Kalkulus I.

Járai Antal

Ezek a programok csak szemléltetésre szolgálnak

1. Halmazok

2. Számok

3. Határérték

```
> restart;
```

3.1. Határérték és folytonosság

3.1.1. Környezetek.

```
> verify(3,Pi+I/2,neighborhood(1)); verify(3,Pi,neighborhood
  (0.1));
```

true

false (3.1.1.1)

3.1.2. Belső, izolált és torlódási pontok.

3.1.3. Nyílt és zárt halmazok.

3.1.4. Állítás.

3.1.5. Állítás.

```
> verify(3,2+I,neighborhood(sqrt(2)));
  verify(3,2+I,neighborhood(sqrt(2),closed));
```

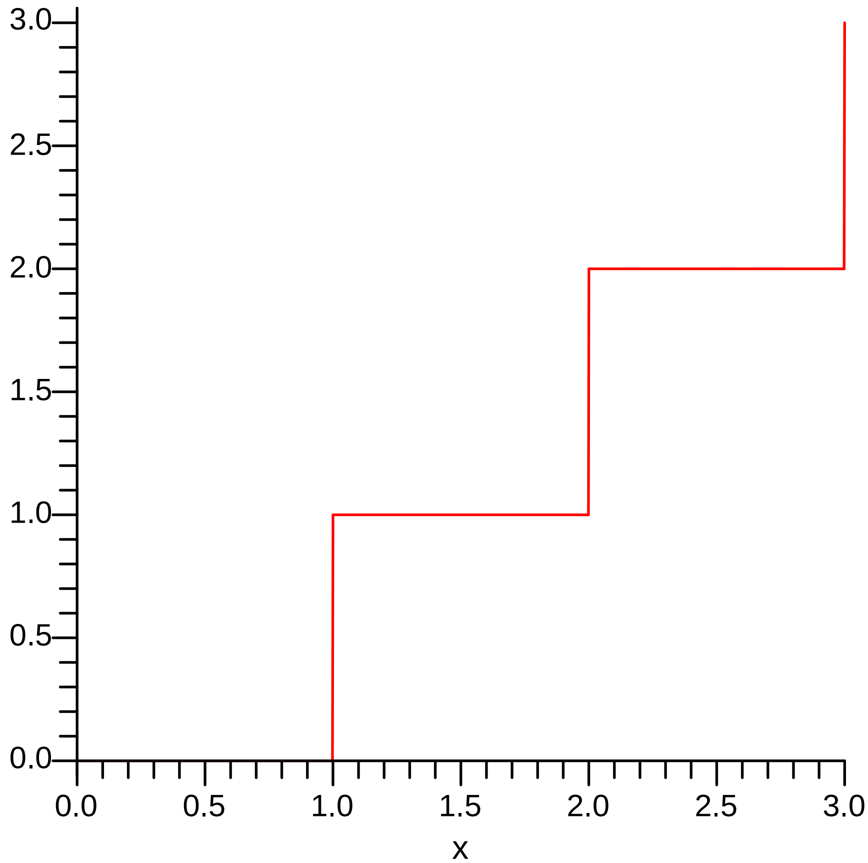
false

true (3.1.5.1)

3.1.6. Példák.

3.1.7. Folytonosság.

```
> plot(floor(x),x=0..3);
  iscont(floor(x),x=1.1..1.9); iscont(floor(x),x=1.1..2.9);
  iscont(floor(x),x=1..2); iscont(floor(x),x=1..2,closed);
```



true
false
true
false (3.1.7.1)

```
> discont(floor(x),x);
```

{_Z5~} (3.1.7.2)

```
> fdiscont(floor(x),x=0..3,10^(-7));
```

$[-5.97247784632044233\ 10^{-9}..6.83703274669646890\ 10^{-8},$ (3.1.7.3)
$0.99999994896109890..1.00000004853139779,$
$1.99999995481924354..2.00000005438954043,$
$2.99999993523787900..3.00000000958068513]$

3.1.8. Példák.

```
> iscont(c,x=-infinity..infinity); iscont(x,x=-infinity..
infinity);
```

```
iscont(abs(x),x=-infinity..infinity);
```

true

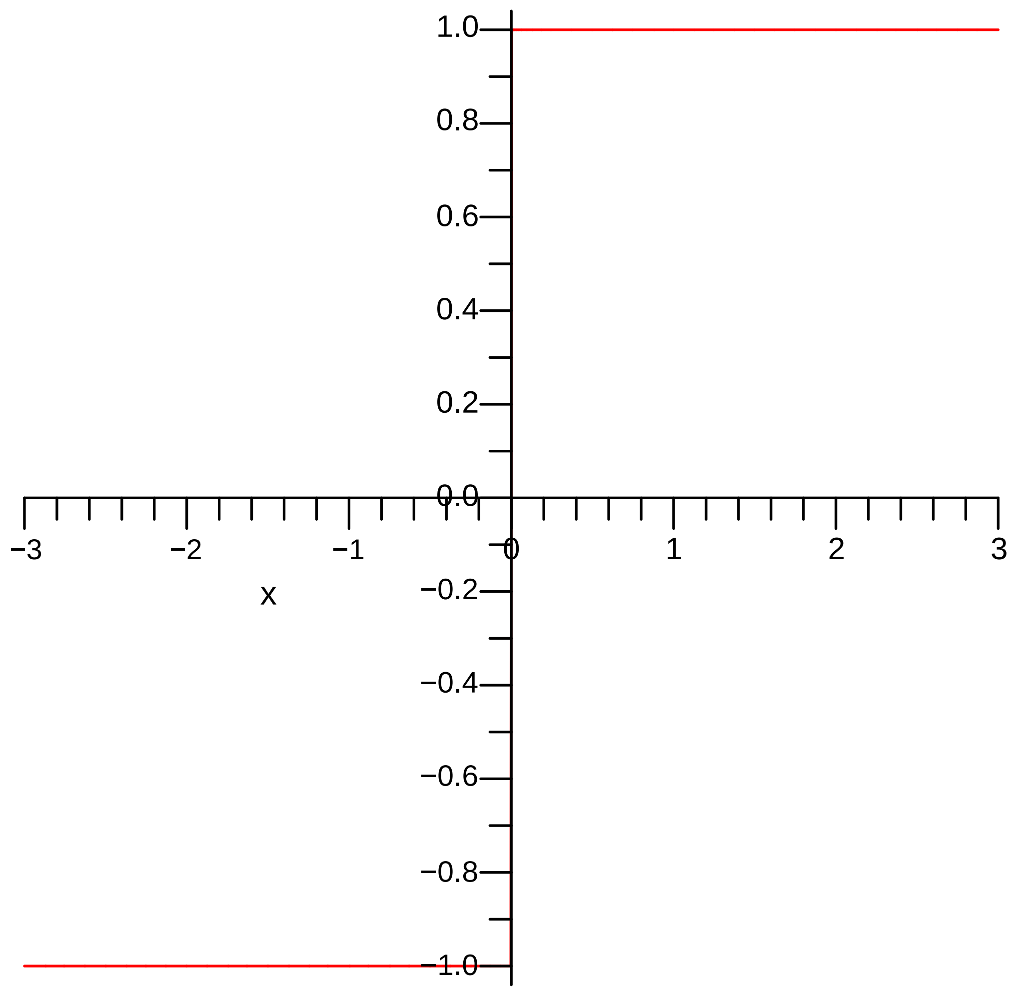
true

true (3.1.8.1)

3.1.9. A Dirichlet-függvény.

3.1.10. Példa.

```
> plot(signum(x),x=-3..3);
```




```
> discont(signum(x),x);
```

$\{0\}$ (3.1.10.1)

