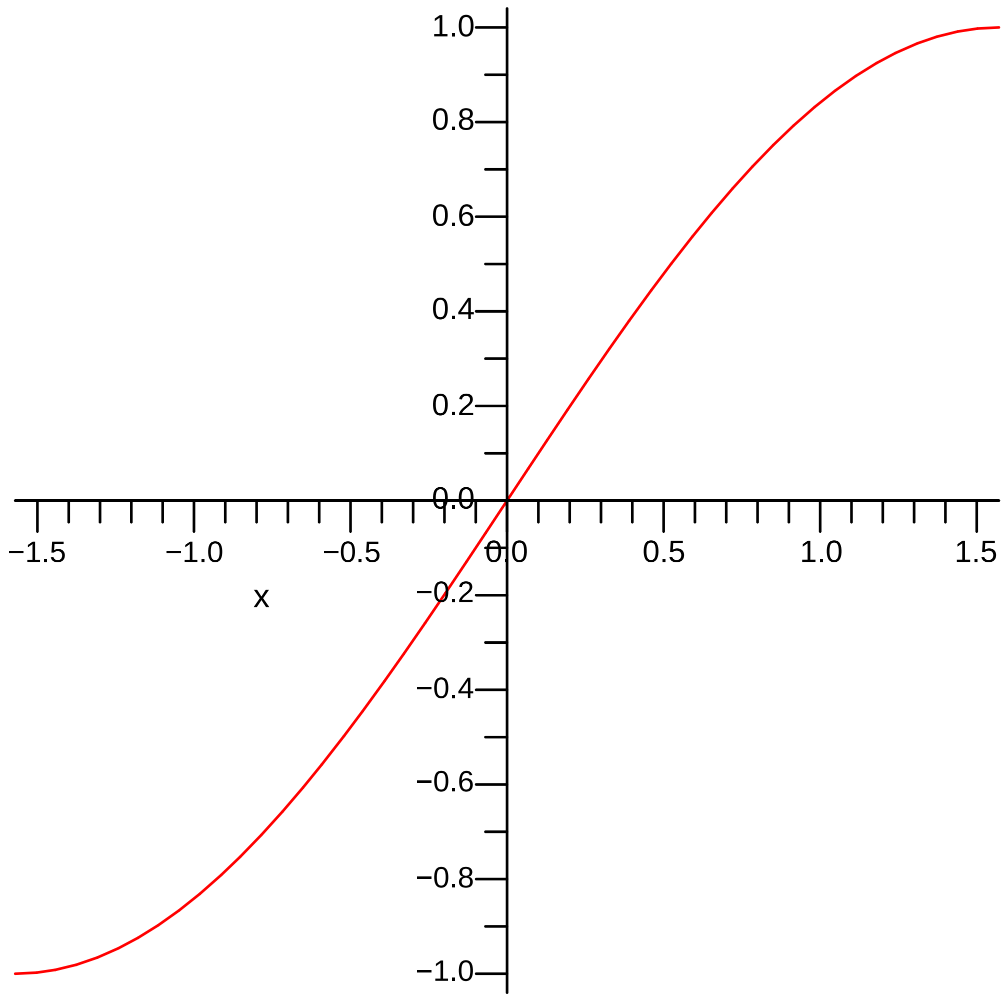


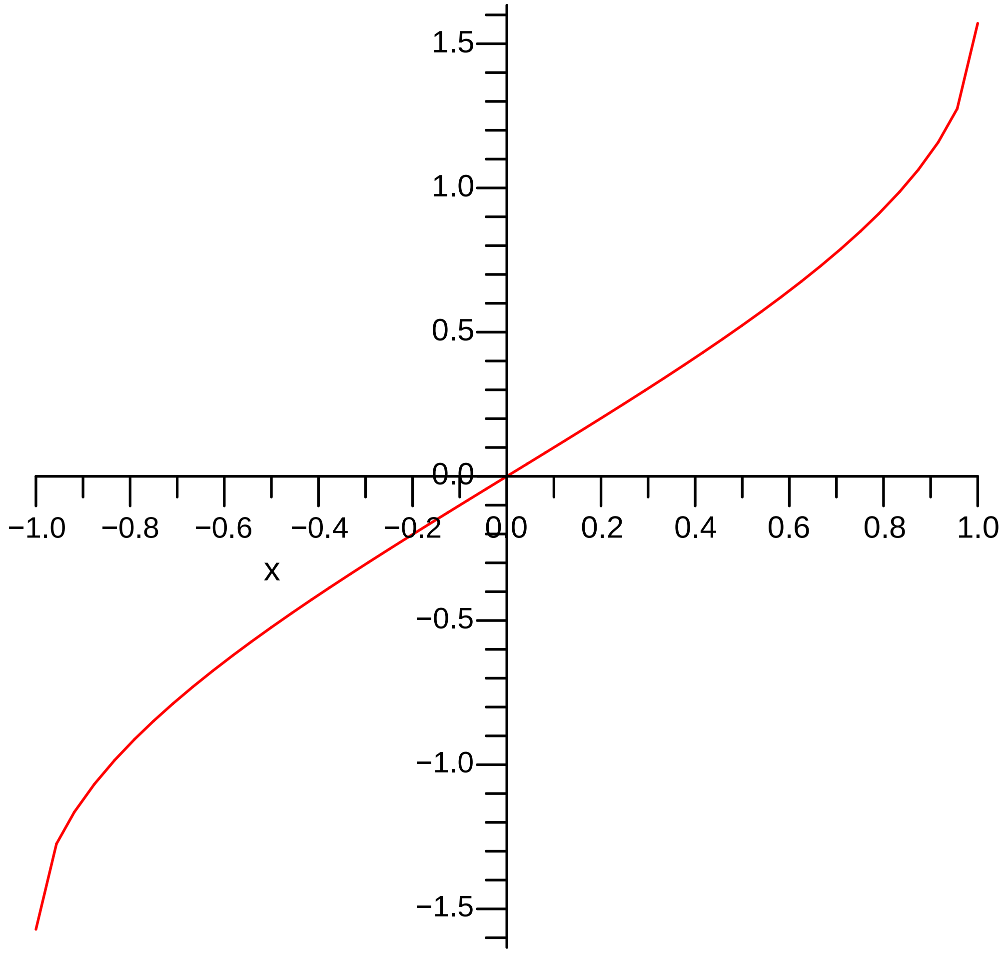

## *4.2.9. Elemi függvények inverzei.*

```
> Student[Calculus1][InverseTutor]();
```

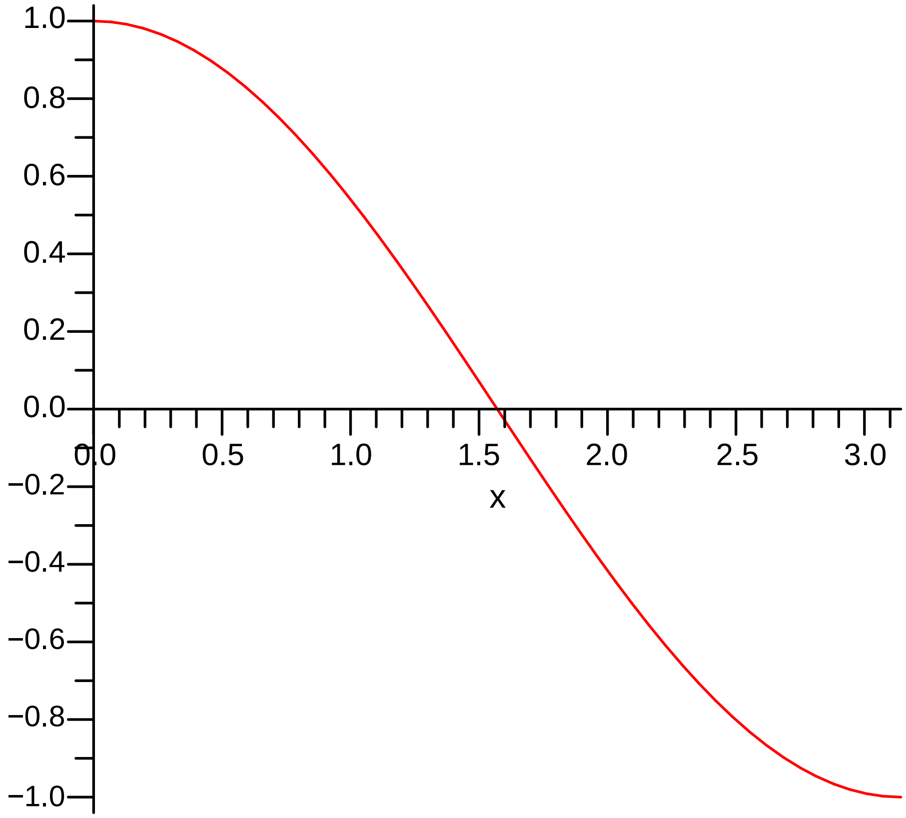


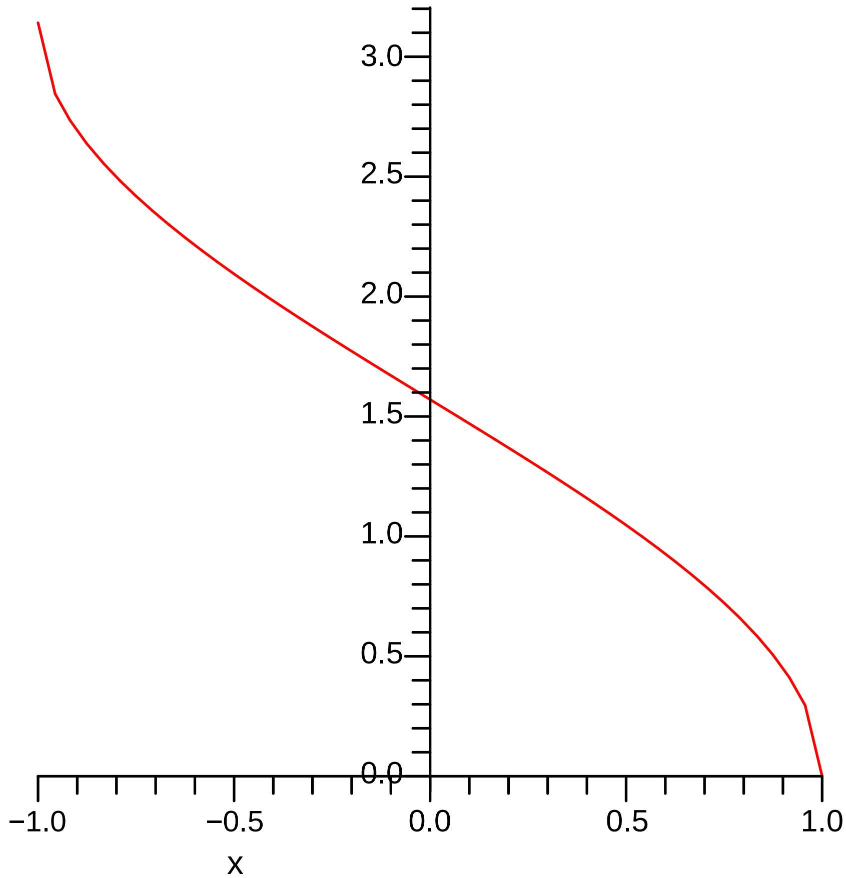

```
> plot(sin(x),x=-Pi/2..Pi/2); plot(arcsin(x),x=-1..1);
```