3.1.11. Tétel.

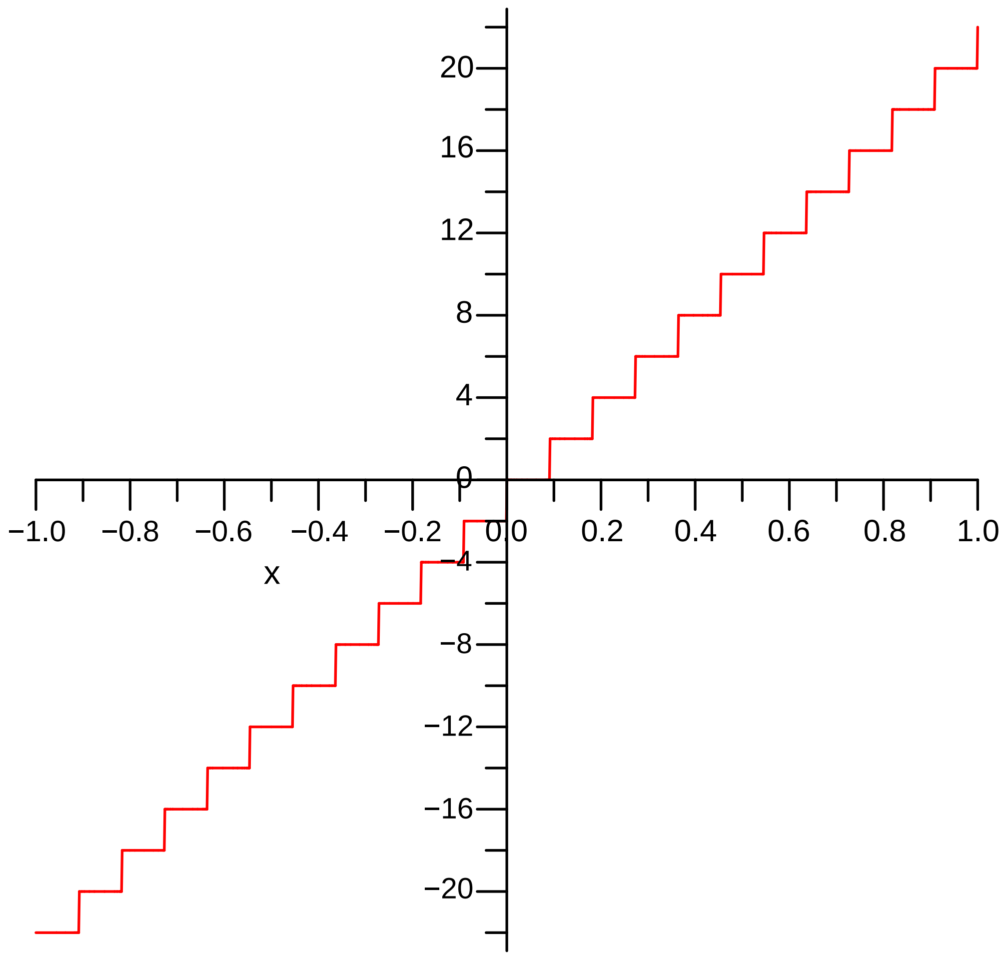
3.1.12. Tétel.

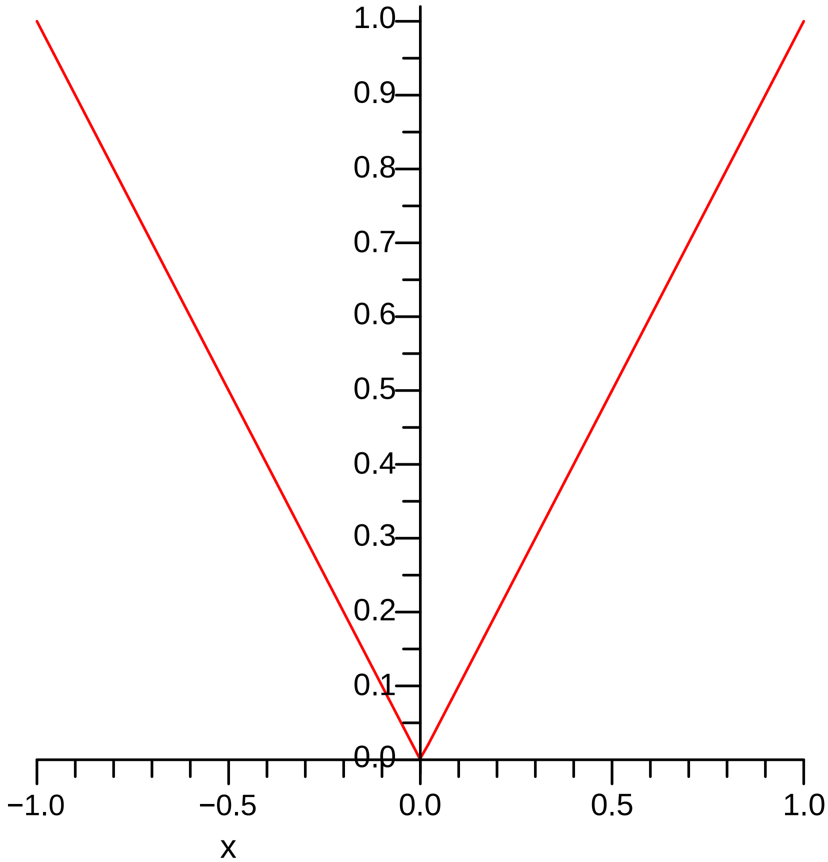
```
> f:=2*floor(11*x); g:=abs(x);
```

```
plot(f,x=-1..1); plot(g,x=-1..1); plot(f*g,x=-1..1);
```