1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
x

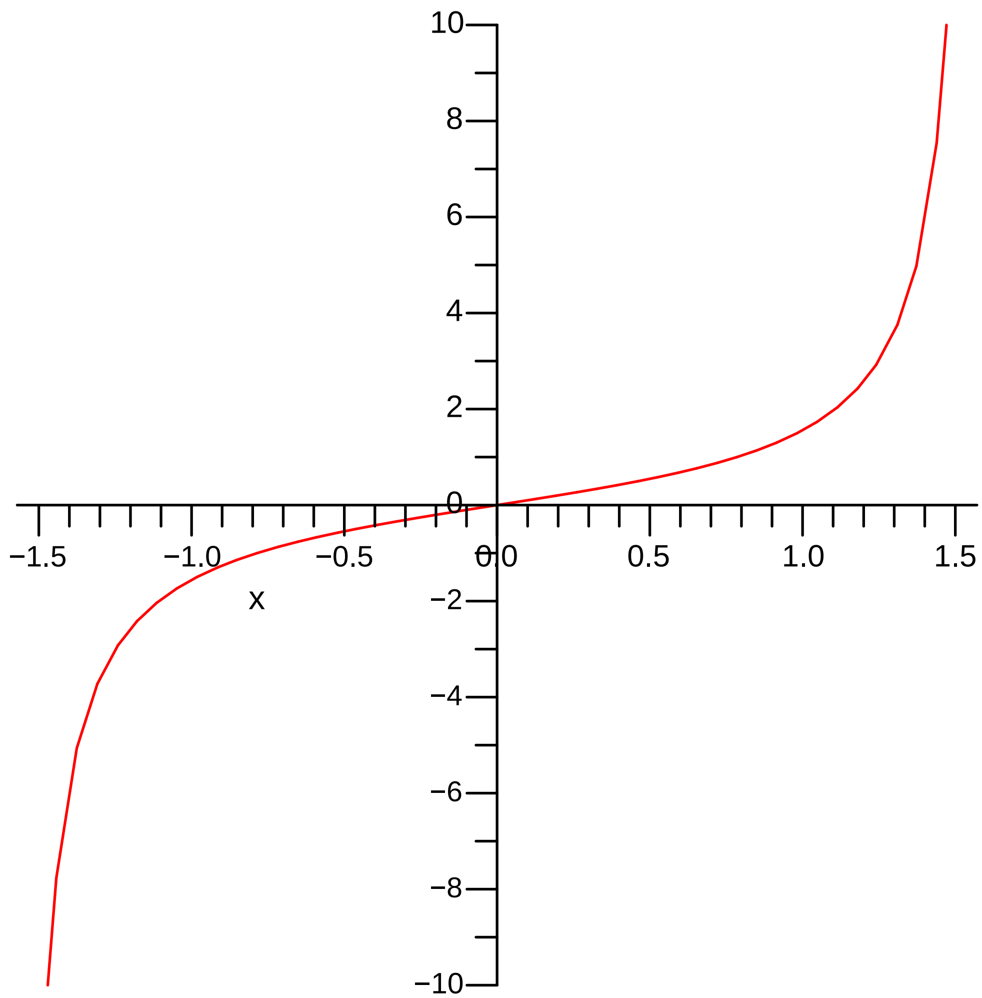



```
> plot(cos(x),x=0..Pi); plot(arccos(x),x=-1..1);
```

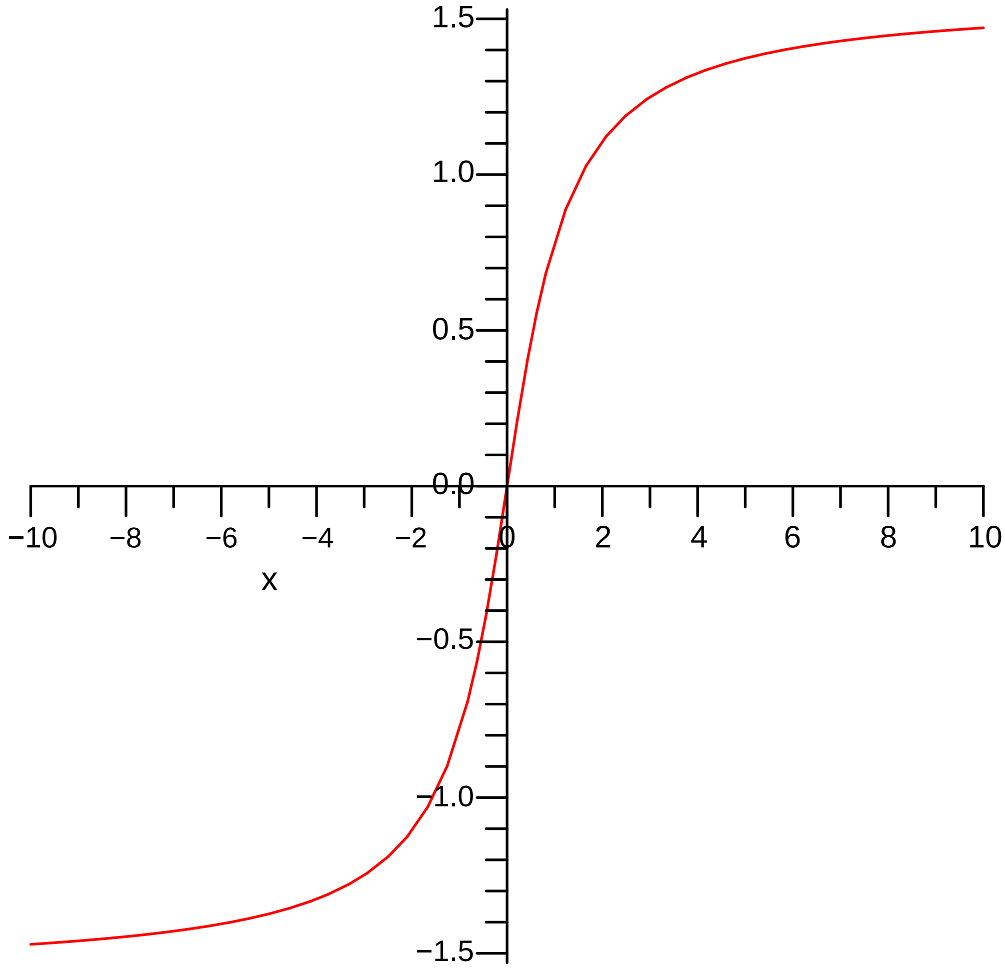

1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
x



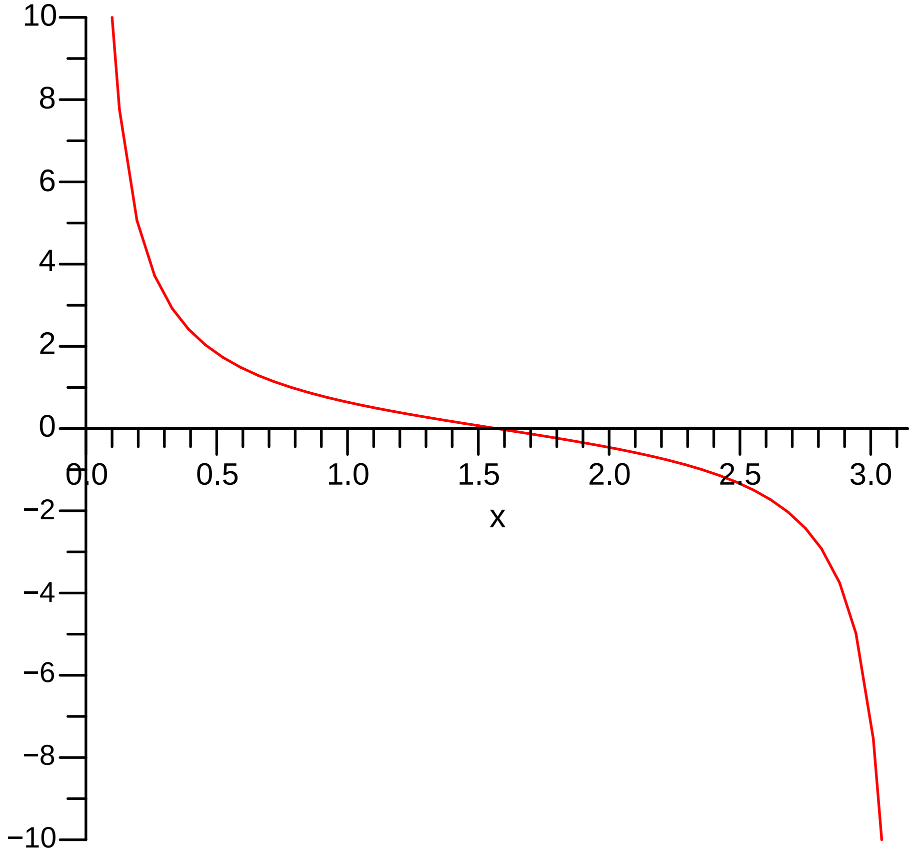

```
> plot(tan(x),x=-Pi/2..Pi/2,-10..10); plot(arctan(x),x=-10.
  .10);
```