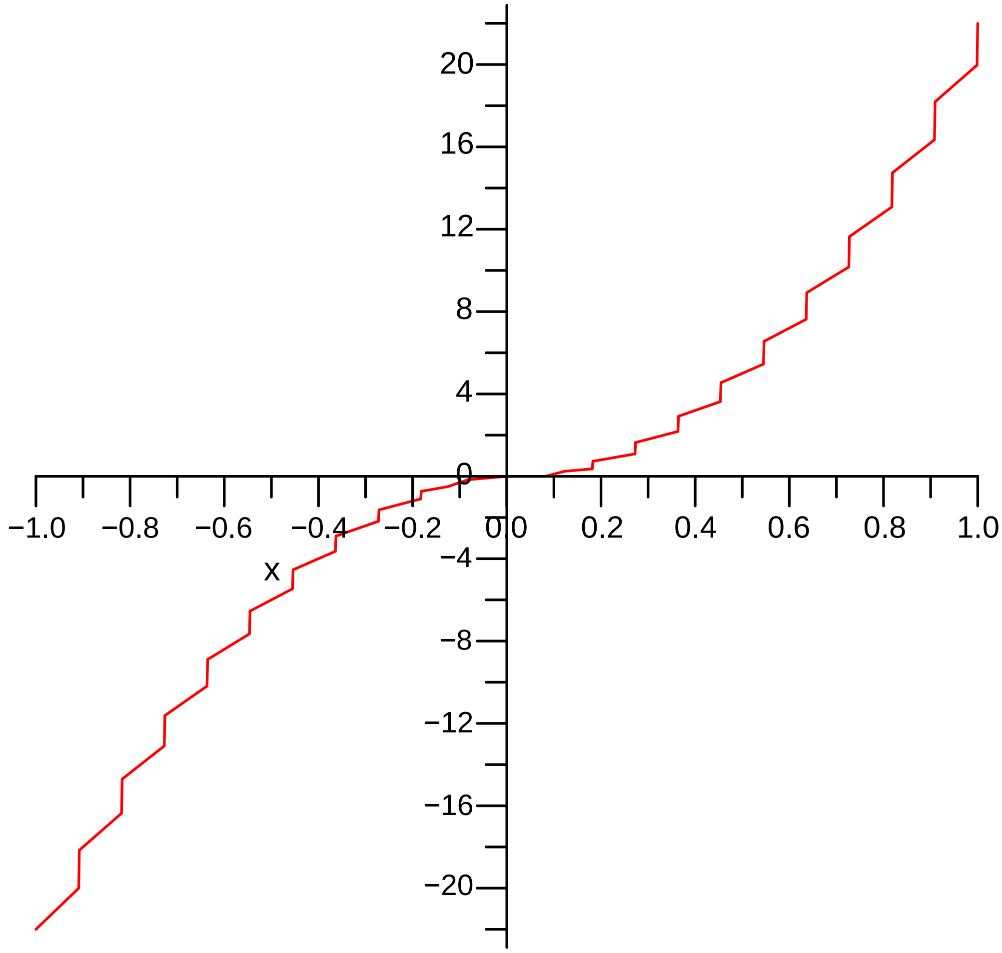
$$f := 2\,\mathrm{floor}(11\,x)$$

$$g := |x|$$





1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
−1.0
−0.5
0.0
0.5
1.0
x

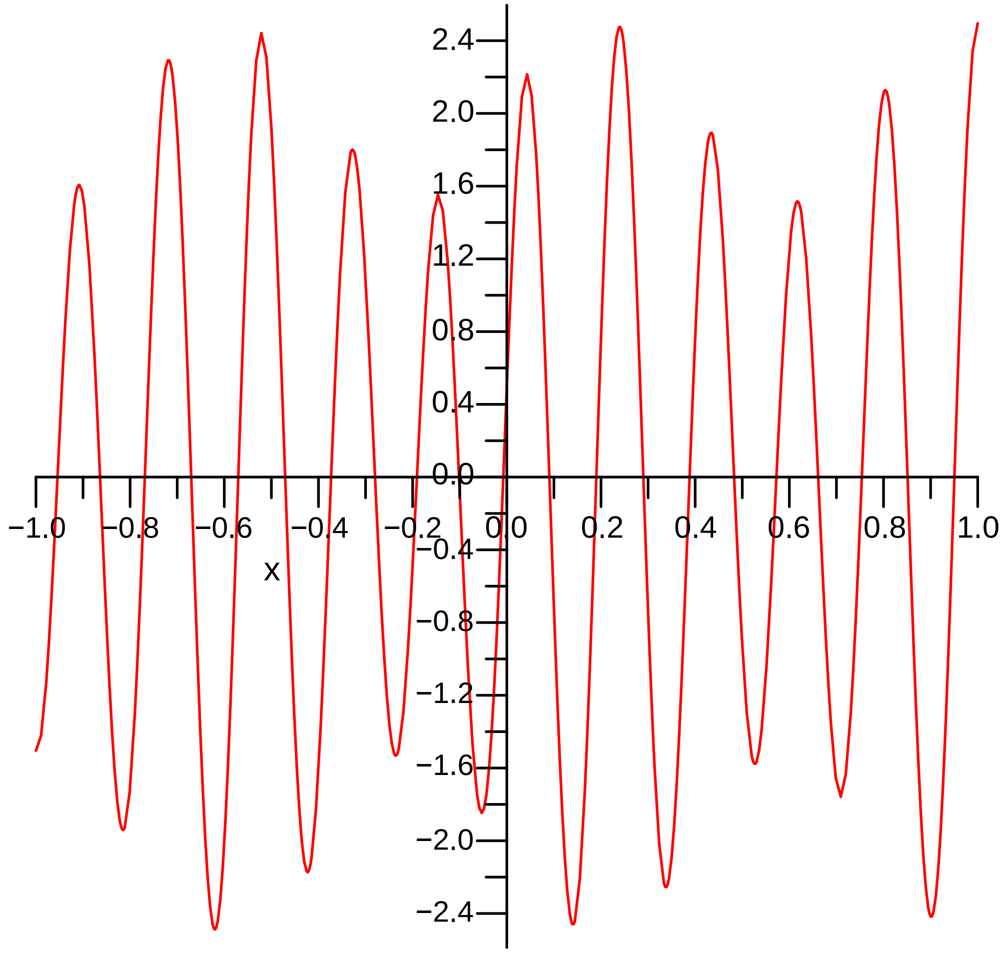


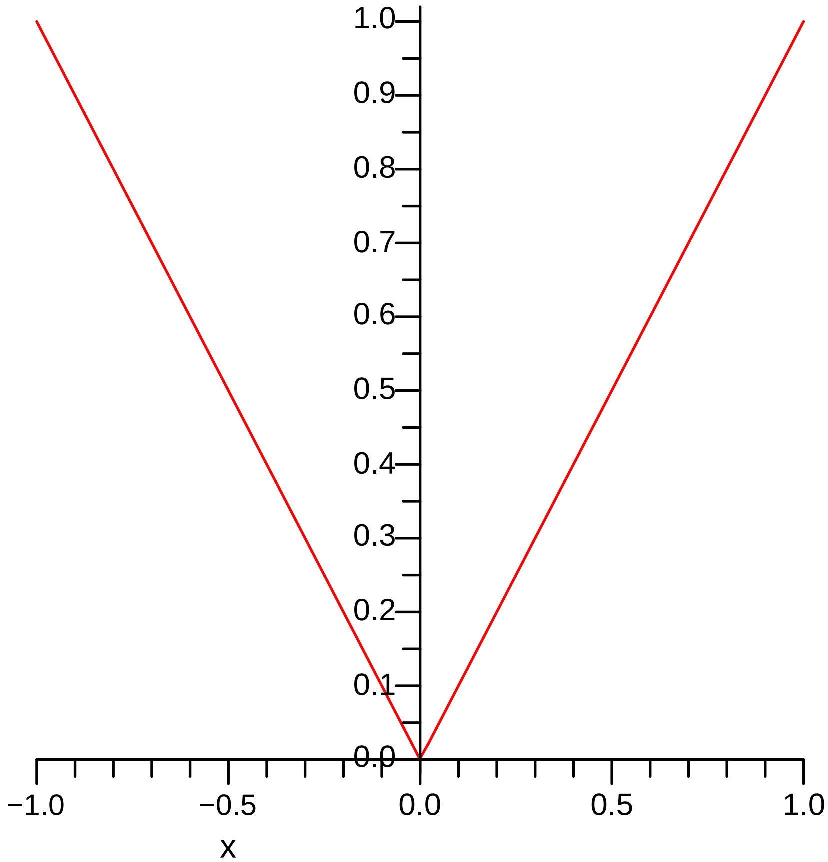
3.1.13. Tétel.

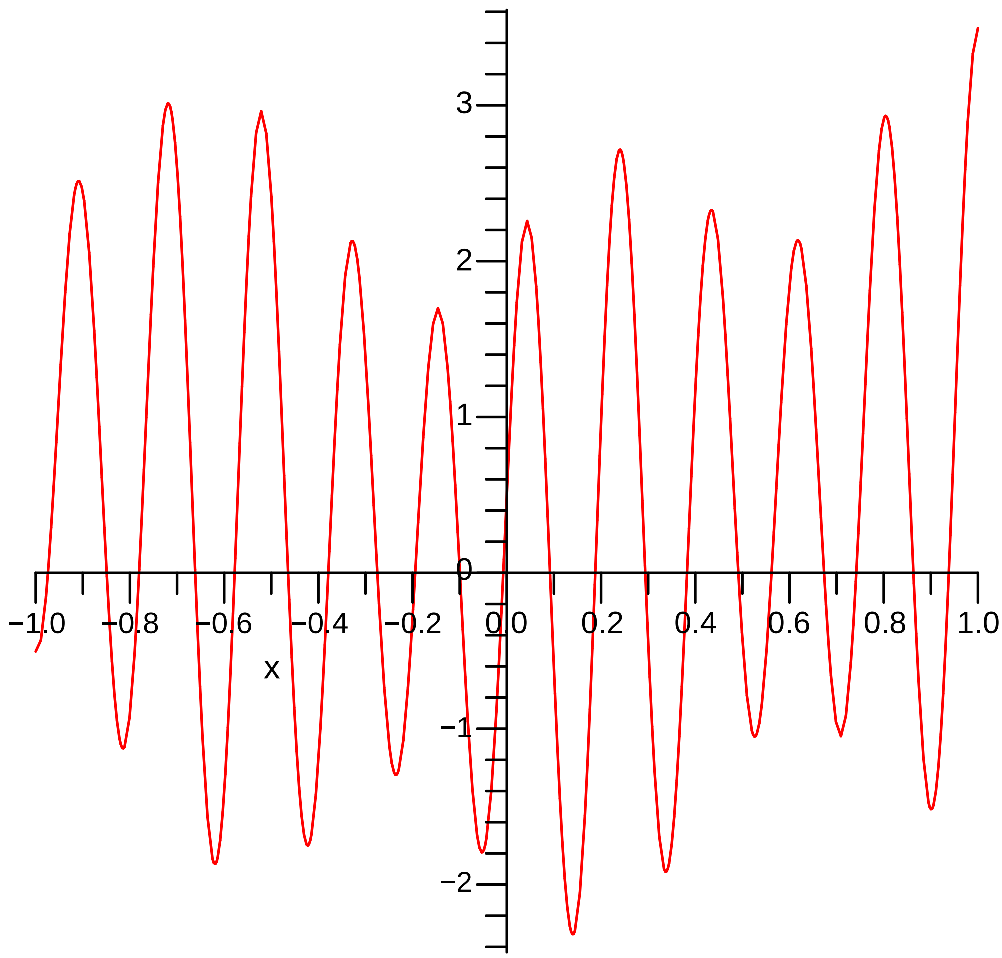
```
> f:=x->2*sin(33*x)+cos(25*x)/2; g:=x->abs(x);
  plot(f(x),x=-1..1); plot(g(x),x=-1..1);
  plot(f(x)+g(x),x=-1..1); plot(f(x)-g(x),x=-1..1);
  plot(f(x)*g(x),x=-1..1); plot(1/g(x),x=-1..1,y=0..10);
```