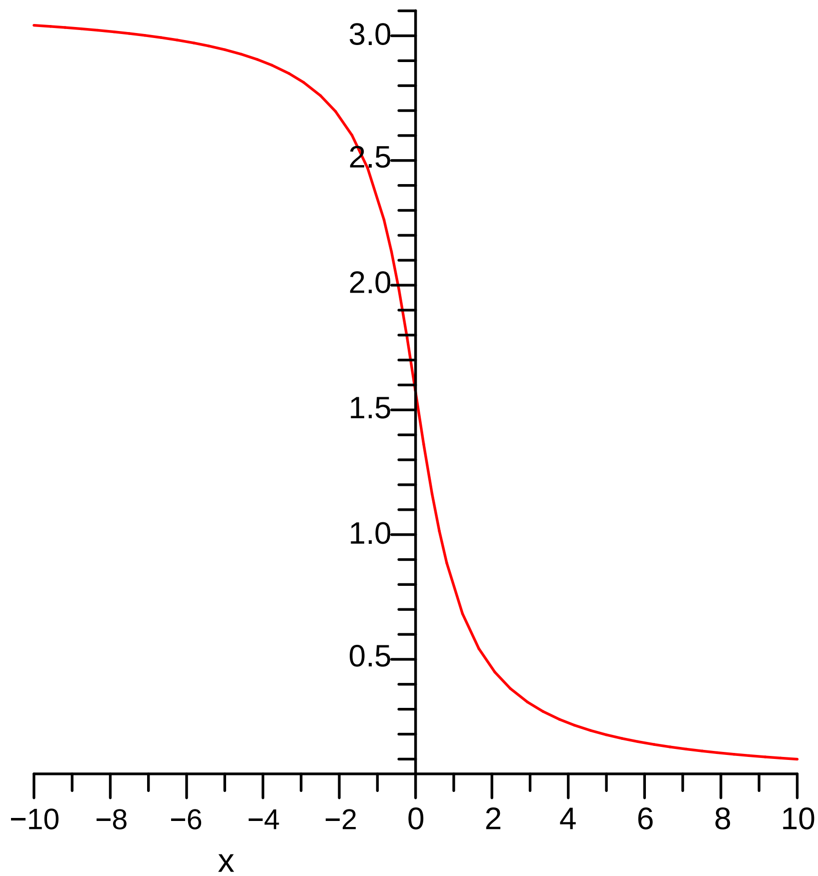
10
8
6
4
2
0
−2
−4
−6
−8
−10
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
x



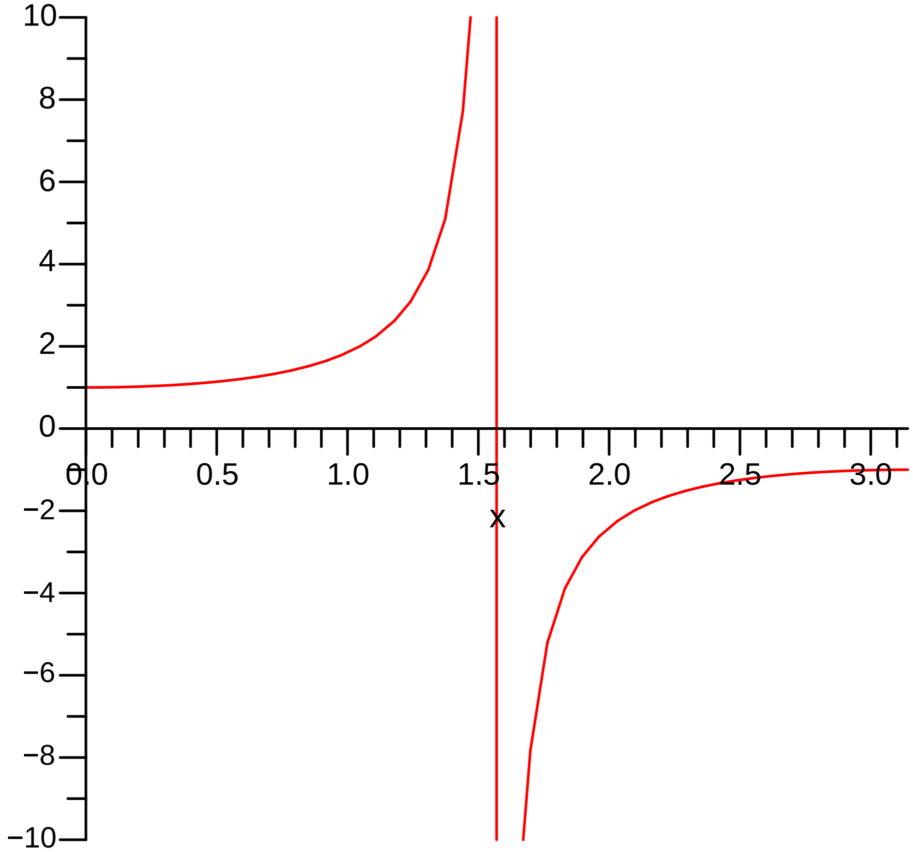

```
> plot(cot(x),x=0..Pi,-10..10); plot(arccot(x),x=-10..10);
```

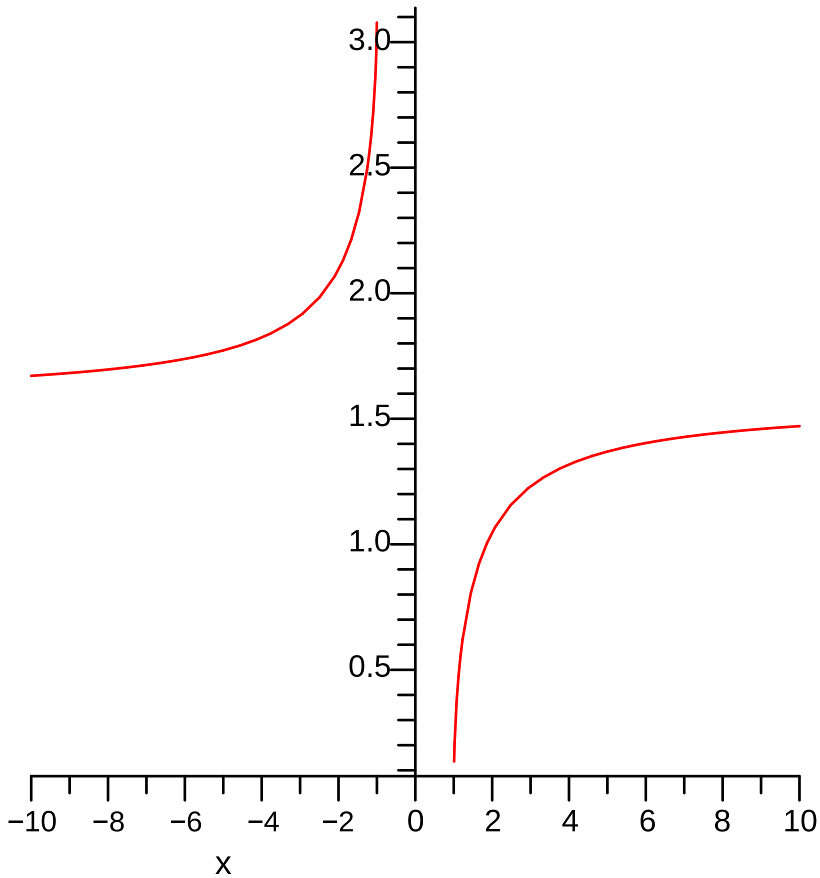


10
8
6
4
2
0
−2
−4
−6
−8
−10
0.0
0.5
1.0
1.5
2.0
2.5
3.0
x

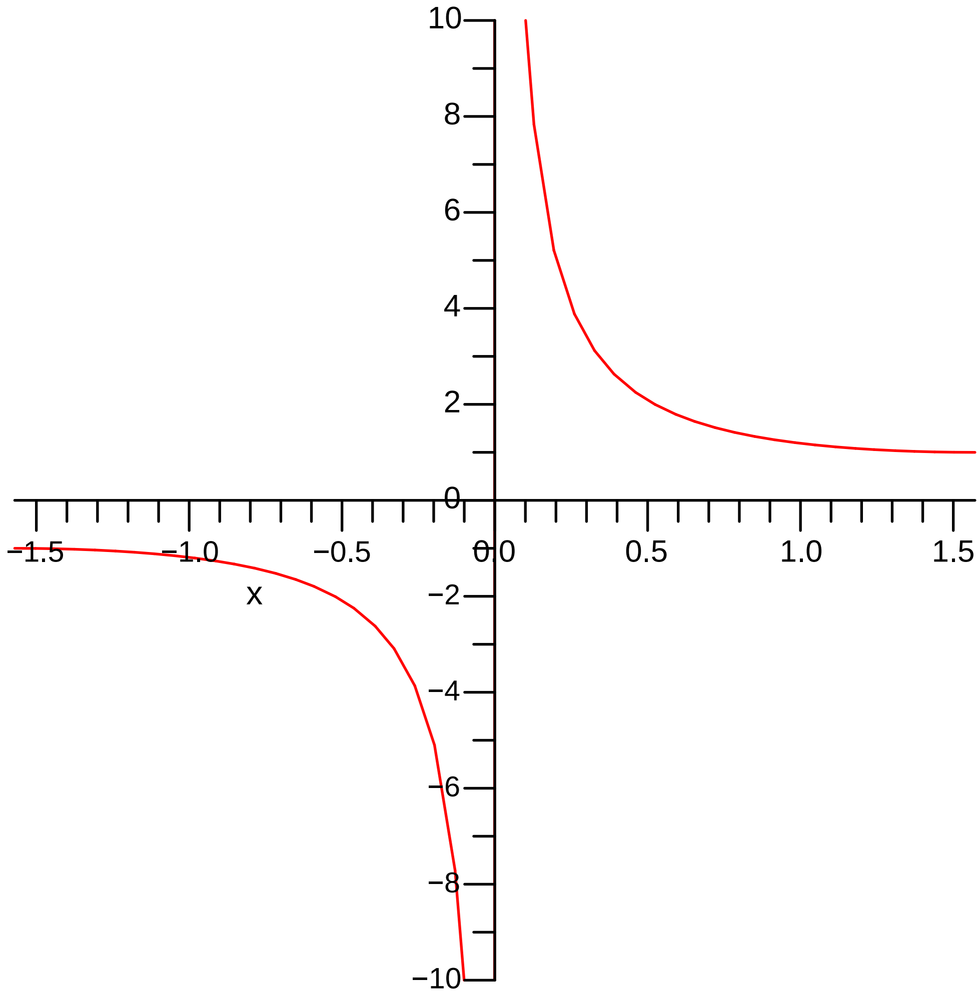



```
> plot(sec(x),x=0..Pi,-10..10); plot(arcsec(x),x=-10..10);
```

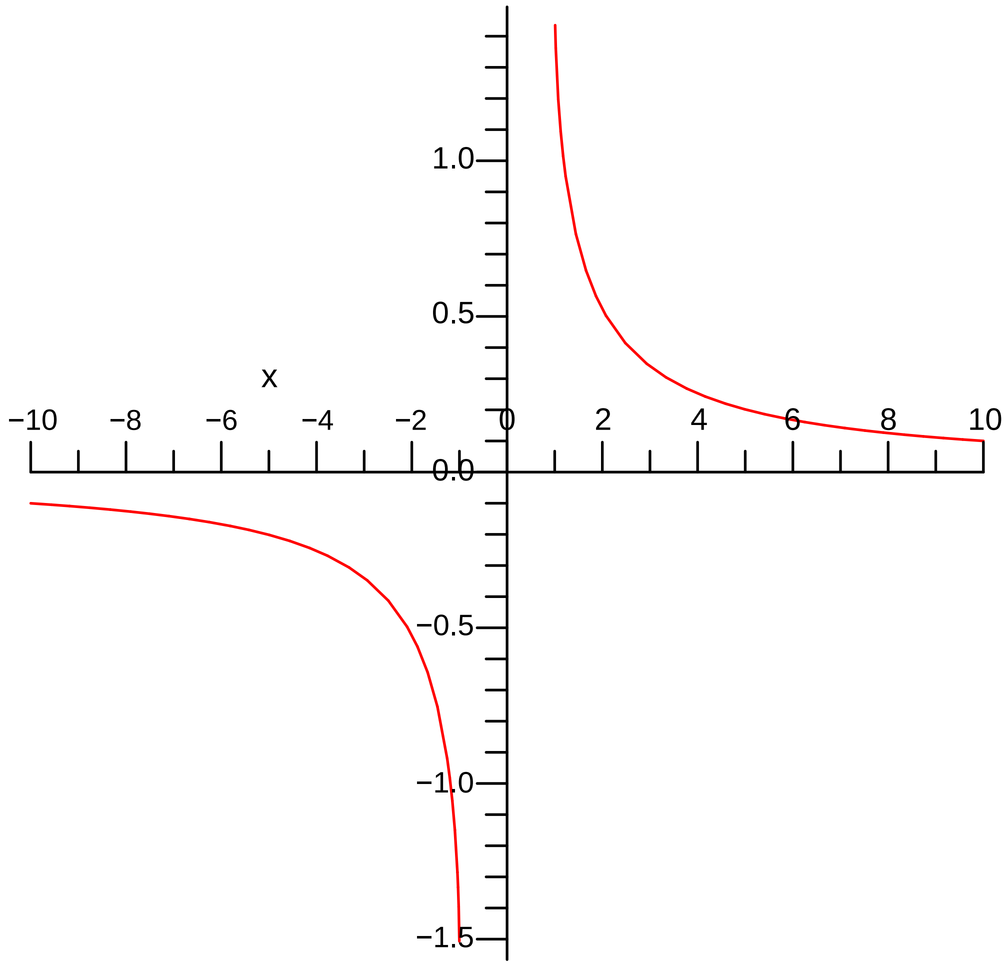

10
8
6
4
2
0
−2
−4
−6
−8
−10
0.0
0.5
1.0
1.5
2.0
2.5
3.0
x



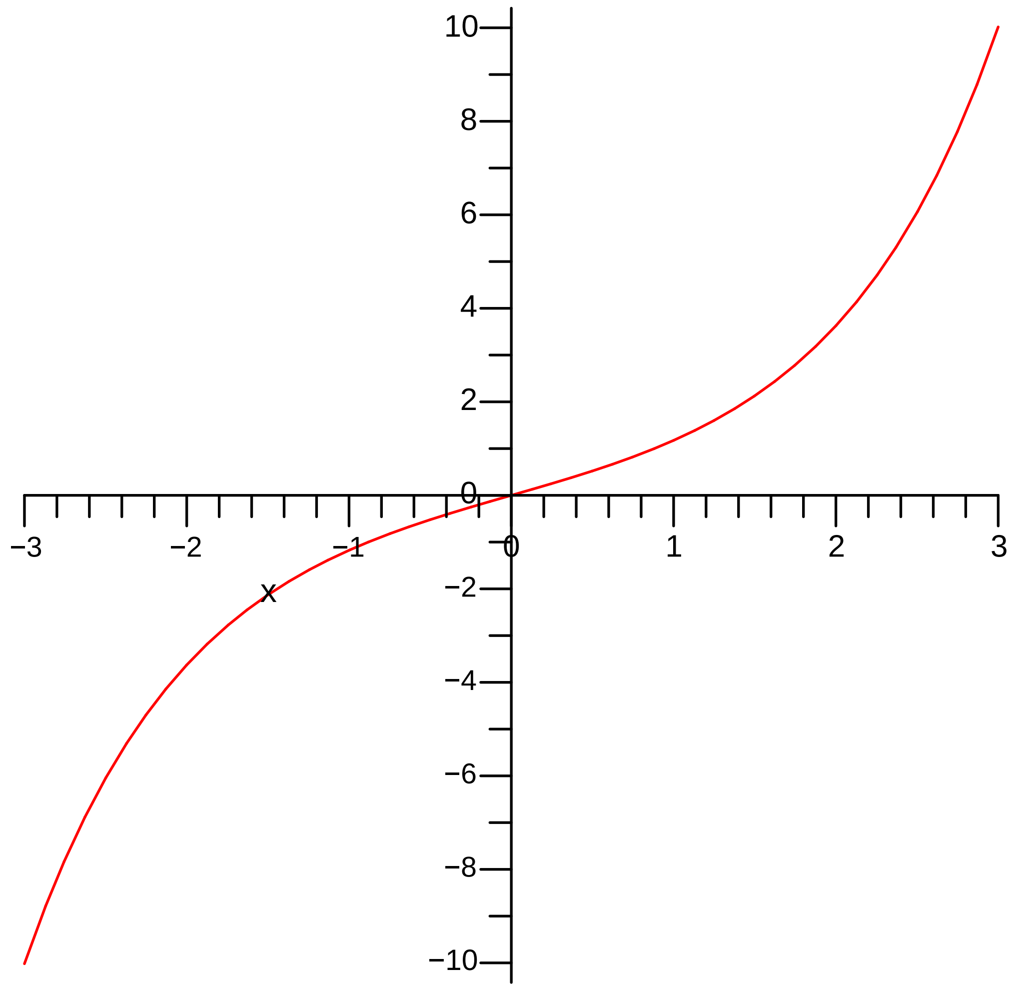

```
> plot(csc(x),x=-Pi/2..Pi/2,-10..10); plot(arccsc(x),x=-10.
  .10);
```

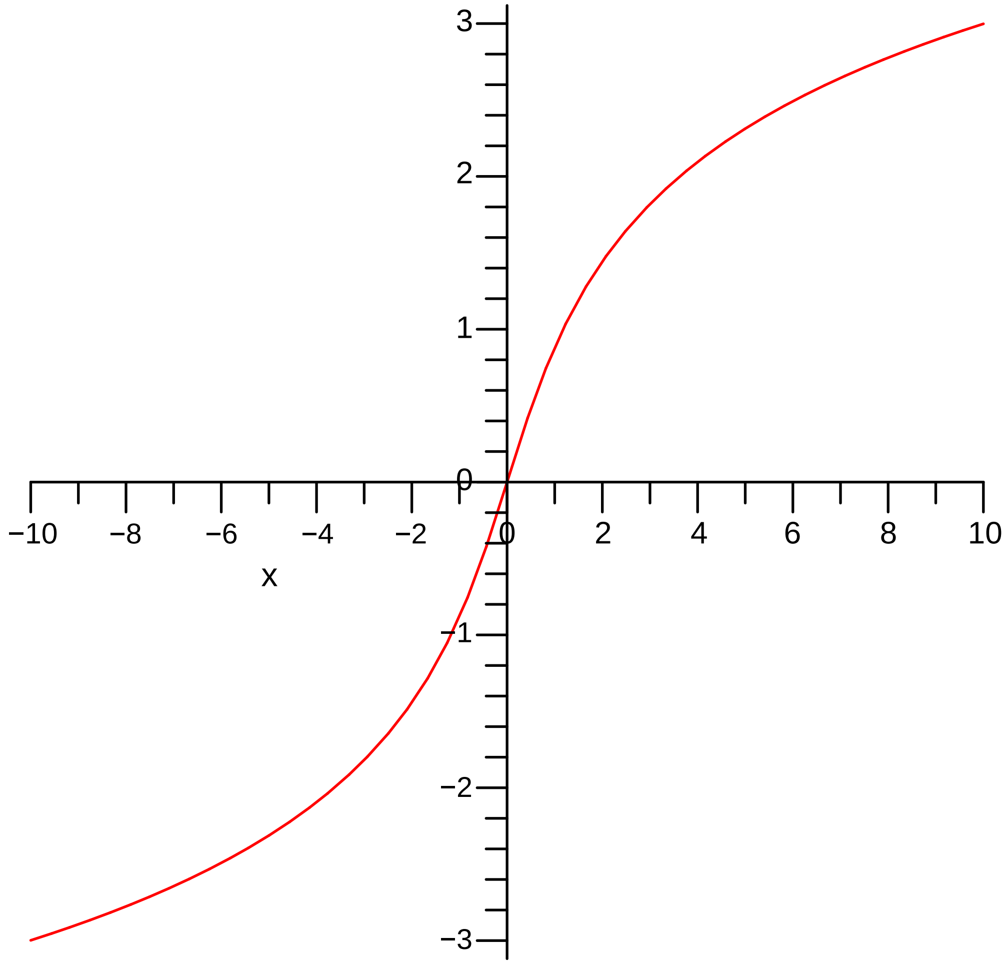


10
8
6
4
2
0
−2
−4
−6
−8
−10
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
x