$$f := x \to 2\sin(33\,x) + \frac{1}{2}\cos(25\,x)$$

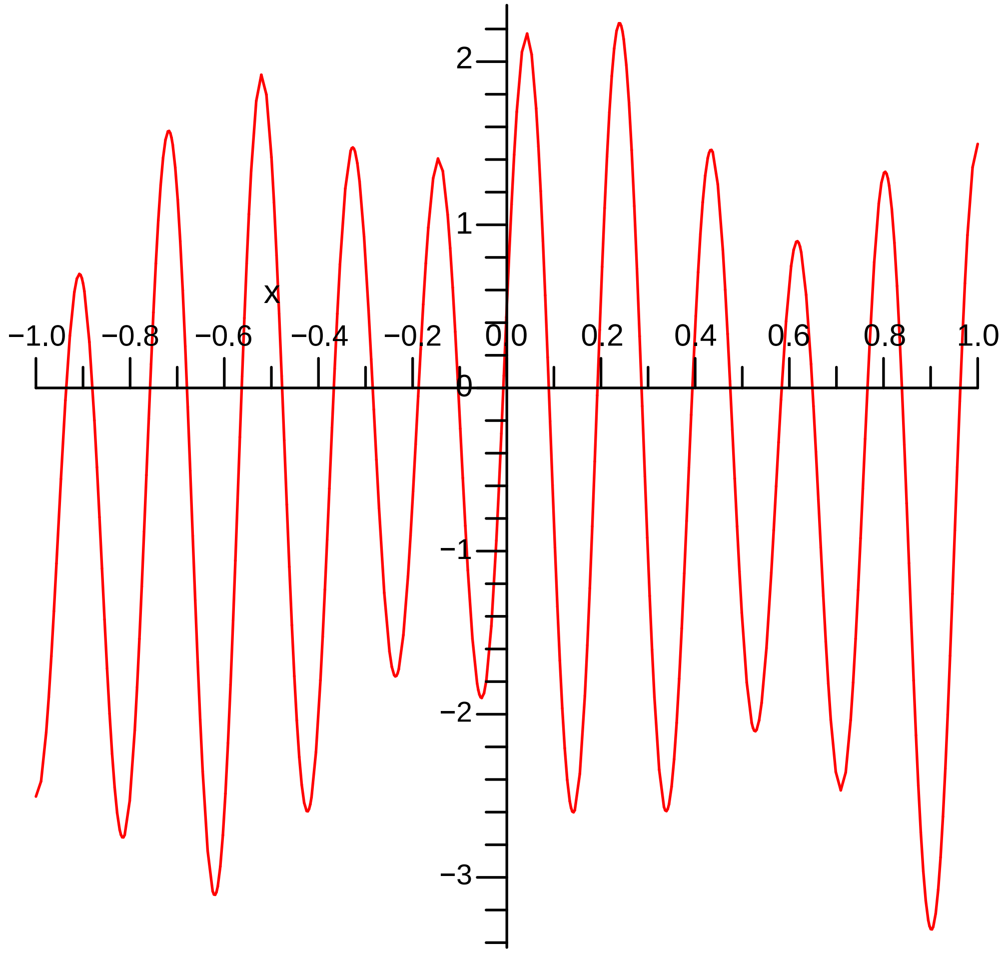
$$g := x \to |x|$$


2.4
2.0
1.6
1.2
0.8
0.4
0.0
−0.4
−0.8
−1.2
−1.6
−2.0
−2.4
−1.0
−0.8
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
x

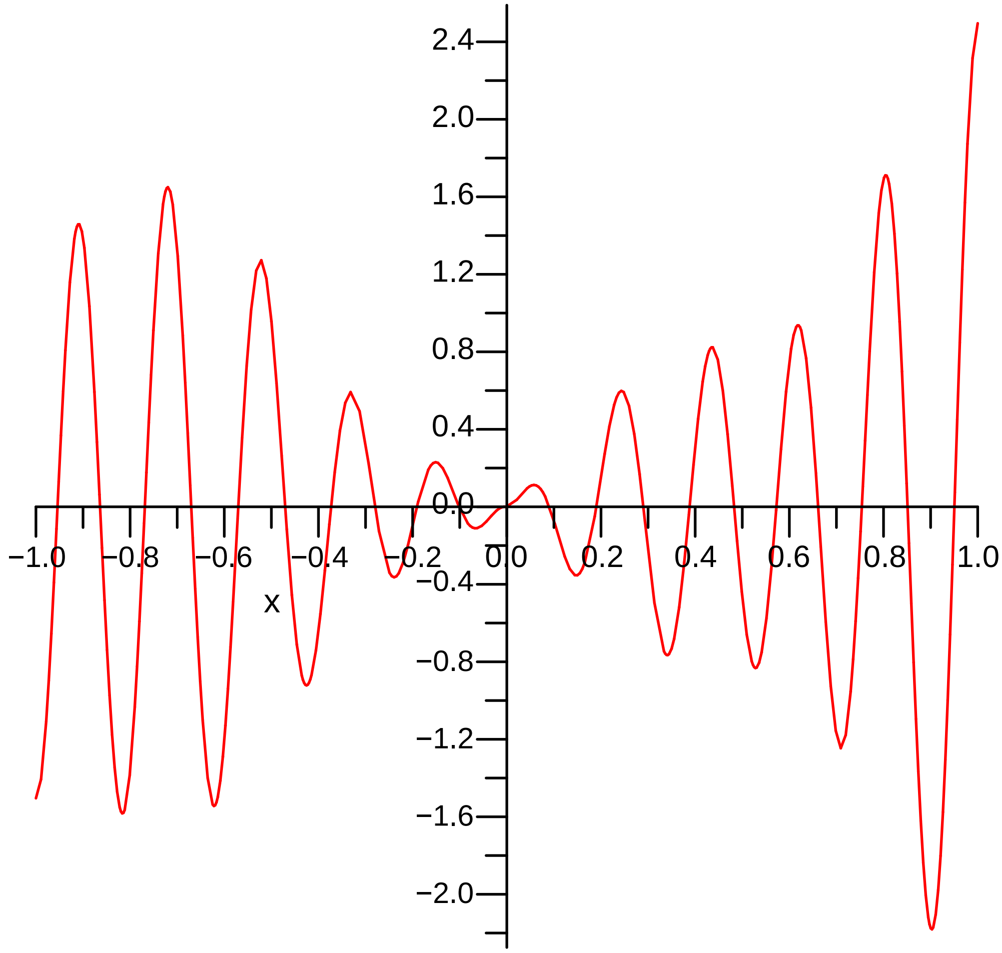

1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
−1.0
−0.5
0.0
0.5
1.0
x



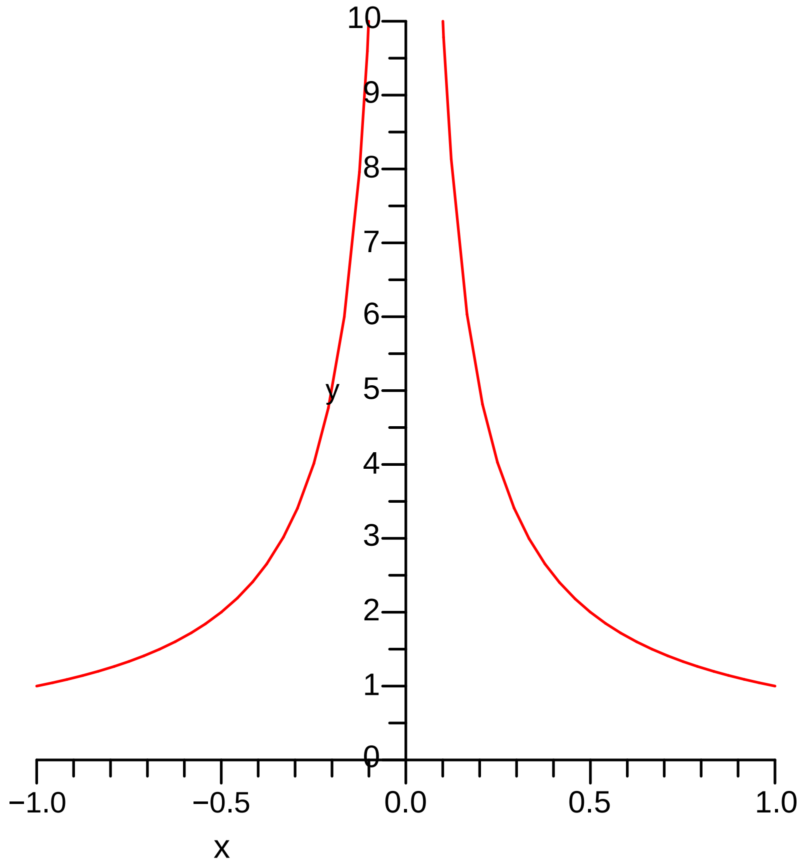
3
2
1
0
−1
−2
−1.0
−0.8
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
x



2
1
x
−1.0
−0.8
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
0
−1
−2
−3



2.4
2.0
1.6
1.2
0.8
0.4
0.0
−0.4
−0.8
−1.2
−1.6
−2.0
−1.0
−0.8
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
x