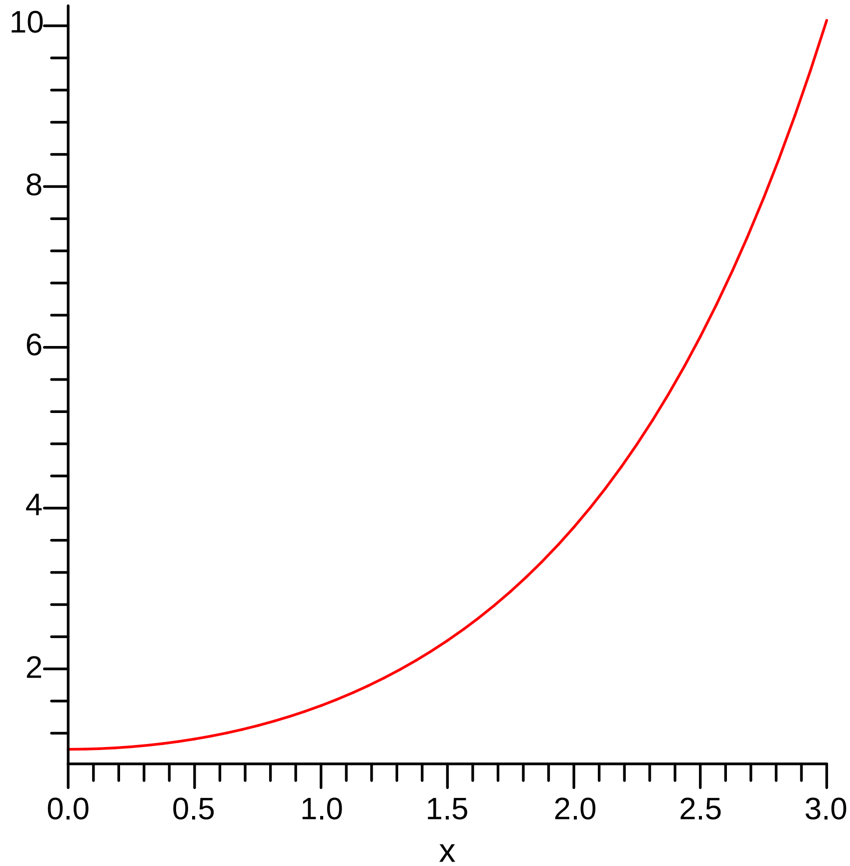


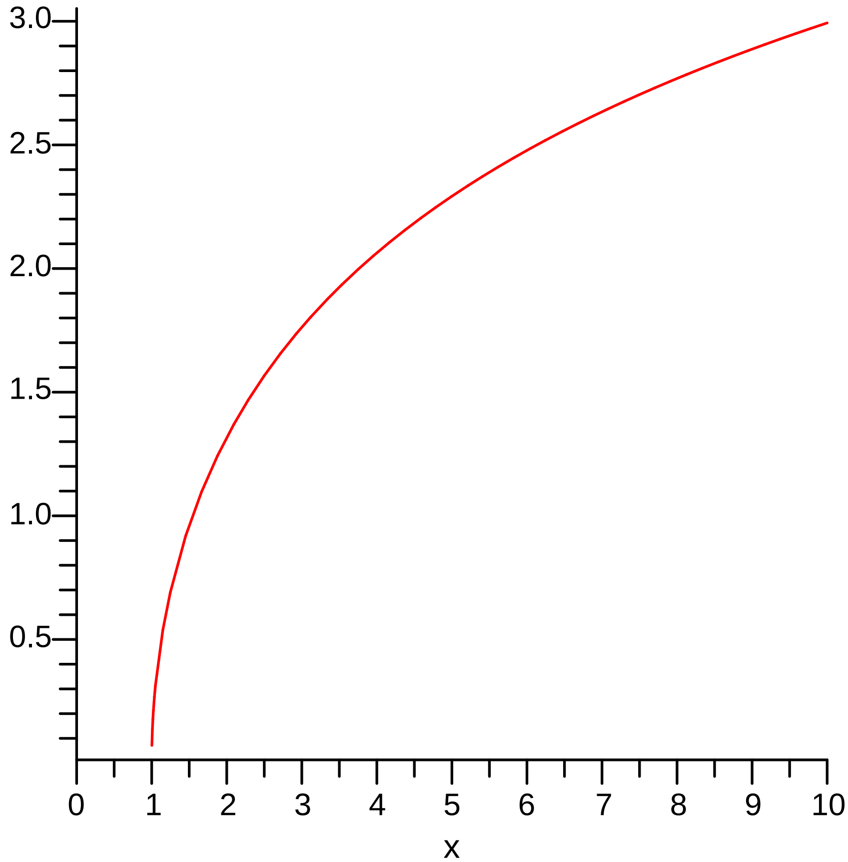

```
> plot(sinh(x),x=-3..3); plot(arcsinh(x),x=-10..10);
```


10
8
6
4
2
0
−2
−4
−6
−8
−10
−3
−2
−1
0
1
2
3
x

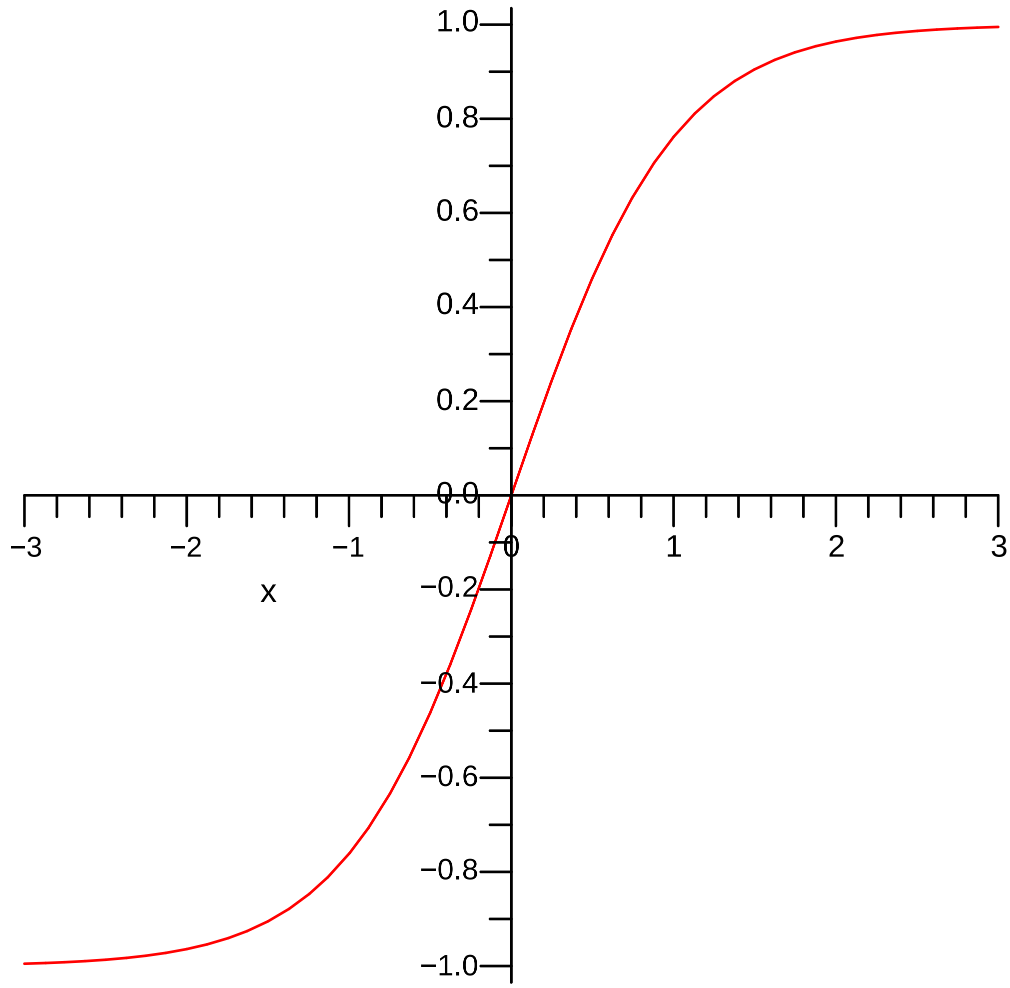


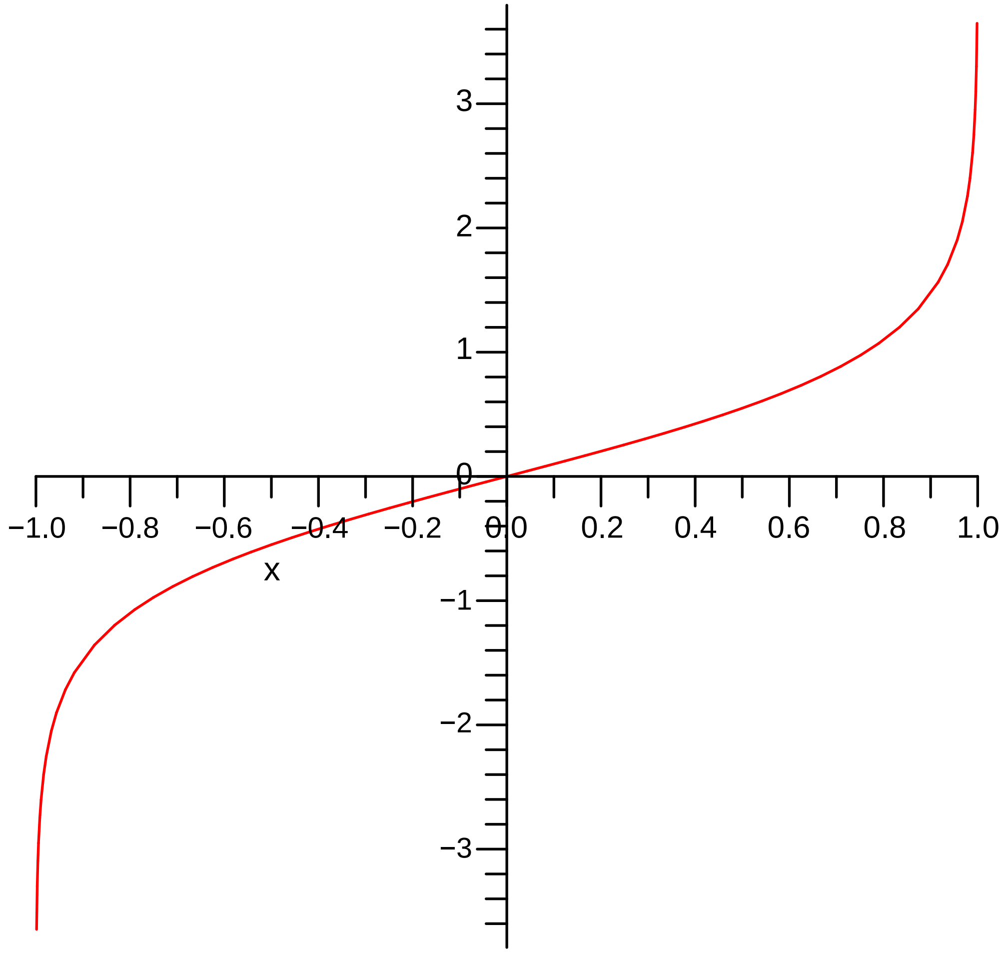

```
> plot(cosh(x),x=0..3); plot(arccosh(x),x=0..10);
```