3.1.14. Következmény.

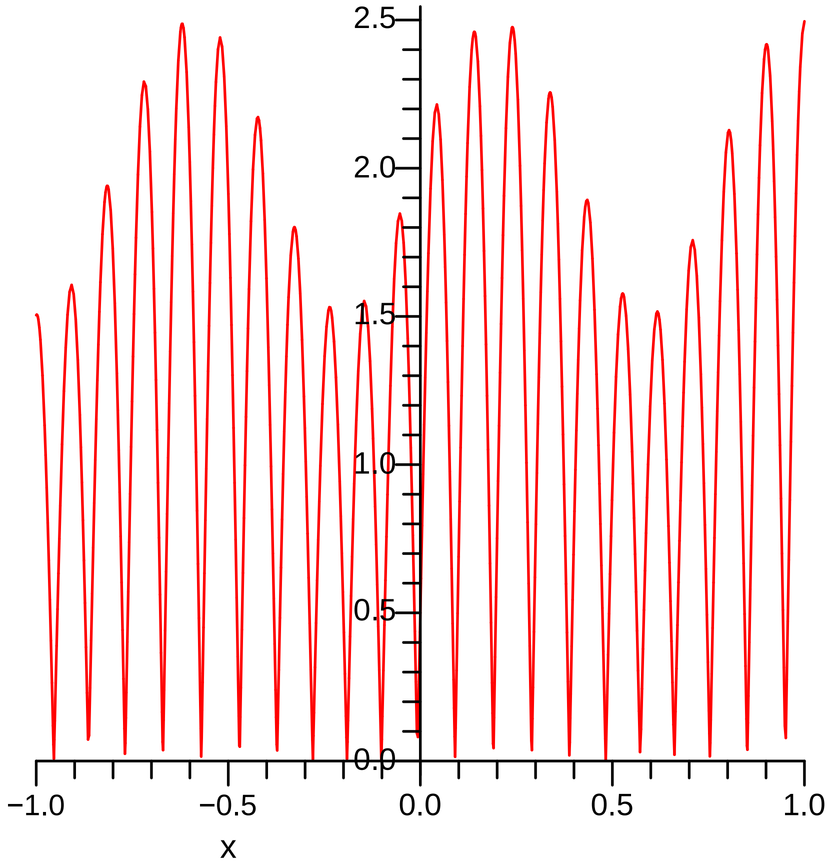
```
> discont(1/(x^2-x),x);
```

$$\{0, 1\} \quad (3.1.14.1)$$

3.1.15. Tétel.

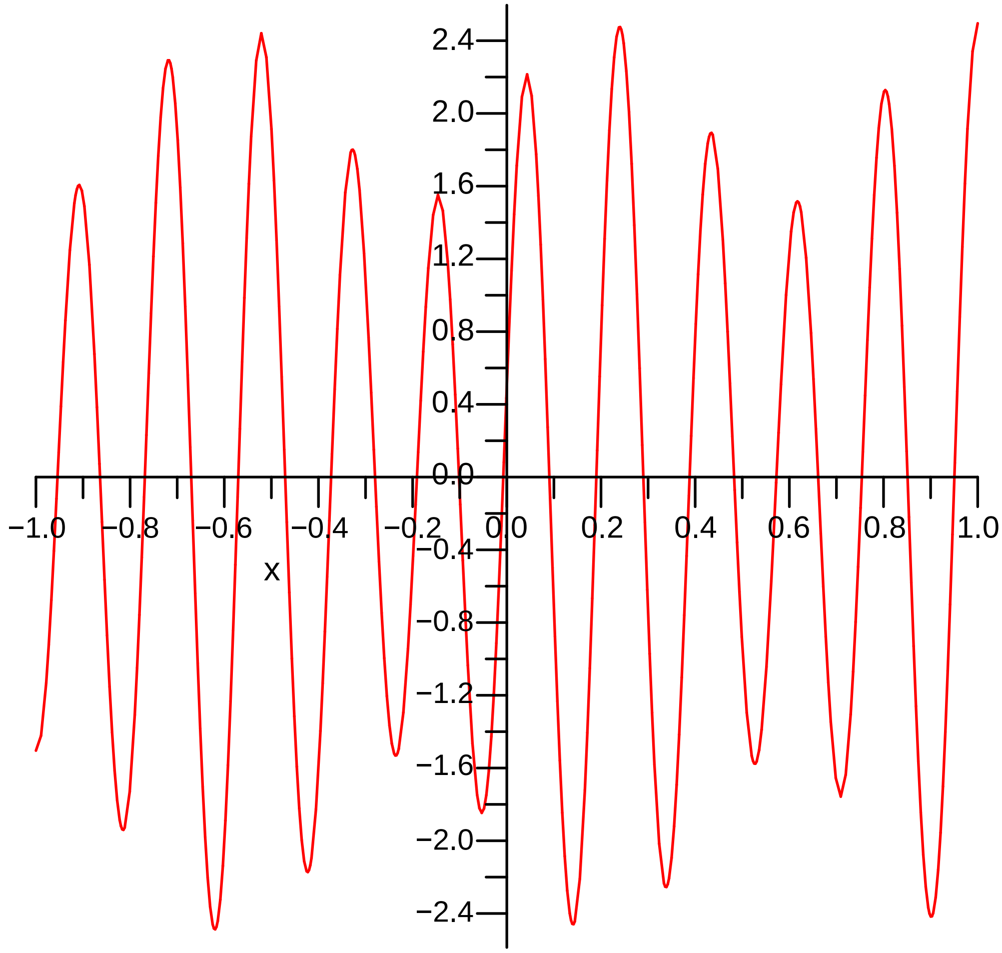
```
> h:=g@f; plot(h(x),x=-1..1);
```

$$h := g@f$$



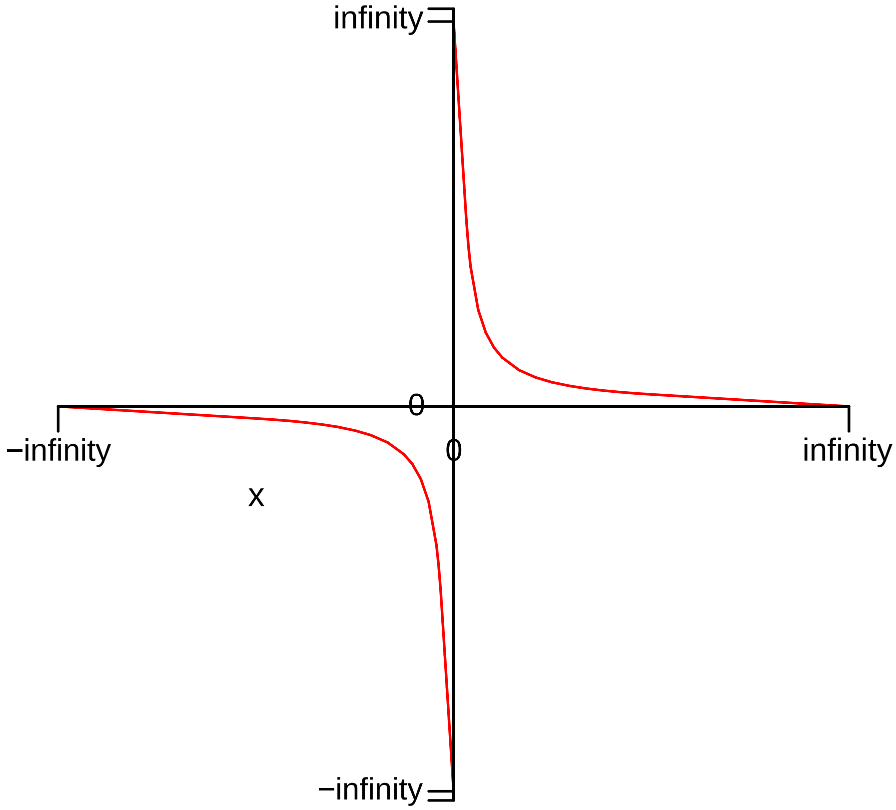
3.1.16. Weierstrass tétele.

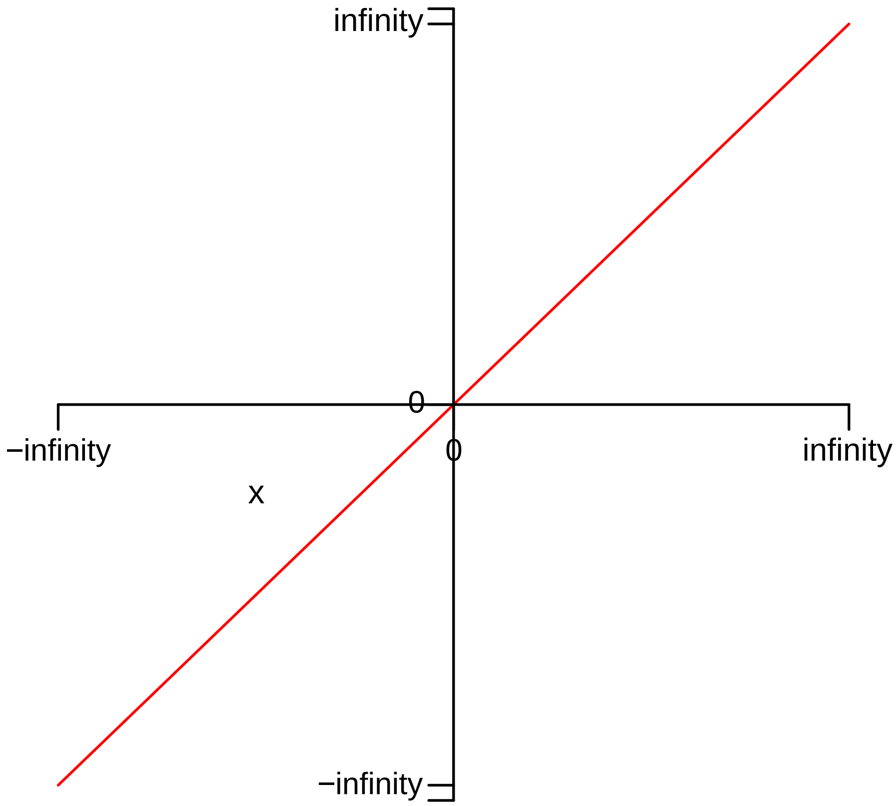
```
> plot(f(x),x=-1..1);
```



3.1.17. Példák.

```
> plot(1/x,x=-infinity..infinity); plot(x,x=-infinity..
  infinity);
```


infinity
0
−infinity
0
infinity
x
−infinity



3.1.18. Bolzano tétele.

```
> Bolzanosolve:=proc(f::procedure,R::range(numeric)) local a,
  b,c;
  a:=op(1,R); b:=op(2,R); if f(a)*f(b)>0 then return FAIL fi;
  do if abs(f(a))<10^(4-Digits) then return a fi;
    if abs(f(b))<10^(4-Digits) then return b fi;
    print(a..b);
    c:=(a+b)/2; if f(a)*f(c)<=0 then b:=c else a:=c fi;
  od; end;

  Bolzanosolve(x->x-cos(x),0.0..1.0);
```