10
8
6
4
2
0.0
0.5
1.0
1.5
2.0
2.5
3.0
x

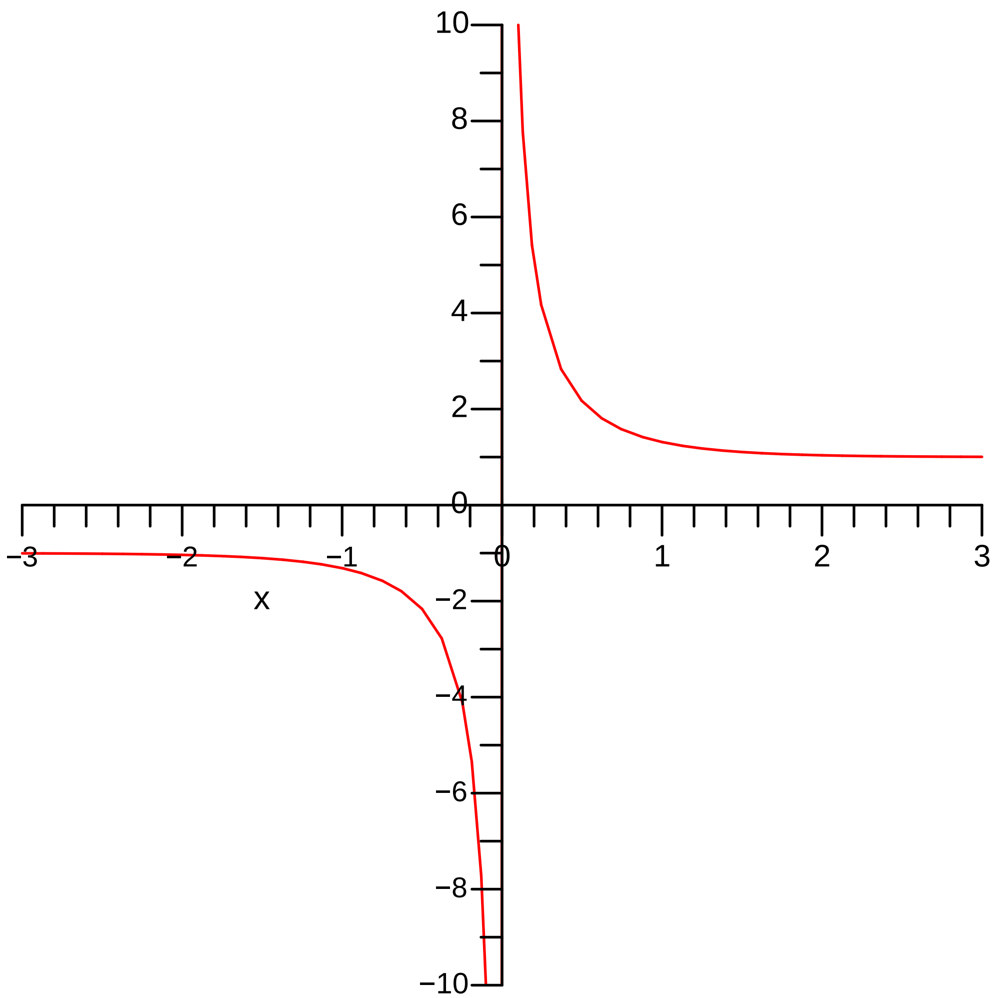


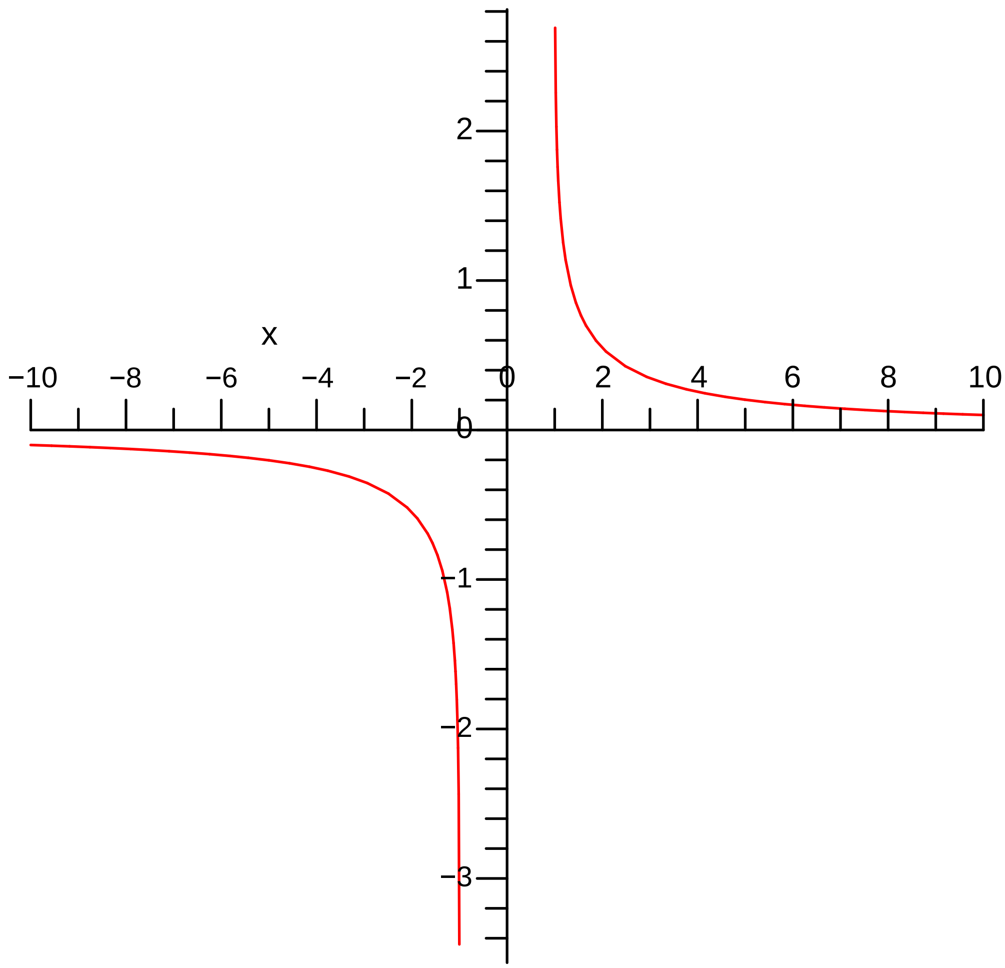

```
> plot(tanh(x),x=-3..3); plot(arctanh(x),x=-1..1);
```


1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
−3
−2
−1
0
1
2
3
x

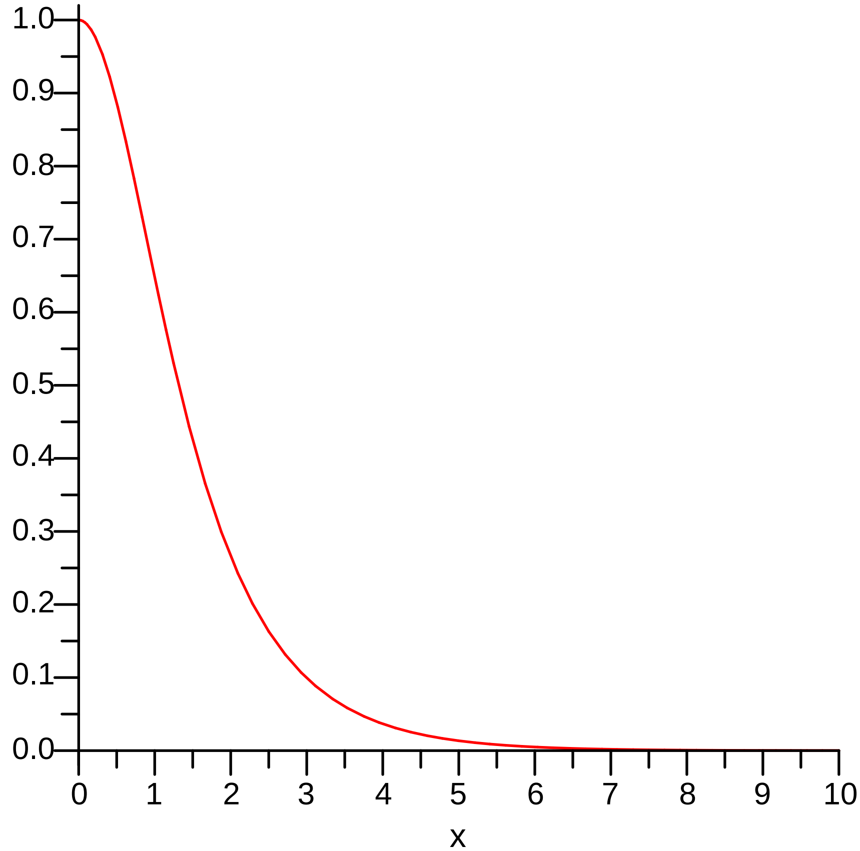


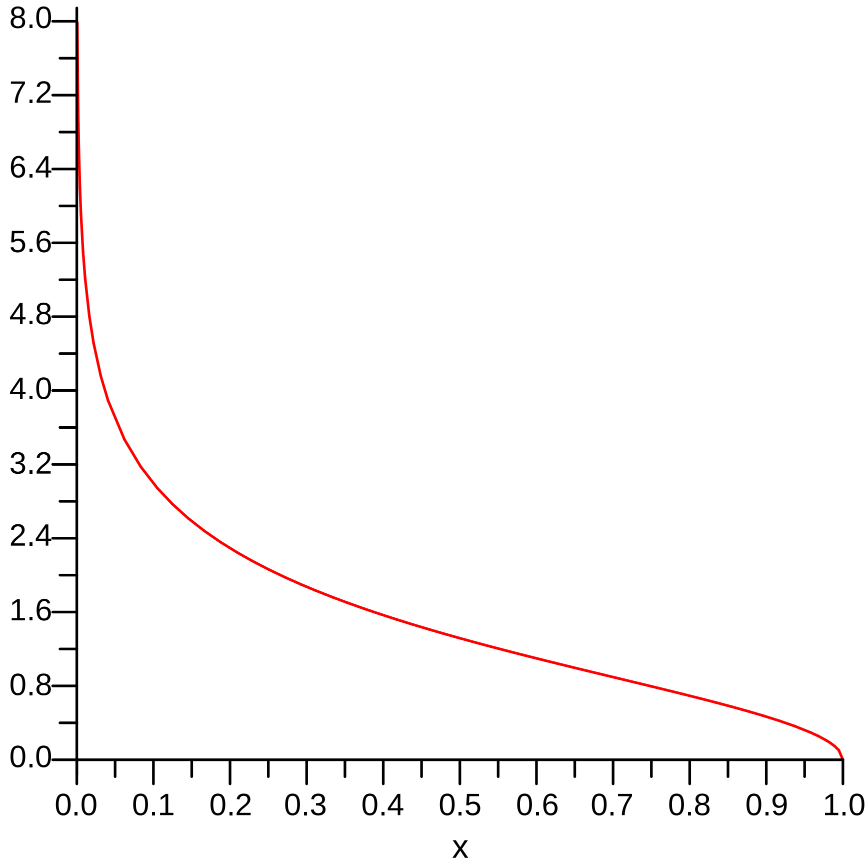

```
> plot(coth(x),x=-3..3,-10..10); plot(arccoth(x),x=-10..10);
```


10
8
6
4
2
0
−2
−4
−6
−8
−10
−3
−2
−1
0
1
2
3
x

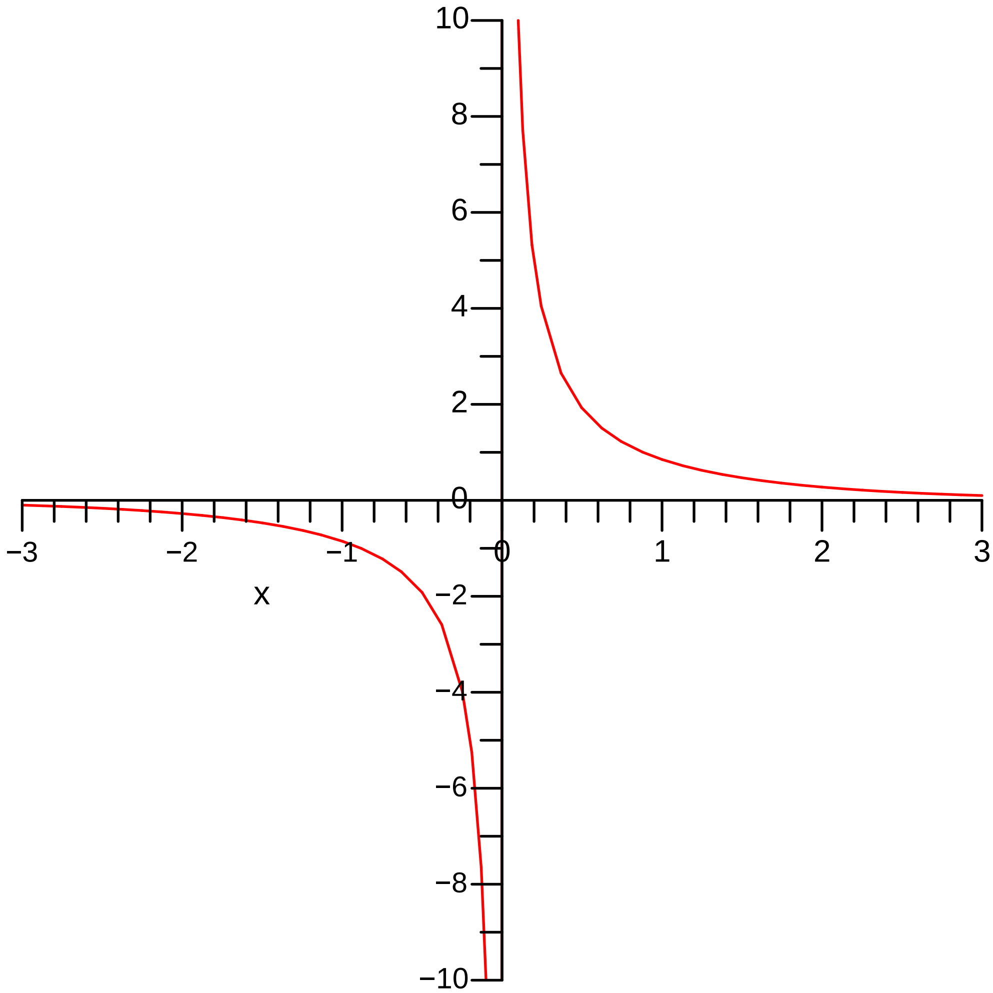



```
> plot(sech(x),x=0..10); plot(arcsech(x),x=0..1);
```

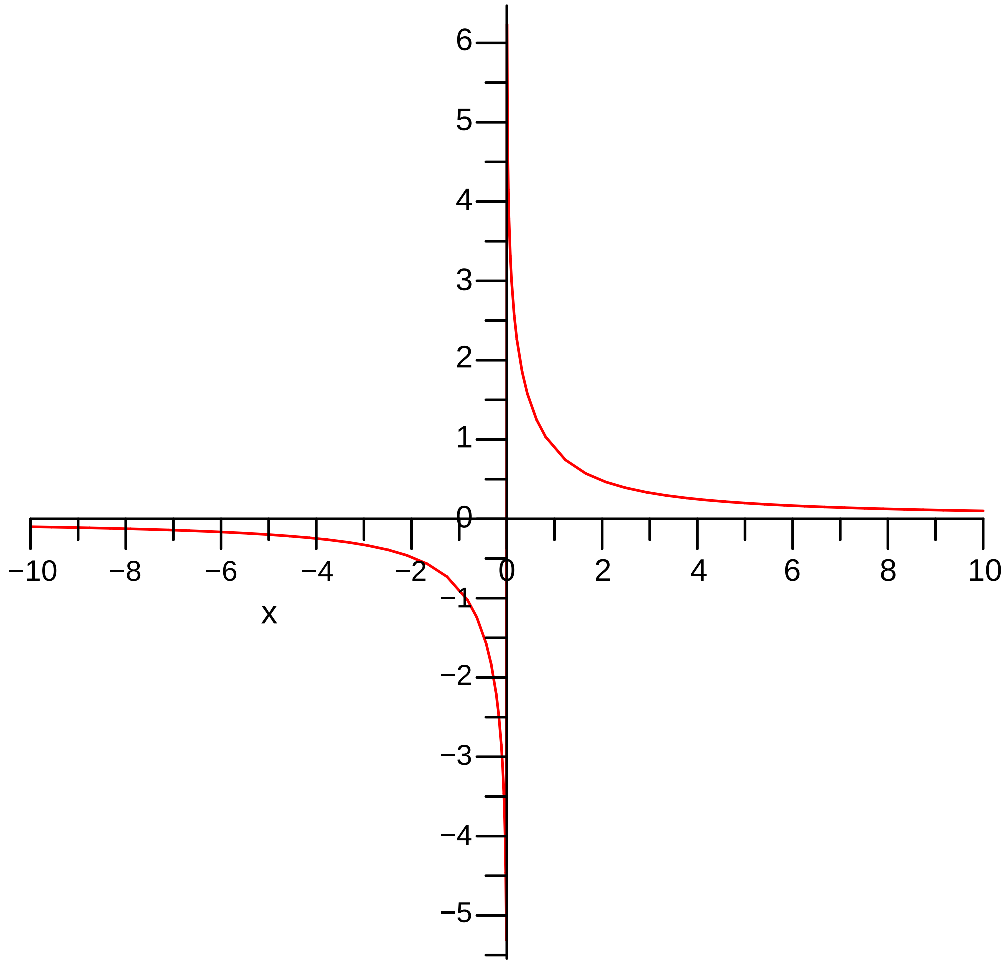

1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7
8
9
10
x




```
> plot(csch(x),x=-3..3,-10..10); plot(arccsch(x),x=-10..10);
```



10
8
6
4
2
0
−2
−4
−6
−8
−10
−3
−2
−1
0
1
2
3
x




**4.2.10. Darboux közbenső érték tétele.*

*4.2.11. L'Hospital-szabály.*

*4.2.12. Példák.*

```
> Student[Calculus1][LimitTutor](ln(1+x)/x,x=0);
```

$$\lim_{x\to 0}\left(\frac{\ln(1+x)}{x}\right)=1 \qquad (4.2.12.1)$$

```
> Student[Calculus1][LimitTutor](ln(1+x)/x,x=+infinity);
```

$$\lim_{x\to \infty}\left(\frac{\ln(1+x)}{x}\right)=0 \qquad (4.2.12.2)$$

```
> limit((1+a/x)^(b*x),x=+infinity);
```