Bolzanosolve := **proc**(*f*::*procedure*, *R*::(*range*(*numeric*)))
local *a*, *b*,
c;
a := *op*(1, *R*);

```
    b := op(2, R);
    if 0 < f(a) * f(b) then
        return FAIL
    end if;
    do
        if abs(f(a)) < 10^(4 - Digits) then
            return a
        end if;
        if abs(f(b)) < 10^(4 - Digits) then
            return b
        end if;
        print(a..b);
        c := 1 / 2 * a + 1 / 2 * b;
        if f(a) * f(c) <= 0 then
            b := c
        else
            a := c
        end if
    end do
end proc
```

0. ..1.0

0.5000000000..1.0

0.5000000000..0.7500000000

0.6250000000..0.7500000000

0.6875000000..0.7500000000

0.7187500000..0.7500000000

0.7343750000..0.7500000000

0.7343750000..0.7421875000

0.7382812500..0.7421875000

0.7382812500..0.7402343750

0.7382812500..0.7392578125

0.7387695312..0.7392578125

0.7390136718..0.7392578125

0.7390136718..0.7391357421

0.7390747069..0.7391357421

0.7390747069..0.7391052244

0.7390747069..0.7390899656

0.7390823362..0.7390899656

$$0.7390823362..0.7390861509$$
$$0.7390842435..0.7390861509$$
$$0.7390851972 \qquad (3.1.18.1)$$

3.1.19. Következmény.

3.1.20. Következmény.

3.1.21. Jobb és bal oldali folytonosság.

3.1.22. Tétel.

3.1.23. Következmény: a gyökvonás folytonossága.

3.1.24. Határérték.

```
> limit(sin(x)/x,x=0); limit((cos(x)-1)/x^2,x=0); limit((exp
(x)-1)/x,x=0);
```

$$1$$
$$-\frac{1}{2}$$
$$1 \qquad (3.1.24.1)$$

```
> limit(sin(x)/x,x=0,complex); limit((cos(x)-1)/x^2,x=0,
complex);
limit((exp(x)-1)/x,x=0,complex);
```

$$1$$
$$-\frac{1}{2}$$
$$1 \qquad (3.1.24.2)$$

3.1.25. Tétel.

3.1.26. Tétel.

3.1.27. Példa.

```
> limit(limit((x-y)/(x+y),y=0),x=0); limit(limit((x-y)/(x+y),
x=0),y=0);
```

$$1$$
$$-1 \qquad (3.1.27.1)$$

3.1.28. Jobb és bal oldali határérték.

```
> limit(floor(x),x=2,right); limit(floor(x),x=2,left);
limit(floor(x),x=2); limit(floor(x),x=2,real);
```

$$2$$
$$1$$

$$undefined$$
$$undefined \qquad (3.1.28.1)$$

3.1.29. Szakadások.

```
> discont(1/(sin(x)-1/2),x);
```

$$\left\{\frac{1}{6}\,\pi+\frac{2}{3}\,\pi\,\_B1{\sim}+2\,\pi\,\_Z6{\sim}\right\} \qquad (3.1.29.1)$$

3.1.30. Példák.

```
> limit(signum(x),x=0,right); limit(signum(x),x=0,left);
  limit(signum(x),x=0); limit(signum(x),x=0,complex);

  limit(abs(signum(x)),x=0); limit(abs(signum(x)),x=0,
  complex);
```

$$1$$
$$-1$$
$$undefined$$
$$undefined$$
$$1$$
$$1 \qquad (3.1.30.1)$$

3.1.31. Példa.

3.1.32. A bővített valós és a bővített komplex számok topológiája.

3.1.33. Példa.

```
> limit(1/x,x=0); limit(1/x,x=0,complex);
  limit(1/x,x=0,right); limit(1/x,x=0,left);

  limit(1/x,x=infinity); limit(1/x,x=-infinity);
  limit(1/x,x=infinity,complex);
```

$$undefined$$
$$\infty-\infty\,\mathrm{I}$$
$$\infty$$
$$-\infty$$
$$0$$
$$0$$
$$0 \qquad (3.1.33.1)$$

3.1.34. *Műveletek bővített valós és komplex számokkal.*

```
> infinity+infinity; infinity-infinity; 1/(infinity+I*
  infinity);
```

$$\infty$$
$$undefined$$
$$0 \qquad (3.1.34.1)$$

3.1.35. *Tétel.*
3.1.36. *Megjegyzés.*
3.1.37. *Tétel.*
3.1.38. *Tétel: rendőr-elv.*
3.1.39. *Tétel.*
3.1.40. *Következmény.*
3.1.41. *Példa.*

3.2. Sorozatok és sorok

```
> restart;
```

3.2.1. *Sorozatok.*

```
> limit(1/n,n=infinity);
```

$$0 \qquad (3.2.1.1)$$

3.2.2. *Tétel.*
3.2.3. *Tétel.*
3.2.4. *Tétel.*
3.2.5. *Tétel.*
3.2.6. *Példák.*

```
> limit(a0+n*d,n=infinity) assuming(d=0);

  limit(a0+n*d,n=infinity) assuming d>0;

  limit(a0+n*d,n=infinity) assuming d<0;
```

$$0$$
$$\infty$$
$$-\infty \qquad (3.2.1)$$

```
> limit(n^2,n=infinity);
```

$$\infty \qquad (3.2.2)$$

```
> limit(1/n,n=infinity);
```

$$0 \qquad (3.2.3)$$

```
> limit(1+(-1)^n/n,n=infinity);
```

$$1 \qquad (3.2.4)$$

```
> limit(1+I^n,n=infinity);
```

$$-\mathrm{I}..2+\mathrm{I} \qquad (3.2.5)$$

3.2.7 Felső és alsó határérték.

```
> limit((-1)^n-1/n+1/n^2,n=infinity);
```

$$-1..1 \qquad (3.2.7.1)$$

3.2.8. Tétel.

3.2.9. Állítás.

3.2.10. Cauchy-sorozatok.

3.2.11. Cauchy-féle konvergencia-kritérium.

** 3.2.12. Megjegyzés.*

```
> i:='i': x:=0:
  for i from 0 do while (x+1)^2<2*10^(2*i) do x:=x+1: od; x;
  x:=x*10: od;
```

$$1$$
$$x := 10$$
$$14$$
$$x := 140$$
$$141$$
$$x := 1410$$
$$1414$$
$$x := 14140$$
$$14142$$
$$x := 141420$$
$$141421$$
$$x := 1414210$$
$$1414213$$
$$x := 14142130$$
$$14142135$$
$$x := 141421350$$
$$141421356$$

$x := 1414213560$

1414213562

$x := 14142135620$

14142135623

$x := 141421356230$

141421356237

$x := 1414213562370$

1414213562373

$x := 14142135623730$

14142135623730

$x := 141421356237300$

141421356237309

$x := 1414213562373090$

1414213562373095

$x := 14142135623730950$

14142135623730950

$x := 141421356237309500$

141421356237309504

$x := 1414213562373095040$

1414213562373095048

$x := 14142135623730950480$

14142135623730950488

$x := 141421356237309504880$

141421356237309504880

$x := 1414213562373095048800$

1414213562373095048801

$x := 14142135623730950488010$

14142135623730950488016

$x := 141421356237309504880160$

141421356237309504880168

$x := 1414213562373095048801680$

$1414213562373095048801688$

$x := 14142135623730950488016880$

$14142135623730950488016887$

$x := 141421356237309504880168870$

$141421356237309504880168872$

$x := 1414213562373095048801688720$

$1414213562373095048801688724$

$x := 14142135623730950488016887240$

$14142135623730950488016887242$

$x := 141421356237309504880168872420$

$141421356237309504880168872420$

$x := 1414213562373095048801688724200$

$1414213562373095048801688724209$

$x := 14142135623730950488016887242090$

$14142135623730950488016887242096$

$x := 141421356237309504880168872420960$

$141421356237309504880168872420969$

$x := 1414213562373095048801688724209690$

$1414213562373095048801688724209698$

$x := 14142135623730950488016887242096980$

$14142135623730950488016887242096980$

$x := 141421356237309504880168872420969800$

$141421356237309504880168872420969807$

$x := 1414213562373095048801688724209698070$

$1414213562373095048801688724209698078$

$x := 14142135623730950488016887242096980780$

$14142135623730950488016887242096980785$

$x := 141421356237309504880168872420969807850$

$141421356237309504880168872420969807856$

$x := 1414213562373095048801688724209698078560$

$1414213562373095048801688724209698078569$

$x := 14142135623730950488016887242096980785690$

$14142135623730950488016887242096980785696$

$x := 141421356237309504880168872420969807856960$

$141421356237309504880168872420969807856967$

$x := 1414213562373095048801688724209698078569670$

$1414213562373095048801688724209698078569671$

$x := 14142135623730950488016887242096980785696710$

$14142135623730950488016887242096980785696718$

$x := 141421356237309504880168872420969807856967180$

$141421356237309504880168872420969807856967187$

$x := 1414213562373095048801688724209698078569671870$

$1414213562373095048801688724209698078569671875$

$x := 14142135623730950488016887242096980785696718750$

$14142135623730950488016887242096980785696718753$

$x := 14142135623730950488016887242096980785696718753\,0$

$14142135623730950488016887242096980785696718753\,7$

$x := 141421356237309504880168872420969807856967187537\,0$

$141421356237309504880168872420969807856967187537\,6$

$x := 1414213562373095048801688724209698078569671875376\,0$

$1414213562373095048801688724209698078569671875376\,9$

```
Warning,  computation interrupted
```