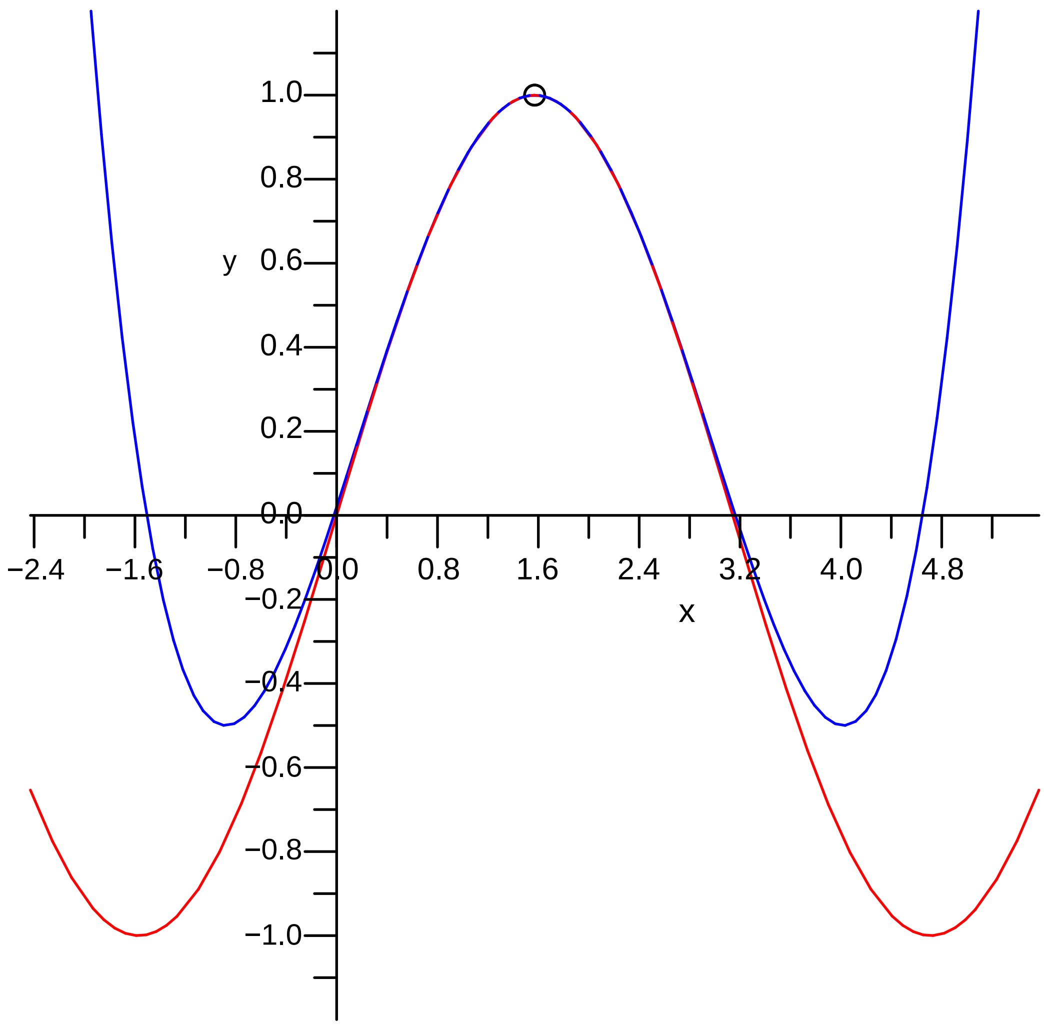
$$e^{a\,b} \qquad (4.2.12.3)$$

*4.2.13. Konvex és konkáv függvények, inflexiós hely.*

**4.2.14. Segédtétel.*

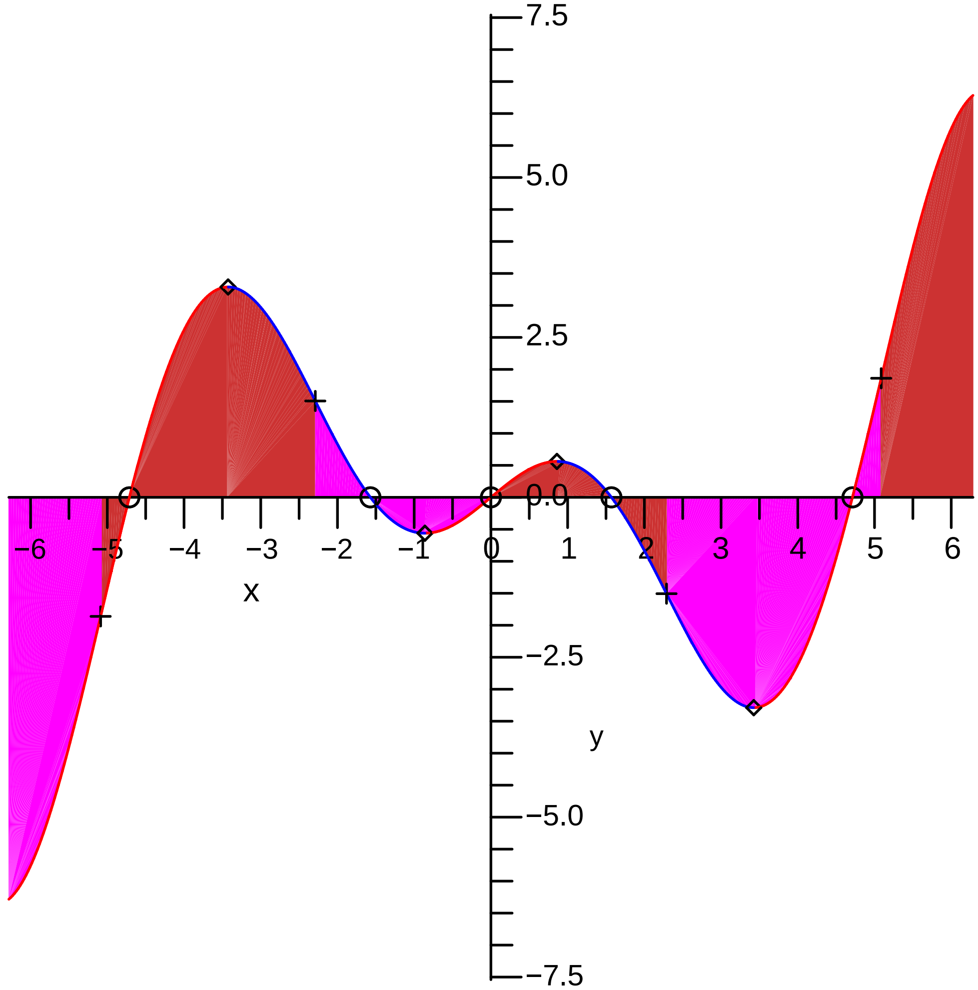
▶ *4.2.15. Tétel.*
▶ *4.2.16. Következmény.*
▶ *4.2.17. Következmény.*
▶ *4.2.18. Következmény.*
▶ *4.2.19. Számtani és mértani ko'zép közti egyenlőtlenség.*
▶ *4.2.20. Taylor-polinom.*
▼ *4.2.21. Taylor-formula maradéktag nélkül.*

```
> Student[Calculus1][TaylorApproximationTutor](sin(x),x=Pi/2)
;
```




▶ *4.2.22. Taylor-formula Lagrange-féle maradéktaggal.*
▼ *4.2.23. Tétel.*

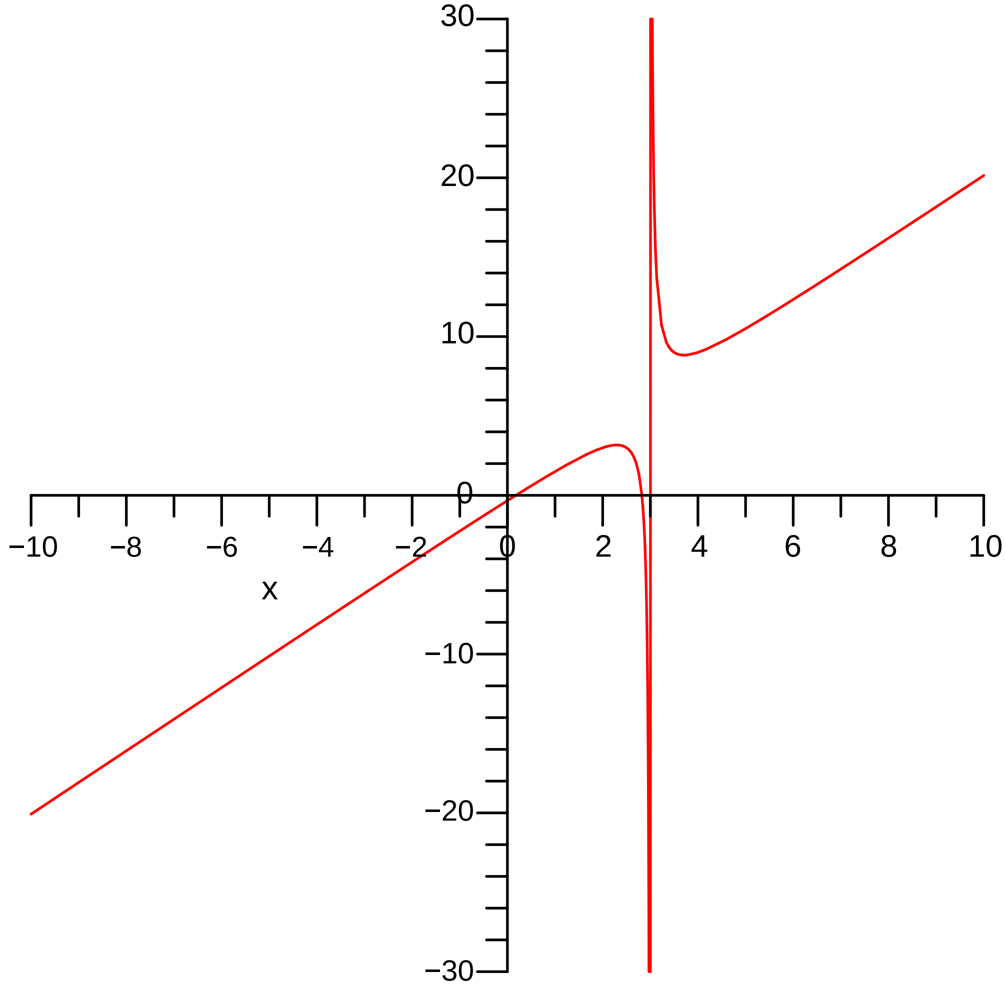
```
> Student[Calculus1][CurveAnalysisTutor](x*cos(x),x=-2*Pi..2*
PI);
```




### 4.2.24. Aszimptóták.

```
> f:=1/(x-3)+2*x; plot(f,x=-10..10,-30..30); with(Student
  [Calculus1]); Asymptotes(f,x);
```

$$f := \frac{1}{x-3} + 2x$$




[*AntiderivativePlot*, *AntiderivativeTutor*, *ApproximateInt*, *ApproximateIntTutor*, *ArcLength*, *ArcLengthTutor*, *Asymptotes*, *Clear*, *CriticalPoints*, *CurveAnalysisTutor*, *DerivativePlot*, *DerivativeTutor*, *DiffTutor*, *ExtremePoints*, *FunctionAverage*, *FunctionAverageTutor*, *FunctionChart*, *FunctionPlot*, *GetMessage*, *GetNumProblems*, *GetProblem*, *Hint*, *InflectionPoints*, *IntTutor*, *Integrand*, *InversePlot*, *InverseTutor*, *LimitTutor*, *MeanValueTheorem*, *MeanValueTheoremTutor*, *NewtonQuotient*, *NewtonsMethod*, *NewtonsMethodTutor*, *PointInterpolation*, *RiemannSum*, *RollesTheorem*, *Roots*, *Rule*, *Show*, *ShowIncomplete*, *ShowSteps*, *Summand*, *SurfaceOfRevolution*, *SurfaceOfRevolutionTutor*, *Tangent*, *TangentSecantTutor*, *TangentTutor*, *TaylorApproximation*, *TaylorApproximationTutor*, *Understand*, *Undo*, *VolumeOfRevolution*, *VolumeOfRevolutionTutor*, *WhatProblem*]

$$[y = 2\,x,\ x = 3] \qquad (4.2.24.1)$$

>

# ▶ 5. Integrálszámítás