3.2.13. Nevezetes sorozatok határértéke.

```
> limit(1/n^(1/p),n=infinity) assuming p>0;
```

$$0 \tag{3.2.13.1}$$

```
> limit(a^(1/n),n=infinity) assuming a>0;
```

$$1 \tag{3.2.13.2}$$

```
> limit(n^(1/n),n=infinity);
```

$$1 \tag{3.2.13.3}$$

```
> assume(a>0); assume(k>=0); limit(n^k/(1+a)^n,n=infinity);
```

$$0 \tag{3.2.13.4}$$

```
> limit(a^n,n=infinity) assuming abs(a)<1;
```

$$\lim_{n\to\infty} a\sim^n \tag{3.2.13.5}$$

```
> limit(a^n/n!,n=infinity);
```

$$0 \tag{3.2.13.6}$$

3.2.14. Sorok.

```
> sum(1/k^2,k=1..infinity);
```

$$\frac{1}{6}\pi^2 \tag{3.2.14.1}$$

3.2.15. Cauchy-féle konvergenciakritérium.

3.2.16. Következmény.

3.2.17. Állítás.

3.2.18. Összehasonlító kritérium.

3.2.19. Következmény.

3.2.20. Néhány nevezetes sor.

```
> sum(a^n,n=0..infinity) assuming abs(a)<1;
  sum(a^n,n=0..infinity) assuming a>=1;
```

$$-\frac{1}{a\sim -1}$$

$$\infty \qquad (3.2.20.1)$$

```
> sum(1/n,n=1..infinity);
```

$$\infty \qquad (3.2.20.2)$$

```
> sum((-1)^(k-1)/k,k=1..infinity);
```

$$\ln(2) \qquad (3.2.20.3)$$

▶ *3.2.21. * Cauchy-féle gyökkritérium.*
▶ *3.2.22. * D'Alambert-féle hányadoskritérium.*
▶ *3.2.23. Tétel.*
▶ *3.2.24. Sorok szorzata.*
▶ *3.2.25. * Kettős sor tétel*
▶ *3.2.26. * Következmény: sorok átrendezése.*
▶ *3.2.27. * Következmény.*
▶ *3.2.28. Tétel.*
▼ *3.2.29. Számrendszerek.*

```
> convert(3.2,binary);
```

$$11.00110011 \qquad (3.2.29.1)$$

▶ *3.2.30. * Következmény.*
▶ *3.2.31. Következmény.*

▼ 3.3. Hatványsorok.

```
> restart;
```

▶ *3.3.1. Hatványsorok.*
▶ *3.3.2. Cauchy-Hadamard-tétel.*
▶ *3.3.3. Konvergenciasugár, konvergenciatartomány.*
▼ *3.3.4. Példák.*

```
> sum(x^n,n=0..infinity); sum(x^n/n,n=1..infinity);
  sum(x^n/n^2,n=1..infinity);

  series(%%%,x=0); series(%%%,x=0); series(%%%,x=0);
```

$$-\frac{1}{x-1}$$

$$-\ln(1-x)$$

$$\mathrm{polylog}(2, x)$$

$$1+x+x^2+x^3+x^4+x^5+\mathrm{O}(x^6)$$

$$x+\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{1}{4}x^4+\frac{1}{5}x^5+\mathrm{O}(x^6)$$

$$x+\frac{1}{4}x^2+\frac{1}{9}x^3+\frac{1}{16}x^4+\frac{1}{25}x^5+\mathrm{O}(x^6) \tag{3.3.4.1}$$

3.3.5. Hatványsorok átrendezése.

```
> series(1/(1-x),x=1/2);
```

$$2+4\left(x-\frac{1}{2}\right)+8\left(x-\frac{1}{2}\right)^2+16\left(x-\frac{1}{2}\right)^3+32\left(x-\frac{1}{2}\right)^4+64\left(x-\frac{1}{2}\right)^5+\mathrm{O}\left(\left(x-\frac{1}{2}\right)^6\right) \tag{3.3.5.1}$$

3.3.6. Analitikus függvények.

3.3.7. Motiváció.

3.3.8. Az exponenciális függvény.

```
> sum(z^n/n!,n=0..infinity); exp(z); exp(z1+z2); expand(%);
```

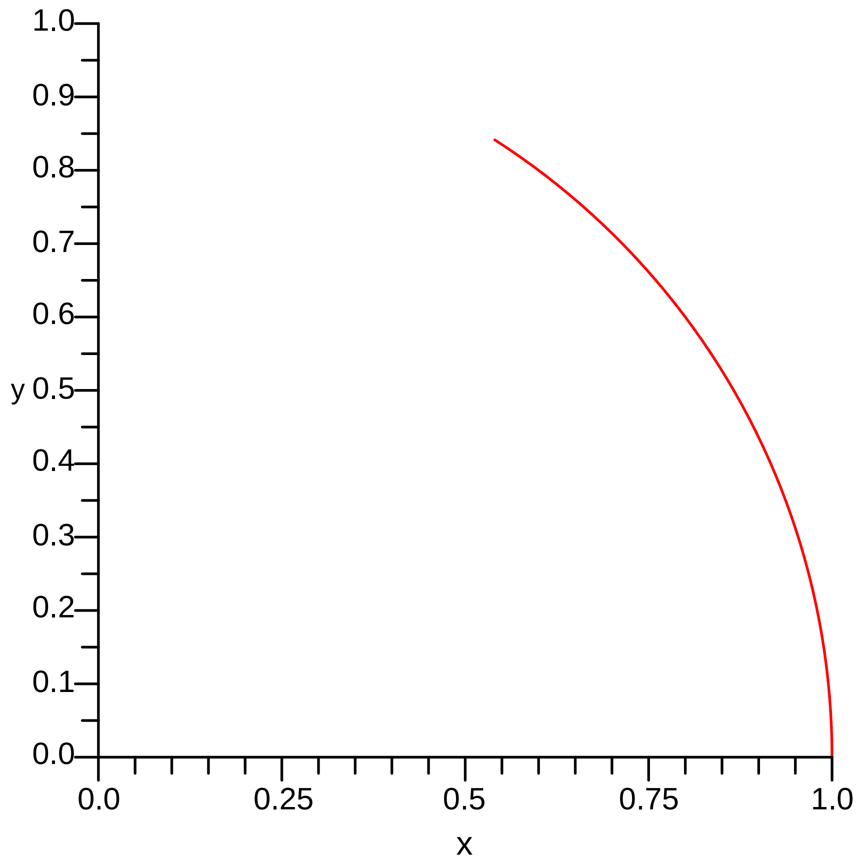
$$\mathrm{e}^z$$

$$\mathrm{e}^z$$

$$\mathrm{e}^{z1+z2}$$

$$\mathrm{e}^{z1}\,\mathrm{e}^{z2} \tag{3.3.8.1}$$

```
> plots[complexplot](exp(I*t),t=0..1,x=0..1,y=0..1);
```



3.3.9. *Megjegyzés.*

3.3.10. *Motiváció.*

3.3.11. *Trigonometrikus és hiperbolikus függvények.*

```
> (exp(I*z)+exp(-I*z))/2; convert(%,trig);
  (exp(I*z)-exp(-I*z))/(2*I); convert(%,trig);
```

$$\frac{1}{2}\,e^{Iz}+\frac{1}{2}\,e^{-Iz}$$

$$\cos(z)$$

$$-\frac{1}{2}\,I\left(e^{Iz}-e^{-Iz}\right)$$

$$\sin(z) \qquad (3.3.11.1)$$

```
> sum((-1)^n*z^(2*n)/(2*n)!,n=0..infinity);
  sum((-1)^n*z^(2*n+1)/(2*n+1)!,n=0..infinity);
```

$$\cos(z)$$

(3.3.11.2)

$$\sin(z) \tag{3.3.11.2}$$

```
> exp(I*t); convert(%,trig);
```

$$e^{It}$$

$$\cos(t) + I\sin(t) \tag{3.3.11.3}$$

```
> cos(z)^2+sin(z)^2; simplify(%);
```

$$\cos(z)^2 + \sin(z)^2$$

$$1 \tag{3.3.11.4}$$

```
> cos(z1+z2); expand(%); sin(z1+z2); expand(%);
```

$$\cos(z1 + z2)$$

$$\cos(z1)\cos(z2) - \sin(z1)\sin(z2)$$

$$\sin(z1 + z2)$$

$$\sin(z1)\cos(z2) + \cos(z1)\sin(z2) \tag{3.3.11.5}$$

```
> (exp(z)+exp(-z))/2; convert(%,trigh);
  (exp(z)-exp(-z))/2; convert(%,trigh);
```

$$\frac{1}{2}e^{z} + \frac{1}{2}e^{-z}$$

$$\cosh(z)$$

$$\frac{1}{2}e^{z} - \frac{1}{2}e^{-z}$$

$$\sinh(z) \tag{3.3.11.6}$$

```
> sum(z^(2*n)/(2*n)!,n=0..infinity);
  sum(z^(2*n+1)/(2*n+1)!,n=0..infinity);
```

$$\cosh(z)$$

$$\sinh(z) \tag{3.3.11.7}$$

```
> cosh(z)^2-sinh(z)^2; simplify(%);
```

$$\cosh(z)^2 - \sinh(z)^2$$

$$1 \tag{3.3.11.8}$$

```
> cosh(z1+z2); expand(%); sinh(z1+z2); expand(%);
```

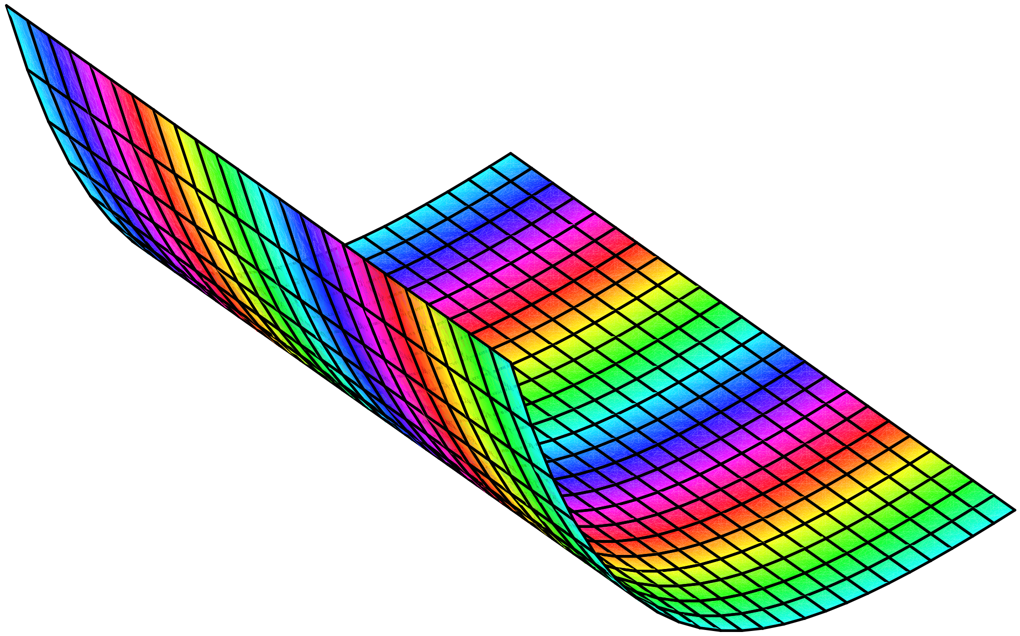
$$\cosh(z1 + z2)$$

$$\cosh(z1)\cosh(z2) + \sinh(z1)\sinh(z2)$$

$$\sinh(z1 + z2)$$

$$\sinh(z1)\cosh(z2) + \cosh(z1)\sinh(z2) \tag{3.3.11.9}$$

```
> plots[complexplot3d](exp(z),z=-2-2*Pi*I..2+2*Pi*I);
```

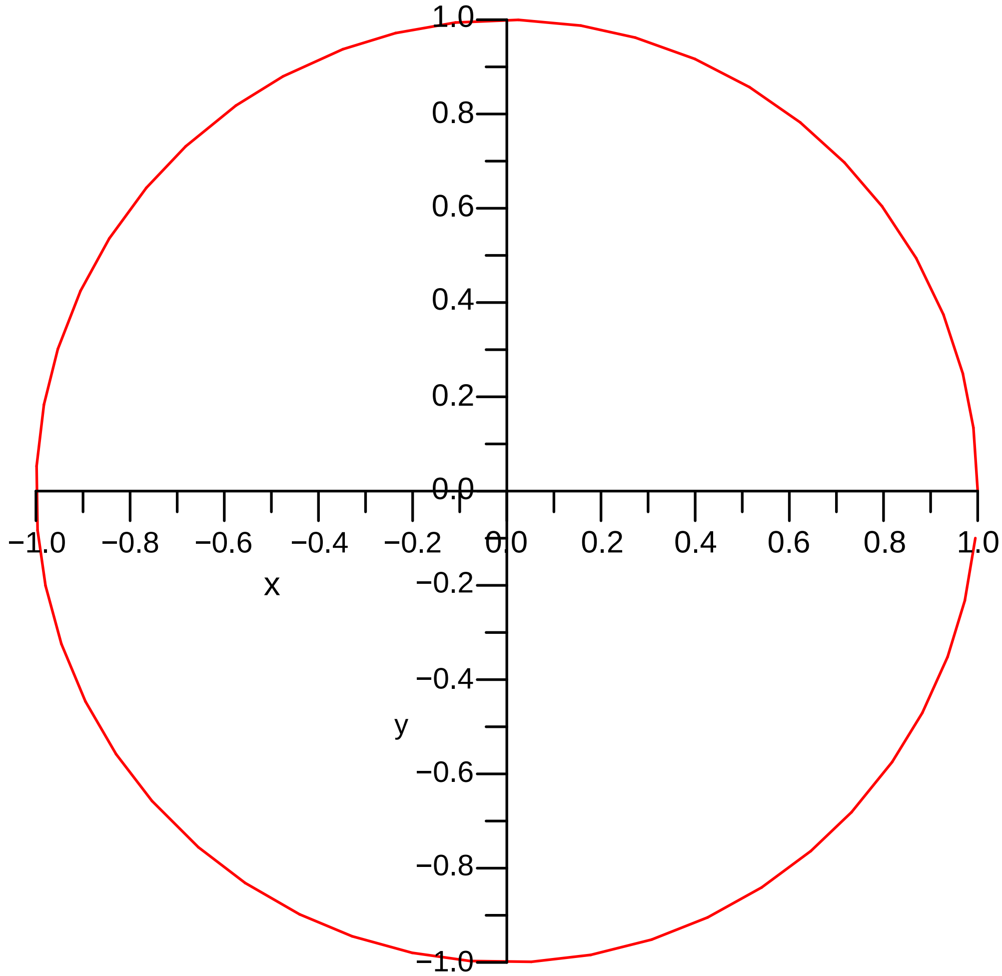


▶ *3.3.12. Tétel.*

▶ *3.3.13. Tétel.*

▼ *3.3.14. A π sz□m.*

```
> plots[complexplot](exp(I*t),t=0..2*Pi-0.1,x=-1..1,y=-1..1);
```



3.3.15. Tétel.

```
> exp(Pi*I/2); exp(Pi*I); cos(z+2*Pi); sin(z+2*Pi);
```

$$I$$
$$-1$$
$$\cos(z)$$
$$\sin(z) \quad (3.3.15.1)$$

```
> exp(z+2*Pi*I); expand(%);
```

$$e^{z+2I\pi}$$
$$e^{z} \quad (3.3.15.2)$$

```
> cosh(z+2*Pi*I); sinh(z+2*Pi*I);
```

$$\cosh(z)$$
$$\sinh(z) \quad (3.3.15.3)$$

3.3.16. Az e szám.

```
> exp(1); evalf(%);
```

$$e$$

$$2.718281828 \qquad (3.3.16.1)$$

3.3.17. Természetes logaritmus.

```
> ln(exp(x)) assuming real; exp(ln(x)) assuming x>0;
```

$$x$$

$$x \qquad (3.3.17.1)$$

3.3.18. Tétel.

```
> ln(1); ln(x)+ln(y); combine(%) assuming positive;
```

$$0$$

$$\ln(x)+\ln(y)$$

$$\ln(x\,y) \qquad (3.3.18.1)$$

3.3.19. Hatványozás.

```
> a^x; convert(%,exp);
```

$$a^x$$

$$e^{\ln(a)\,x} \qquad (3.3.19.1)$$

3.3.20. Tétel.

```
> a^(x+y); expand(%); a^(-x); expand(%); a^0; a^1; 1^x;
```

$$a^{x+y}$$

$$a^x\,a^y$$

$$a^{-x}$$

$$\frac{1}{a^x}$$

$$1$$

$$a$$

$$1 \qquad (3.3.20.1)$$

3.3.21. Logaritmus.

3.3.22. Tétel.

```
> log[a](a); log[a](1); log[exp(1)](x);
  log[a](x)+log[a](y); combine(%) assuming positive;
```

$$1$$

$$0$$

$$\ln(x)$$

$$\frac{\ln(x)}{\ln(a)} + \frac{\ln(y)}{\ln(a)}$$

$$\frac{\ln(x\,y)}{\ln(a)} \qquad (3.3.22.1)$$

3.3.23. Komplex logaritmus és hatványozás.

```
> ln(x+I*y); evalc(%);
```

$$\ln(x + \mathrm{I}\,y)$$

$$\frac{1}{2}\ln\left(x^2 + y^2\right) + \mathrm{I}\arctan(y, x) \qquad (3.3.23.1)$$

4. Differenciálszámítás

5. Integrálszámítás